Goh *et al.* Reply: We introduced in a recent Letter [1] the load distribution following a power law on scale-free (SF) networks. In addition, it was conjectured that the load exponent δ is universal as long as the degree exponent γ is in $2 < \gamma \leq 3$, based on real-world networks and *in silico* models. In the preceding Comment [2], Barthélemy argues that δ is not universal, sensitive to the details of SF networks. In this Reply, we notice that the discrepancy is mainly caused by different usages of definition of load in [1,2]. Following the definition used in [2], we agree with the result of [2]; however, we find that the question of the universality of the load exponent is not settled yet.

In [1], the load ℓ_k of a vertex k includes $N - 1$ packets leaving and another $N - 1$ packets arriving at the vertex, where N is the total number of vertices. However, those $2(N - 1)$ packets are not included in [2]. While the difference of $2(N - 1)$ can be neglected for vertices with large load ℓ in the limit of $N \to \infty$, however, in finite-size systems, particularly those compatible with most real-world networks comprising $N = 10^3 \sim 10^4$ vertices, this difference could produce a different value of δ. We perform extensive numerical simulations on a larger scale $N = 5 \times 10^5$ than the size $N = 10^4$ previously used in [1] for the static model with $\gamma \approx 2.5$, following the definition in [1], and find that indeed δ turns out to be lower than $\delta \approx 2.2$ beyond the error bar as argued in [2]. This behavior also occurs in the model introduced by Barabási and Albert (BA) when the number of edges emanating from a newly added vertex is $m \geq 2$ in finite-size systems. However, we will show that the universal behavior of the load exponent is still likely as far as SF networks are sparse.

The load exponent for the SF tree has been obtained analytically to be $\delta = 2.0$, independent of the degree exponent γ [3,4]. We investigate how the exponent value $\delta = 2.0$ changes as the number of loops increases. We modify the BA model in such a way that a new vertex attaches one or two edges to the existing network with probability $1 - p$ or p, respectively. The mean number of edges emanating from a new vertex is then $\langle m \rangle = 1 + p$. We investigate how the load distribution changes as $\langle m \rangle$ varies. When $p = 0$, the network is tree, and the load exponent is confirmed to be $\delta \approx 2.0$. We find that δ increases to $\delta \approx 2.2$ by increasing $\langle m \rangle$ to $\langle m \rangle \approx 1.1$ at which the edges connecting different branches of the tree structure form sparse loops in a nontrivial manner. The value $\delta \approx 2.2$ turns out to be robust, independent of the degree exponent γ for $2 < \gamma < 3$. Such behavior persists as long as $\langle m \rangle$ is smaller than a γ-dependent critical value, $\langle m \rangle_c$, beyond which δ depends on γ as observed in [2]. Moreover, we find that the plateau region of $\delta \approx 2.2$ is extended as the system size N increases as shown in Fig. 1. These data suggest that the universal behavior of δ may hold in some finite region of parameter space in the thermodynamic limit, at least for the sparse BA model.

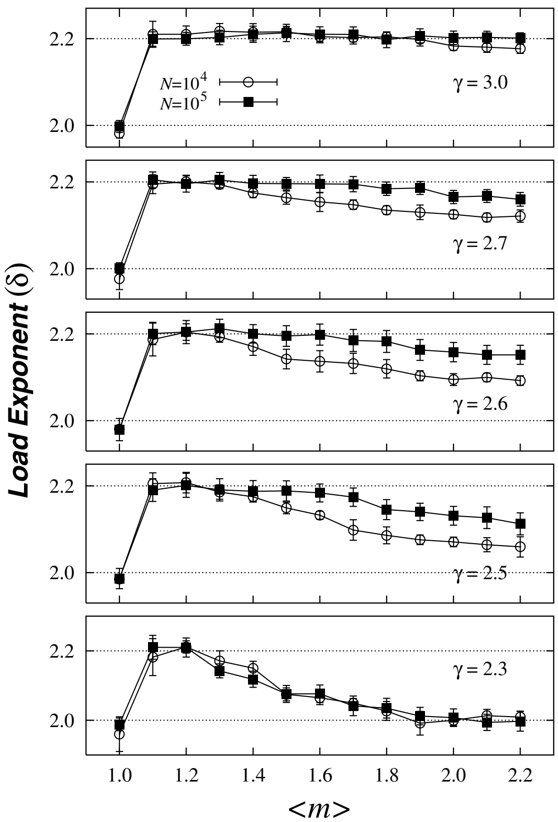

FIG. 1. The load exponent as a function of the mean number of edges $\langle m \rangle$ emanating from a new vertex for various degree exponents γ in the BA model and different system sizes, $N = 10^4$ (○) and $N = 10^5$ (■).

Thus, the possibility of the universal behavior of the load exponent is still an open question. Further details will be published elsewhere [5].

This work is supported by the ABRL program of the KOSEF.

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Received 30 May 2003; published 28 October 2003
DOI: 10.1103/PhysRevLett.91.189804
PACS numbers: 89.75.Hc, 05.10.–a, 89.70.+c, 89.75.Da

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