Experimental Application of Decoherence-Free Subspaces in an Optical Quantum-Computing Algorithm

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For a practical quantum computer to operate, it is essential to properly manage decoherence. One important technique for doing this is the use of "decoherence-free subspaces" (DFSs), which have recently been demonstrated. Here we present the first use of DFSs to improve the performance of a quantum algorithm. An optical implementation of the Deutsch-Jozsa algorithm can be made insensitive to a particular class of phase noise by encoding information in the appropriate subspaces; we observe a reduction of the error rate from 35% to 7%, essentially its value in the absence of noise.

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One of the great stumbling blocks to building quantum computers, with their oft-touted ability to resolve certain problems more efficiently than any classical algorithm [1,2], is the ubiquity of decoherence. Coupling of any element of a quantum computer to an environment destroys its unitary evolution and introduces uncontrollable noise; at first, it was thought by many [3] that these errors would make quantum computation impossible in practice. Since then, a variety of techniques for correcting errors and/or building in immunity to decoherence have been developed [4,5] and it has been proved that if errors are kept below a certain threshold, arbitrarily large quantum computers are possible [6]. One important technique involves computing within subspaces of the full system's Hilbert space which remain unaffected by interaction with the environment; these are known as decoherence-free subspaces (DFSs) [5]. Such DFSs exist when the interaction Hamiltonian has an appropriate symmetry. DFSs have been demonstrated in a linear-optical experiment [7] and in NMR [8] and recently to help circumvent the technical noise which had previously plagued ion-trap quantum computers [9]. To date, no demonstration has been made of the use of DFSs in the context of the implementation of an actual quantum-computing algorithm [10]. In this paper, we present a linear-optical implementation of the two-qubit Deutsch-Jozsa algorithm [2,11] and demonstrate that when a certain class of noise is introduced into the system, greatly increasing the error rate of the algorithm, it is possible to "encode" one logical qubit into two physical qubits and take advantage of DFSs, reducing the error rate to close to zero.

Optics is well known to be an extremely powerful arena for the transportation and manipulation of quantum information [12]. Although due to the linearity of optics, this arena does not allow for scalable construction of quantum gates [13], the incorporation of detection and postselection may render all-optical quantum computers an attractive possibility [14]. Work also proceeds on nonlinearities which would allow for the development of natural two-qubit gates in optics [15]. While we do not yet have access to a truly scalable optical quantum-computer architecture, many of the elements of any such system would be identical to those used in simple linear-optical geometries [13]. For this reason, linear optics remains an important domain for the study of quantum coherence and error correction, even while the ultimate fate of optical quantum computing is uncertain. Recently, striking demonstrations of quantum search algorithms have been carried out using linear optics and linear atom-photon interactions [16], as has the first verification of DFSs [7]. Additionally, it is already clear that even if quantum computation never becomes truly practical, quantum-information processing may have a great effect on the practice of communications and cryptography [17]. Although some information processing will be necessary in this area as well, the question of scalability is not crucial, and linear-optical quantum computation could well prove applicable for elements such as quantum repeaters [18]. In this context, we have chosen to study the applicability of DFSs to a linear-optical implementation, despite the nonscalable nature of the present architecture.

The Deutsch-Jozsa algorithm is designed to distinguish between two classes of functions ("oracles") on N-bit binary inputs. "Constant" functions return the same value (0 or 1) for all 2^n possible inputs, while "balanced" functions return 0 for half the possible inputs and 1 for the other half. Clearly, a classical algorithm would on some occasions require as many as $2^{n-1}+1$ queries to unambiguously determine to which class a given oracle belongs. By contrast, Deutsch and Jozsa showed [11] that a quantum algorithm requires only one such query. In the two-qubit Deutsch-Jozsa algorithm [2], the oracle is a function f on a single bit. It takes as input a query bit x and a signal bit y; its action is to perform the unitary mapping $|x, y\rangle \rightarrow |x, y \oplus f(x)\rangle$. To perform the algorithm, the input is prepared in $H|0\rangle \otimes H|1\rangle = \frac{1}{2}[|0\rangle + |1\rangle] \otimes [|0\rangle - |1\rangle]$, where H is a Hadamard gate. This state is mapped by the oracle to $\frac{1}{2}[|0\rangle \otimes (|f(0)\rangle - |\overline{f(0)}\rangle) + |1\rangle \otimes (|f(1)\rangle - |\overline{f(1)}\rangle)] = \frac{1}{2}[|0\rangle e^{i\pi f(0)} + |1\rangle e^{i\pi f(1)}] \otimes H|1\rangle$. A Hadamard on the query qubit then transforms it into

$|f(0) \oplus f(1)\rangle$, which is equal to $|0\rangle$ for constant and $|1\rangle$ for balanced functions. Thus measurement in the computational basis allows one to determine a global property of $f(x)$, namely, $f(0) \oplus f(1)$, in a single evaluation of the function. Furthermore, the signal qubit is in fact superfluous after the oracle. If some source of decoherence is present during the propagation from the oracle to the final Hadamard, one may encode the query qubit in some DFS of the two physical qubits.

In this experiment we represent the four basis states of two logical qubits ($|00\rangle$, $|01\rangle$, $|10\rangle$, and $|11\rangle$, where the first bit corresponds to the query and the second to the signal) by a photon traveling down one of four optical rails numbered 1–4, respectively. It is possible to implement a universal set of one- and two-qubit operations in a four-rail representation [13]. For example, a NOT gate on the query qubit can be realized by simultaneously swapping rails 1 and 3 and rails 2 and 4. A CNOT gate on the signal qubit is implemented by swapping rails 3 and 4. To perform a Hadamard gate on the query qubit, we combine rails 1 and 3 and rails 2 and 4 at two 50/50 beam splitters; a π phase shift is also needed on two of the arms. Analogous gates can be constructed for the other qubit. The transformations introduced by the four possible functions can also be implemented in this representation by four different settings of an oracle operating as follows: if $f(0)$ is 1, rails 1 and 2 are swapped; if $f(1)$ is 1, rails 3 and 4 are swapped. Thus the task of distinguishing balanced from constant oracles reduces to that of determining whether the number of swaps was odd or even. Note that this oracle is capable of generating entanglement between the two qubits, even though entanglement need not arise for the specific input states demanded by the algorithm.

The schematic diagram of the interferometer is shown in Fig. 1. Consider a photon traveling down rail 2, representing the state $|01\rangle$. The two pairs of 50/50 beam splitters $A1$, $A2$ and $B1$, $B2$ implement the two

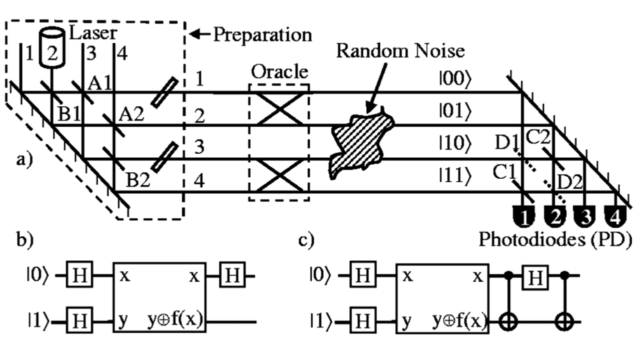

FIG. 1. Schematic of an interferometer which implements the two-qubit Deutsch-Jozsa algorithm (a). All beam splitters are 50/50. With beam splitters $C1$ and $C2$ in place, the standard algorithm (b) is performed. We show that an alternate encoding (c) is preferable in the presence of random collective noise on rails 2 and 3; replacing beam splitters $C1$ and $C2$ with beam splitters $D1$ and $D2$ implements this modified algorithm.

Hadamard gates on the query and signal qubits, preparing the qubits for the oracle. Beam splitters $C1$ and $C2$ realize the Hadamard gate on the query qubit after the oracle. Rails 1–4 illuminate photodiodes $PD1$–$PD4$. A photon reaching $PD1$ or $PD2$ indicates that the value of the query qubit after the algorithm, $f(0) \oplus f(1)$, is 0. This constitutes a determination that the oracle is constant, while $PD3$ and $PD4$ indicate balanced oracles.

One source of decoherence in such systems is the phase noise introduced by fluctuating optical path lengths, created either by variations in distance or by temperature variations and turbulent air flow. In real optical systems, the stability of certain path-length differences may be larger than that of others, either because of the physical proximity of certain paths to one another or because of the particular sources of mechanical or thermal noise. This may lead to a situation where the dominant source of decoherence exhibits a symmetry which can be exploited for computing within DFSs. To simulate the effects such processes could have in larger-scale, distributed quantum-information systems, we introduced a high degree of turbulence by placing the tip of a hot soldering iron below two of the optical rails. These two optical paths (rails 2 and 3) were spatially superposed in this region, distinguished only by their polarization; for this reason, they experienced essentially the same random phase shifts. Since the outputs of the optical Deutsch-Jozsa setup are the outputs of two parallel interferometers, which measure the phase of rail 2 with respect to that of rail 4 and rail 3 with respect to rail 1, this phase noise destroys the interference on which the success of the algorithm relies. On the other hand, inspection of the optical schematic makes the physical process behind the algorithm evident: rails 1 and 3 are prepared in phase with one another, while rails 2 and 4 are also prepared in phase, but 180° out of phase with the former pair. Thus, constructive interference is observed either between 1 and 3 or between 2 and 4. If a single pair (1 and 2 or 3 and 4) is swapped by the oracle, destructive interference is instead observed at both interferometers, while if an even number of swaps occurs, constructive interference is restored. So long as each interferometer compares an output of each of the potential swap regions in the oracle with one from the other, it is possible to distinguish a balanced oracle (one swap) from a constant oracle (zero or two swaps). The strategy to deal with phase noise impressed symmetrically on paths 2 and 3 now becomes clear: instead of interfering 2 with 4 and 1 with 3, one can instead interfere 2 with 3 and 1 with 4. In this way, the random phase appears at both inputs to the same interferometer and has no effect on the outcome.

This modification can be expressed as an encoding of the data into a pair of DFSs. Since our engineered phase noise has identical effects on the two states of odd parity ($|01\rangle$ and $|10\rangle$, stored on rails 2 and 3, respectively) and on the two states of even parity ($|00\rangle$ and $|11\rangle$, stored on rails 1 and 4), each fixed-parity subspace can store a single logical qubit in a decoherence-free fashion. The parity can be written in standard Pauli matrix notation [2] as $\sigma_1^z\sigma_2^z$, where σ_i^z is $+1$ or -1 for bit i in state 0 or 1, respectively ($\sigma_1^z\sigma_2^z$ is $+1$ if both bits are the same and -1 otherwise). The action of the soldering iron therefore may be modeled by the operator $\exp(i\sigma_1^z\sigma_2^z\delta\phi)$, where $\delta\phi$ is a fluctuating phase. In a subspace with a definite eigenvalue of $\sigma_1^z\sigma_2^z$, the random phase $\delta\phi$ only impresses an overall phase on the quantum state, leaving the information within the subspace unaffected. Since the two-qubit Deutsch-Jozsa algorithm relies on only the query qubit after the oracle has completed its action, this qubit may be encoded in either of these DFSs, providing immunity to parity-dependent phase noise which occurs between the oracle and the final Hadamard gates. As shown in Fig. 1(c), a CNOT after the oracle encodes the query qubit into these DFSs, and a second CNOT after the final Hadamard can be used for decoding. The decoding CNOT is in fact unnecessary since measurements are performed only on the query qubit. Swapping rails 3 and 4 performs the encoding; or equivalently, beam splitters $C1$ and $C2$ may be replaced by $D1$ and $D2$.

The experimental setup is shown in Fig. 2. The device was characterized with a large ensemble of identical photons in a coherent state produced by a 780 nm diode laser. To implement the four different oracle settings a variable beam splitter (VBS) was designed. This VBS consists of a half wave plate between two polarizing beam splitters (PBS); any desired reflectivity can be obtained with this optical arrangement. To construct our oracles, a pair of these VBSs was used. Each VBS was adjusted to act either as a swap or the identity. Hadamards were constructed using similar VBSs. After the oracle, rails 2 and 3 were combined into the same spatial mode in a PBS to guarantee the collective phase shift for these beams in the presence of decoherence, and then separated out by another PBS. The transformation between two different encodings was performed by applying another VBS to either swap rails 3 and 4 or not. The experimental setup was designed such that in all of these interferometers the spatial path lengths are always balanced. The average fringe visibility for all four output ports and all possible settings of the oracle and encoding was measured to be about 95%.

The experiment was performed by measuring the signals at detectors $PD1$ through $PD4$ as the half wave plates were cycled through all four oracles and both encodings. The intensities at detectors $PD1$–$PD4$ were normalized to their sum, to yield the probabilities of a photon reaching each of the detectors. These normalized intensities are plotted in Fig. 3 for all four oracle settings, in both encodings. In the DFS encoding, for the constant functions, all photons should arrive at detectors $PD1$ or $PD2$, the "constant pair" of detectors; for the balanced functions, all photons arrive at $PD3$ and $PD4$, the "balanced pair." In the standard encoding, the roles of $PD2$ and $PD4$ are interchanged; thus for this encoding, the constant (balanced) pair is $PD1$ and $PD4$ ($PD3$ and $PD2$). In Fig. 4, for constant and balanced functions and in each encoding, we plot the probability of a photon reaching either detector in the corresponding constant or balanced pair. The average error rates were measured to be about 8% in the absence of added noise. The sources of errors in this experiment were mostly due to imperfect visibility (due to alignment and wave plate setting), and uncertainty and drift in the optical phase settings, where

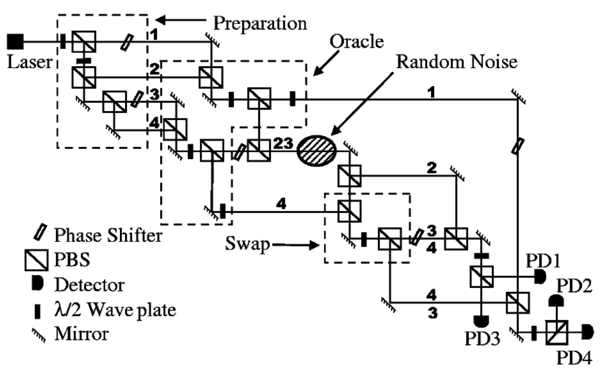

FIG. 2. Experimental setup. Variable-reflectivity beam splitters are implemented using a pair of polarizing beam splitters (PBS) and a half wave plate. The "preparation" portion of the interferometer produces the same superposition as the pair of Hadamards in Fig. 1. The oracle consists of two variable beam splitters which can each be set to exchange two rails or leave them unchanged, plus two half wave plates used to induce π phase shifts on two of the outputs. The random noise is generated by inducing turbulent airflow under rails 2 and 3 while they are spatially superposed.

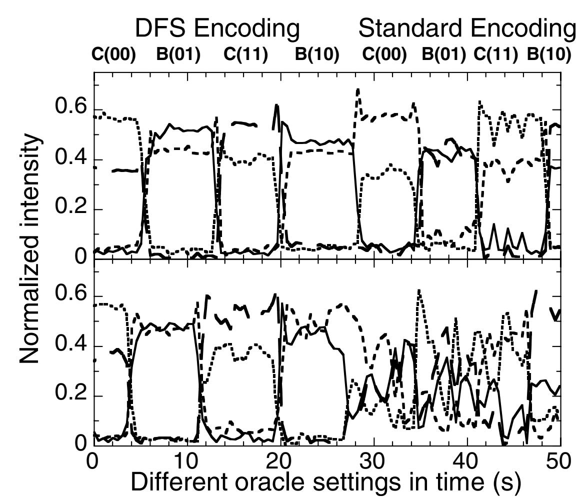

FIG. 3. Experimental data: Normalized intensity is a measure of the fraction of photons reaching each detector, $PD1$ through $PD4$, denoted by dotted, long dashed, solid, and short dashed lines, respectively. Data are shown for both the DFS and standard encoding, for each of the four oracles (00, 01, 10, and 11); C indicates "constant" oracles while B indicates "balanced" oracles. The bottom plot shows the same data in the presence of noise. Note that the noise has a much more significant effect in the case of the standard encoding.

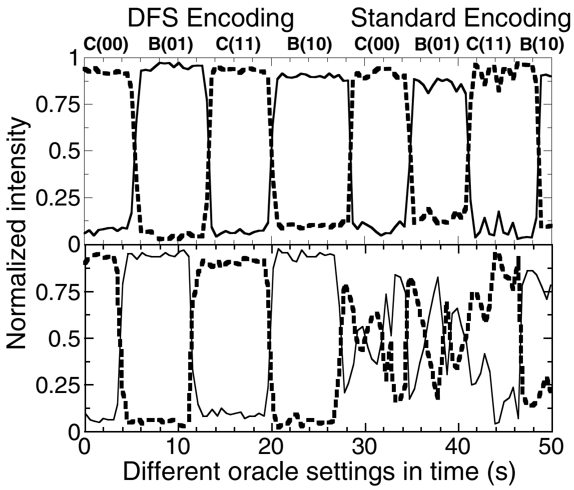

FIG. 4. The probability of the algorithm returning a 0 or a 1 for each of the oracles, in each encoding, with (bottom plot) and without (top plot) the addition of phase noise. The data are extracted by summing the normalized intensities from Fig. 3 for the pair of detectors indicating the constant functions (dashed line), and the balanced functions (solid line). Note that the success rate is close to unity even in the presence of noise when the DFS encoding is used.

the 12° phase uncertainty in our adjustments contributes 2% to the error rate. The drift of the interferometer during measurement was kept low by balancing all path lengths and enclosing the interferometer. In the standard algorithm, introduction of the turbulent airflow increased the error rate (averaged over all oracle settings for the duration of the acquisition) to 35%. When the DFS encoding was used in the presence of turbulence, however, the error rates dropped to 7%, essentially equal to the value in the absence of noise.

We have implemented the Deutsch-Jozsa algorithm in an optical interferometer, and provided the first experimental demonstration of how decoherence-free subspaces can be used to make such a quantum algorithm insensitive to noise with appropriate symmetry properties. We engineered realistic random phase noise for our optical system and demonstrated a significant reduction in error rate by encoding information into decoherence-free subspaces. Phase noise is an ever present issue in coherent optical systems and often exhibits certain correlations which can be exploitable in this manner. The DFS encoding presented here is one promising way of controlling decoherence in quantum systems and could be directly incorporated into scalable quantum computation schemes; it is also applicable to the area of quantum communications and cryptography. The appropriate application of such error-avoidance techniques will be essential in order for quantum-information processing to achieve its great promise.

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