Vortex State in a Strongly Coupled Dilute Atomic Fermionic Superfluid

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We show that in a dilute fermionic superfluid, when the fermions interact with an infinite scattering length, a vortex state is characterized by a strong density depletion along the vortex core. This feature can make a direct visualization of vortices in fermionic superfluids possible.

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The existence of stable vortex states is one of the most spectacular manifestations of superfluidity in both Bose and Fermi systems. In Bose dilute atomic gases isolated quantized vortices [1] and subsequently large arrays of vortices [2] have been observed already. The quest for the observation of superfluidity in dilute Fermi gases started as soon as the first experimental evidence of an atomic dilute Fermi degenerate gas was published [3]. In Bose systems methods for establishing unambiguously both the superfluidity and the quantization of vortices exists [4], while most of the suggestions made so far for Fermi systems are rather indirect. The presence of vortices in Bose-Einstein condensation (BEC) was confirmed by analyzing the density variations of an expanding BEC cloud [2,4]. The BEC creation was confirmed by studying the character of the expansion of the atomic cloud after the trap was removed [5]. A recent experimental result in Fermi systems [6] suggests that a hydrodynamic expansion of a Fermi superfluid is a plausible scenario [7].

In Fermi systems significant density variations due to the presence of vortices are not expected [8]. We have reasons to expect, however, that under certain conditions the density variations induced by the presence of one or more vortices could be akin to those in the case of BEC, as a similar feature was recently established in the analysis of the spatial structure of a vortex in low density nuclear matter [9]. At densities significantly smaller than nuclear saturation densities, the superfluid gap in homogeneous neutron matter can attain values rather large by normal standards, $\Delta \approx 0.25\varepsilon_F$, where Δ is the value of the gap and ε_F is the Fermi energy. Even though the gap is still smaller than the Fermi energy, such values proved sufficient in the case of low density neutron matter to lead to major density depletions in the vortex core. As Pitaevskii and Stringari note [6], the observation of quantized vortices in a dilute Fermi gas would provide the ultimate proof that the system has undergone a transition to a superfluid state. *A vortex is just about the only phenomenon in which a true stable superflow is created in a neutral system.* Other phenomena would only somewhat indirectly be affected by the onset of superfluidity.

Recently, a new *ab initio* calculation of the properties of low density nuclear matter became available [10]. A particular result of this analysis, which is valid in principle for any fermion system, concerns the properties of a two fermion species interacting with an infinite scattering length. These authors have shown that the energy per particle of such a normal Fermi system is

$$\mathcal{E}_N = \alpha_N \frac{3}{5}\varepsilon_F, \tag{1}$$

with $\alpha_N \approx 0.54$. This result was obtained for a system of fermions interacting with a short range attractive potential with a zero-energy bound state. As long as $k_F r_0 \ll 1$, where r_0 is of the order of the radius of the potential, one expects on general grounds that the energy per particle of such a system is proportional to the energy per particle of a noninteracting system. The scattering length a and the effective range r_0 parametrize the low energy behavior of the s-wave scattering phase of two particles $k\cot\delta(k) \approx -1/a + r_0k^2/2$, where k is the wave vector of the relative motion. A second result of this *ab initio* calculation concerns the energy per particle of the superfluid phase of such a system [11], namely, that

$$\mathcal{E}_S = \alpha_S \frac{3}{5}\varepsilon_F, \tag{2}$$

with $\alpha_S \approx 0.44$. A simple estimate of the corresponding value of the superfluid gap, using for the condensation energy the weak coupling BCS value $-3\Delta^2/8\varepsilon_F$ leads to $\Delta \approx 0.4\varepsilon_F$. Such large values for the gap Δ are not compatible with the BCS weak coupling limit, when $k_F|a| \ll 1$ and $a < 0$. It is well-known that in this limit the BCS approximation [12] leads to too large a value of the gap and that polarization corrections lead to a reduction of the gap [13], namely, to

$$\Delta = \left(\frac{2}{e}\right)^{7/3}\varepsilon_F \exp\left(\frac{\pi}{2k_F a}\right).$$

A recent analysis [8] of the vortex state in a dilute superfluid Fermi gas, using the simple BCS value for the gap (which exceeds by a factor of ≈ 2.2 the true gap value) shows a relatively modest density depletion in the vortex core of about 10% at most.

The possibility that the value of the superfluid gap can attain large values was raised more than two decades ago in connection with the BCS $\rightarrow$ BEC crossover [14,15]. One can imagine that one can increase the strength of

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the two-particle interaction in such a manner that at some point a real two-bound state forms, and in that case $a \to -\infty$. By continuing to increase the strength of the two-particle interaction, the scattering length becomes positive and starts decreasing. When $a \gg r_0$ and $a > 0$ the energy of the two-particle bound state is $\epsilon_2 \approx -\hbar^2/ma^2$. A dilute system of fermions, when $nr_0^3 \ll 1$, will thus undergo a transition from a weakly coupled BCS system, when $a < 0$ and $a = \mathcal{O}(r_0)$, to a BEC system of tightly bound fermion pairs, when $a > 0$ and $a = \mathcal{O}(r_0)$ again. In the weakly coupled BCS limit the size of the Cooper pair is given by the so-called coherence length $\xi \propto \frac{\hbar^2 k_F}{m\Delta}$, which is much larger than the interparticle separation $\approx \lambda_F = 2\pi/k_F$. In the opposite limit, when $k_F a \ll 1$ and $a > 0$, and when tightly bound pairs/dimers of size a are formed, the dimers are widely separated from one another. Surprisingly, these dimers also repel each other with an estimated scattering length $\approx(0.6$–$2)a$ [12,16] and thus the BEC phase is also (meta)stable. The bulk of the theoretical analysis in the intermediate region where $k_F|a| > 1$ was based on the BCS formalism [12,14,15,17,18] and thus is highly questionable. All these authors have considered only the simple ladder diagrams in the particle-particle channel. The inclusion of the additional "bosonic" degrees of freedom [18], which represent nothing else but a two-atom bound state, falls into the same approximation scheme, which includes only ladder diagrams in the atom-atom channel. Even the simplest polarization corrections have not been included into this type of analysis so far. In particular, it is well-known that in the low density region, where $a < 0$ and $k_F|a| \ll 1$ the polarization corrections to the BCS equations lead to a noticeable reduction of the gap [13]. Only a truly *ab initio* calculation could really describe the structure of a many fermion system with $k_F|a| \gg 1$. In the limit $a = \pm\infty$, when the two-body bound state has exactly zero energy, and if $k_F r_0 \ll 1$, one can expect that the energy per particle of the system is proportional to $\varepsilon_F = \hbar^2 k_F^2/2m$, as was recently confirmed by the variational calculations of Refs. [10,11].

As in any other fermion systems [19], the knowledge of the energy per particle as a function of density in the homogeneous phase, Eq. (1), allows us to construct the normal part of the local density approximation (LDA) for the energy density functional (EDF). The extension of the LDA to superfluid systems presented in Ref. [20] in conjunction with Eq. (2) lead us to a rather well-defined EDF, appropriate for fermion systems with infinite scattering length, namely,

$$\begin{aligned}\mathcal{E}(\boldsymbol{r})n(\boldsymbol{r}) &= \frac{\hbar^2}{m}\left[\frac{m}{2m^*}\tau(\boldsymbol{r}) + \beta n(\boldsymbol{r})^{5/3} + \gamma\frac{|\nu(\boldsymbol{r})|^2}{n(\boldsymbol{r})^{1/3}}\right],\\ n(\boldsymbol{r}) &= \sum_\alpha |v_\alpha(\boldsymbol{r})|^2, \qquad \tau(\boldsymbol{r}) = \sum_\alpha |\boldsymbol{\nabla} v_\alpha(\boldsymbol{r})|^2,\\ \nu(\boldsymbol{r}) &= \sum_\alpha v_\alpha^*(\boldsymbol{r})u_\alpha(\boldsymbol{r}).\end{aligned} \tag{3}$$

Here $n(\boldsymbol{r})$ and $\nu(\boldsymbol{r})$ are the normal and anomalous densities and $\tau(\boldsymbol{r})$ is the kinetic energy density, all of them expressed through the quasiparticle wave functions $[u_\alpha(\boldsymbol{r}), v_\alpha(\boldsymbol{r})]$. The summation over α should be interpreted as a sum or integral when appropriate, over all quasiparticle states with the Bogoliubov–de Gennes eigenvalues $E_\alpha > 0$. The dimensionless couplings m/m^*, β, and γ have to be chosen so as to reproduce in the case of infinite homogeneous matter the expressions (1) and (2) for the energy per particle. Unfortunately the parametrization of the EDF is not unique and its form has to be restricted using some additional considerations. One possible choice corresponds to $m/m^* = \alpha_N$, $\beta = 0$, and $\gamma \approx -6.64$. We refer to this case as parameter set I. Another possible choice of parameters, referred to as parameter set II, corresponds to $m/m^* = 1$, $\beta = 3(\alpha_N - 1)(3\pi^2)^{2/3}/5$, and $\gamma \approx -7.20$. Since the anomalous density $\nu(\boldsymbol{r})$ is a diverging quantity, a regularization procedure of the above expression (3) is required. This was performed according to the formalism described in great detail by us elsewhere [20]. The renormalization procedure amounts to replacing γ with a new well-defined running coupling $\gamma_{\text{eff}}(\boldsymbol{r})$. Even though in this renormalization procedure there is an explicit energy cutoff appearing in the formalism, no observable is affected by its presence, when this energy cutoff is chosen appropriately. Up to the overall factor $\hbar^2/m$, the energy density in Eq. (3) has the overall scaling $\mathcal{E}(\boldsymbol{r}) \propto [n(\boldsymbol{r})]^{5/3}$, which is expected from dimensional arguments for a system with an infinite scattering length and $k_F r_0 \ll 1$. In principle, there are an infinite number of possible local energy densities satisfying this requirement. In particular, one could have considered for the superfluid part of the functional, for example, a contribution of the form $\propto|\nu(\boldsymbol{r})|^{5/3}$. Since a term $\propto|\nu(\boldsymbol{r})|^2$ works in both BCS and BEC limits and since the emerging pairing field $\Delta(\boldsymbol{r})$ has the simplest form and the renormalization procedure is also the simplest when using the EDF Eq. (3), we did not consider explicitly any other forms. Moreover, nobody ever suggested other forms for the contribution of the pairing correlations to the energy density in the past, as far as we are aware. We do not expect, however, any qualitative changes in our results resulting from considering other forms for the superfluid contribution to the energy density.

We looked for a self-consistent solution of the Bogoliubov–de Gennes equations corresponding to the EDF in relation (3), with a vortex in the pairing field along the Oz axis and $\hbar/2$ of net angular momentum per particle, namely, $\Delta(\boldsymbol{r}) = \Delta(\rho)\exp(i\phi)$, where $\boldsymbol{r} = (\rho, \phi, z)$ are cylindrical coordinates. The mean field depends only on the radial coordinate ρ. The quasiparticle wave functions have the structure

$$\begin{pmatrix} u_\alpha(\boldsymbol{r}) \\ v_\alpha(\boldsymbol{r}) \end{pmatrix} = \begin{pmatrix} u_s(\rho)\exp[in\phi + ikz] \\ v_s(\rho)\exp[i(n-1)\phi + ikz] \end{pmatrix}, \tag{4}$$

where s labels the radial part of the quasiparticle states, n is an integer, k is the wave vector of the quasiparticle state along the vortex symmetry axis Oz, and $\alpha = (s, k, n)$. The presence of the vortex implies a net (super)flow of the fermion superfluid around the vortex axis with the velocity profile

$$\mathrm{V}_s(\rho)\hat{\boldsymbol{e}}_{\boldsymbol{\phi}} = \frac{\hbar}{m\rho}\frac{1}{n(\rho)}\sum_{\alpha} \boldsymbol{v}_{\alpha}^{*}(\boldsymbol{r}) i\hat{\boldsymbol{e}}_{\boldsymbol{\phi}}\frac{\partial}{\partial\phi}\boldsymbol{v}_{\alpha}(\boldsymbol{r}),$$

where $\hat{\boldsymbol{e}}_{\boldsymbol{\phi}} = (y, -x, 0)/\rho$.

The most salient feature emerging from this analysis is the behavior of the density as a function of the radial coordinate ρ, the prominent density depletion on the vortex axis; see Fig. 1. The density at the vortex core is lowered to a value of $\approx 0.3\rho_\infty$, while the pairing field vanishes at the vortex axis as expected; see Fig. 2. By comparing the actual flow profile with that of an ideal vortex, see Fig. 3, one can see that only the fraction $f_s(\rho) = \mathrm{V}_s(\rho)/V_v(\rho)$ of the system is superfluid at a distance ρ from the vortex axis. We have performed also calculations by imposing the artificial constraint that the pairing field in the homogeneous phase has a larger value than the one expected from the self-consistent solution, corresponding to the EDF Eq. (3). In that case the density depletion at the vortex core becomes even larger. In hindsight this result could have been expected. Large values of the pairing field correspond to the formation of atom pairs/dimers of relatively small sizes. When these dimers are relatively strongly bound and when they are also widely separated from one another, they undergo a Bose-Einstein condensation. For a vortex state in a 100% BEC system the density at the vortex axis vanishes identically. Therefore, by increasing the strength of the two-particle interaction, the fermion system simply approaches more and more an ideal BEC system, for which a density depletion of the vortex core is expected.

The value of the coupling constant, and correspondingly the value of the scattering length, can nowadays be routinely controlled in experiments [21], following earlier theoretical suggestions [22]. One can then expect that it would be feasible to create a BCS state as in the experiment of Ref. [6], stir the system so as to create one or several vortices in a manner similar to the experiments with dilute Bose gases [1,2,4], and change the ambient magnetic field so that the system is brought close to the Feshbach resonance. The first quantized vortex will appear only at some critical angular velocity, if the system is gradually spun up. The vortex quantization could be demonstrated, for example, as in Ref. [1]. At zero temperature, the single vortex state is the lowest energy state with the total angular momentum $N\hbar/2$ (here N is the total number of particles) and its decay into lower energy states with total angular momenta differing by a few number of $\hbar/2$ is strongly suppressed. The next vortex will appear at a larger critical angular velocity, and so forth. The density profiles of the vortices now develop significant density depletions at the cores, which could hopefully be subsequently visualized, if they survive the expansion when the trap is removed. As discussed in Ref. [7], a fluid with a power law equation of state (the present case) will have a simple scaling expansion; see also Refs. [5,23]. As a consequence, the density depletion along the vortex core should survive the atomic cloud expansion upon trap release. Note that a system in which the scattering length is infinite is highly unusual, as, even when it expands and its density decreases, the relative role of the interaction does not change, unlike any other interacting system, and such a system remains strongly coupled at all densities.

By increasing further the strength of the interaction, two atoms will form a bound state. In the limit when the size of this bound state becomes significantly smaller then the mean interparticle separation the system becomes a BEC of atom pairs. These pairs repel each other with an estimated scattering length $\approx(0.6\text{–}2)a$ [12,16]. The BEC phase can in principle be described as well within the framework of the same formalism outline above. Such an approach would be unreasonable, since an accurate description of the relatively strongly bound

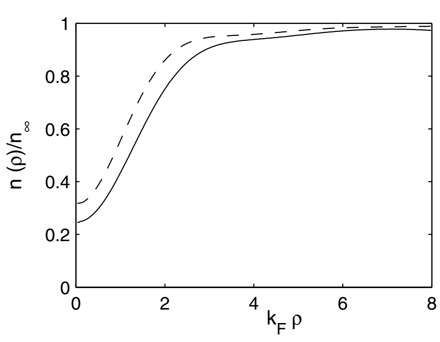

FIG. 1. The density profile of a vortex with the symmetry axis Oz as a function of $k_F\rho$, where k_F is the value of the Fermi wave vector in the homogeneous phase and n_∞ is the asymptotic value of the density. The solid line is for parameter set I, while the dashed line is for set II.

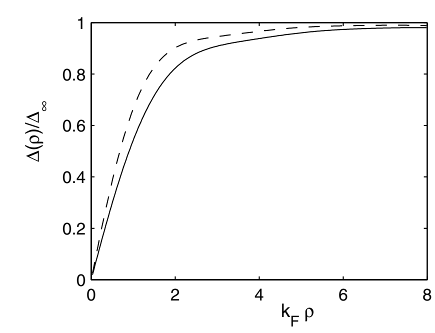

FIG. 2. The pairing field profile, where Δ_∞ is the asymptotic value of the pairing field and the rest of the notations are identical to those in Fig. 1.

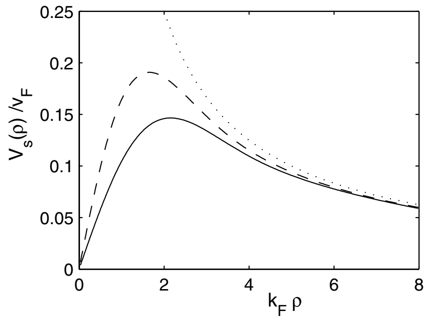

FIG. 3. The velocity profile in units of the Fermi velocity in the homogeneous phase $v_F = \hbar k_F/m$ and the rest of the notations are identical to those in Fig. 1. The dotted line corresponds to the ideal vortex velocity profile $V_v(\rho) = \hbar/2m\rho$.

two-atom system in terms of extended mean-field quasiparticle states would require an exceedingly large number of quasiparticle states. The ratio of the quantum states required for a reasonable accuracy to the number of particles per particle would be at least of order $1/(na^3) \gg 1$. The natural description in this limit is in terms of bosonic dimers, which weakly repel each other. The magnitude of the coherence length in the BEC phase is exceeding considerably the mean separation between pairs, $\xi_b \approx \sqrt{na} \gg 1/n^{1/3}$. This coherence length ξ_b determines the radius of the vortex core. Thus along with the two-atom scattering length a, the size of the vortex core could be controlled as well by means of the Feshbach resonance.

In summary, we have shown that the vortex in a dilute atomic superfluid Fermi gas interacting with an infinite scattering length develops a significant density depletion along its core, which could be visualized after expanding the atomic cloud.

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