Evidence for Instanton-Induced Dynamics from Lattice QCD

Pietro Faccioli[1,2,*] and Thomas A. DeGrand[3,4,†]

[1]*E.C.T.*, Strada delle Tabarelle 286, I-38050 Villazzano (Trento), Italy*

[2]*INFN, Sezione di Trento, Trento, Italy*

[3]*Department of Physics, University of Colorado, Boulder, Colorado 80309, USA*

[4]*Max-Planck-Institut für Physik (Werner-Heisenberg Institut), Föhringer Ring 6, 80805 Müncnen, Germany*

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We perform a study of the nonperturbative dynamics of the light-quark sector of QCD, based on some recent results of lattice simulations with chiral fermions. We analyze some correlators that are designed to probe the Dirac structure of the quark-quark interaction at different scales. We show that, in the nonperturbative regime, such an interaction contains very large scalar and pseudoscalar components. We observe quantitative agreement between lattice QCD results and the predictions of the instanton liquid model. Moreover, we study how the quark-quark interaction is modified, when quark loops are suppressed. We observe a dramatic effect related to the loss of unitarity, which is naturally explained in the instanton picture. Such an effect cannot be explained in a Dyson-Schwinger equations (DSE) approach, if one assumes a vector quark-gluon coupling.

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The physics of the light-quark sector of QCD is strongly influenced by the nonperturbative structure of the vacuum, and, in particular, by the spontaneous breaking of chiral symmetry (SCSB). Hence, identifying the microscopic mechanism responsible for such a phenomenon represents a fundamental step toward the comprehension of the strong interaction. Unfortunately, this dynamics resides in the nonperturbative sector of QCD and has not been completely understood from first principles.

The typical energy scale of phenomena related to the breaking of chiral symmetry is $4\pi f_\pi \simeq 1.2$ GeV, considerably larger than the confinement scale, $\Lambda_{\rm QCD} \sim 1\ {\rm fm}^{-1} \sim 0.2$ GeV. From a theoretical perspective, such a separation of scales is crucial, because it justifies attempting model descriptions of the nonperturbative physics of SCSB, without needing to simultaneously take into account the dynamics of quark confinement. The common feature in all these semiphenomenological approaches is a strong attraction in the flavor-singlet O^+ channel, leading to a quark condensate. On the other hand, some of the models which have been proposed in the literature rely on drastically different microscopic descriptions of the nonperturbative quark-quark interaction.

Historically, the first attempt to explain the breaking of chiral symmetry predates QCD and was developed in the Nambu–Jona-Lasinio model [1], where a chirally symmetric effective Lagrangian, characterized by a scalar and pseudoscalar four-fermion interaction, was postulated. Later on, the same structure was recovered in the context of the instanton liquid model (ILM) [2]. The latter approach has the advantage of being formulated in terms of quark and gluon degrees of freedom and to be motivated from QCD through a semiclassical argument. Moreover, it explains in a very natural way the structure of the spectrum of lowest-lying eigenvalues of the Dirac operator.

An alternate model description of the nonperturbative sector of QCD which encodes the physics of SCSB has been developed in the context of Dyson-Schwinger equations (DSE). In such an approach, one parametrizes the low-energy behavior of QCD through an ansatz of the infrared structure of the quark-gluon vertex and of the propagators [3]. DSE are then solved numerically, in a given truncation scheme.

Although both DSE, on the one hand, and the ILM, on the other hand, give comparable phenomenology in the light hadron sector, they rely on drastically different microscopic pictures of the nonperturbative interaction, at the 1 GeV scale. Most applications of the DSE developed so far assume a simple vector ansatz for the quark-gluon vertex function, $\Gamma_\mu \propto \gamma_\mu$. (For an example of a DSE model with a more general ansatz for the quark-gluon vertex, see Fischer and Alkhofer [4].) In other words, the nonperturbative dynamics is mediated by the exchange of one (albeit nonperturbative) gluon at the time.

On the other hand, in the instanton picture, the nonperturbative dynamics is dominated by the 't Hooft interaction. Through standard bosonization of the 't Hooft vertex, such an instanton-induced interaction can be thought of as being mediated by fields carrying the quantum numbers of scalar and pseudoscalar bosons.

The goal of this Letter is to identify which one of these two alternate microscopic pictures is closer to QCD. We shall present evidence for the existence of a large scalar and pseudoscalar component of the nonperturbative quark-quark low-energy interaction. This evidence is built using some recent results from lattice simulations with chiral fermions [5]. On the one hand, these results agree on a quantitative level with the predictions of the

ILM. On the other hand, they rule out any picture in which the nonperturbative quark-quark interaction is assumed to have a vector structure, like in present DSE models.

Our analysis is based on the study of the flavor nonsinglet (NS) chirality-flip ratio, introduced in [6]:

$$R^{\rm NS}(\tau) := \frac{A^{\rm NS}_{\rm flip}(\tau)}{A^{\rm NS}_{\rm nonflip}(\tau)} = \frac{\Pi_\pi(\tau) - \Pi_\delta(\tau)}{\Pi_\pi(\tau) + \Pi_\delta(\tau)}, \qquad (1)$$

where $\Pi_\pi(\tau)$ and $\Pi_\delta(\tau)$ are pseudoscalar and scalar NS two-point correlators (notice that the above correlators are defined in coordinate representation, they are not zero-momentum projections) related to the currents $J_\pi(\tau) := \bar{u}(\tau) i\gamma_5 d(\tau)$ and $J_\delta(\tau) := \bar{u}(\tau) d(\tau)$. If the propagation is chosen along the (Euclidean) time direction, $A^{\rm NS}_{\rm flip(nonflip)}(\tau)$ represents the probability amplitude for a $|q\,\bar{q}\rangle$ pair with isospin 1 to be found after a time interval τ in a state in which the chirality of the quark and antiquark is interchanged (not interchanged). Notice that the ratio $R^{\rm NS}(\tau)$ must vanish as $\tau \to 0$ (no chirality flips), and must approach 1 as $\tau \to \infty$ (infinitely many chirality flips).

In [6] it was shown that the correlator (1) is a particularly useful theoretical tool for studying the nonperturbative dynamics of the light-quark sector of QCD. In fact, $R^{\rm NS}(\tau)$ receives no leading perturbative contribution and probes directly the chirality-mixing interaction. A spectral analysis of $R^{\rm NS}(\tau)$ indicated that such an interaction is mediated by topological fields. In particular, it was found that the rate of chirality flips in a quark-antiquark system is proportional to the mass of the η' meson. Moreover, below we shall see that $R^{\rm NS}(\tau)$ is very sensitive to the Dirac structure of the nonperturbative quark-quark interaction.

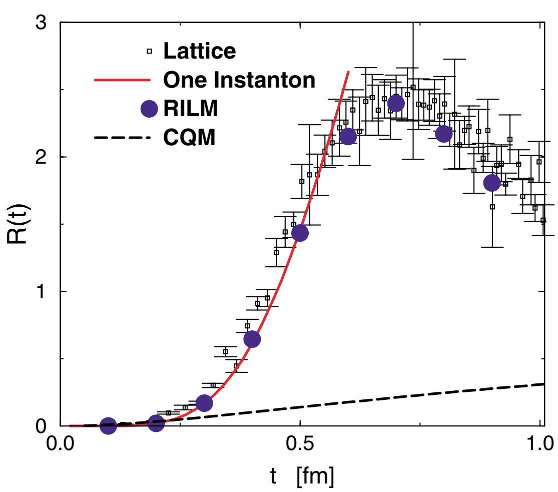

FIG. 1 (color online). The chirality-flip ratio, $R^{\rm NS}(\tau)$, in lattice and in two phenomenological models. Squares are lattice points of [5]. Circles are RILM points obtained numerically from an ensemble of 100 instantons of 1/3 fm size in a 5.3×2.65^3 fm^4 box. The solid line is the contribution of a single instanton, calculated analytically in [6]. The dashed curve was obtained from two free "constituent" quarks with a mass of 400 MeV. Such a curve qualitatively resembles the prediction of a model in which chiral symmetry is broken through a vector coupling (like in present DSE approaches).

The NS scalar and pseudoscalar two-point functions appearing in (1) have been first calculated in the quenched approximation by the MIT group [7] with Wilson fermions and more recently by one of the authors, using chiral (overlap) fermions [5]. The curves for $R^{\rm NS}(\tau)$ obtained from the result of the latter calculation are the square points plotted in Fig. 1. (We recall that, with overlap fermions, the lattice to continuum renormalization factors of the pseudoscalar and scalar correlators are equal and drop out in the ratio.)

As expected, the lattice data interpolate between 0 and 1. Notice that the curve has a maximum at about 0.7 fm, where the ratio is considerably larger than 1. This implies that, after a few fractions of a fermi, the quarks are more likely to be found in a configuration in which their chiralities are flipped, than to be found in their initial configuration. Below we shall see that the presence of such a maximum is a signature of a chirality-mixing component of the quark-quark effective interaction vertex.

We recall that these lattice results have been obtained in the quenched approximation. It is important to ask what differences should be expected in full QCD. Using general QCD inequalities [8], it is immediate to show that $R^{\rm NS}(\tau) > 1$ if and only if $\Pi_\delta(\tau) < 0$. The negativity of such a two-point function represents a severe failure of the quenched approximation which appears only at sufficiently small values of the quark mass. (In the large mass limit, the quenched approximation becomes exact.) It is a reflection of the fact that, in the quenched approximation, the unitarity of the theory is lost.

In terms of chirality flipping amplitudes, we see that the $\Pi_\delta(\tau) \geq 0$ constraint implies that quarks must never be more likely to be found in the flipped chirality configuration than that in the original configuration, $A_{\rm flip}(\tau) \leq A_{\rm nonflip}(\tau)$. Hence, we can conclude that the fermionic determinant suppresses some chirality flipping events, which are otherwise allowed in the quenched approximation. Indeed, the correlators appearing in (1) have recently been evaluated in full QCD, with Wilson fermions [9]. It was observed that the condition $\Pi_\delta(\tau) > 0$ [or, equivalently, $R^{\rm NS}(\tau) < 1$] is restored in going from quenched to full QCD. Such a dramatic qualitative difference between quenched and full QCD calculations of (1) can be used to test phenomenological descriptions of the nonperturbative dynamics. Indeed any realistic model must reproduce a dramatic enhancement of the chirality flipping amplitude, when quark loops are suppressed.

Let us now discuss how $R^{\rm NS}(\tau)$ looks in the ILM and DSE models mentioned above. In both approaches chiral symmetry is spontaneously broken, albeit through two very different microscopic mechanisms. As a consequence, the quarks acquire dynamically an effective mass, which triggers chirality mixing. Because of such a mass generation process, in both cases $R^{NS}(\tau)$ interpolates between 0 and 1. However, one expects crucial qualitative differences between the prediction of the ILM and that of the DSE models. (More generally, we expect differences between the prediction of any model with a vector structure of the vertex on the one hand, and any model with a scalar and pseudoscalar coupling on the other hand.) Let us first consider the result for $R^{NS}(\tau)$ obtained in the random instanton liquid model (RILM). (In the following discussion one can regard the RILM as a prototype of a model with a scalar and a pseudoscalar nonlocal interaction between quasizero modes. The strength of the coupling is tuned by the instanton density, while the delocalization of the quark-quark interaction is provided by the instanton size.) In such a version of the ILM quarks are assumed to propagate in a vacuum populated by an ensemble of randomly distributed instanton and anti-instantons with a density of $\bar{n} = 1\ \text{fm}^{-4}$ and size $\bar{\rho} = 1/3$ fm. It can been shown that the RILM accounts for the 't Hooft interaction to all orders, but neglects quark loops (quenched approximation).

The RILM prediction for $R^{NS}(\tau)$ is presented in Fig. 1. The agreement with the lattice results is impressive. (The RILM points presented in Fig. 1 have been obtained using a quark mass of about 30 MeV, that is the same as the same bare mass used in one of the author's lattice simulations.) It is quite remarkable that not only does the RILM curve display a maximum in $R^{NS}(\tau)$, but also its position agrees quantitatively with the lattice results.

The presence of a maximum in $R^{NS}(\tau)$ and the subsequent falloff towards 1 have a very simple explanation in the RILM: if quarks propagate in the vacuum for a time comparable with the typical distance between two neighbor instantons (i.e., two consecutive 't Hooft interactions), they have a large probability of crossing the field of the closest instanton. If this happens, they must necessarily flip their chirality, due to the scalar and pseudoscalar structures of the 't Hooft vertex. So, after some time, the quarks are most likely to be found in the configuration in which their chirality is flipped. On the other hand, if one waits for a time much longer than 1 fm, the quarks will "bump" into many other pseudoparticles, experiencing several more chirality flips. Eventually, either chirality configurations will become equally probable and $R^{NS}(\tau)$ will approach 1.

The position of the maximum in $R^{NS}(\tau)$ carries information about the interplay between one-body and many-body effects generating chirality mixing. It is interesting to compare the above numerical RILM results with the single-instanton contribution (solid line in Fig. 1), which was derived analytically by one of the authors in [6]. From such a comparison, one can see that one-body effects, i.e., chirality flips induced at the level of a single interaction, dominate the ratio up to Euclidean times of the order of 0.5–0.7 fm. The onset of many-body (many-instanton) effects is governed by the numerical value of the instanton density: the less dilute is the system, the earlier many-instanton effects become important. From the agreement between ILM and lattice data one may argue that the phenomenological value $\bar{n} \simeq 1\ \text{fm}^{-4}$ is indeed realistic.

The prediction for $R^{NS}(\tau)$ would be drastically different in the DSE models and in general in all approaches based on a vector quark-gluon coupling. In this case, the chirality mixing is due only to the dynamical mass generation (i.e., by a genuine many-body effect). In fact, unlike in the ILM, a single quark-antiquark interaction will not interchange the chirality of quarks, because the vector Dirac structure of the quark-gluon vertex is chirality conserving. As a result, even in the quenched approximation, quarks are never more likely to be found in the flipped chirality state than in the initial chirality state, i.e., $R^{NS}(\tau) < 1$ for all τ. On a qualitative level, the DSE prediction for $R^{NS}(\tau)$ will be similar to that obtained considering the propagation of a free but massive "constituent" quark and antiquark pair in the vacuum (dashed line in Fig. 1).

From the above analysis we can conclude that the presence of a maximum in the lattice results for $R^{NS}(\tau)$ implies that the nonperturbative quark-quark interaction contains a strong scalar and pseudoscalar component. In other words, the chirality is mixed already at the level of a single quark-quark interaction, and not only through many-body effects, such as the mass generation induced by SCSB. Therefore, these lattice simulations strongly support the ILM picture against the DSE models.

Additional evidence in this direction comes from comparing quenched and unquenched predictions. In the DSE model, neglecting quark loops affects only the speed of the running of the coupling, but does not generate additional chirality flips. On the contrary, we already mentioned that a dramatic qualitative difference between quenched and unquenched results is expected and observed on the lattice. Such a difference is naturally explained in the ILM. (P. F. acknowledges a clarifying discussion with Shuryak on this point.) If quark loops are allowed, then instantons and anti-instantons can

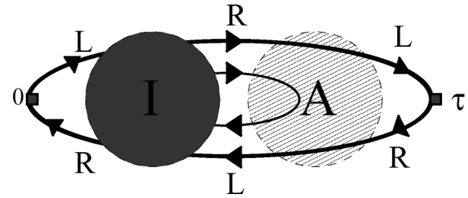

FIG. 2. Suppression of chirality flips due to the topological screening induced by fermion loops in the ILM ($N_f = 3$).

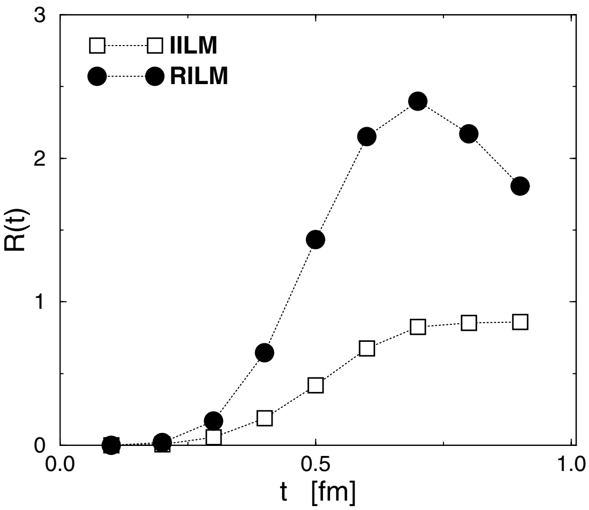

FIG. 3. Suppression of chirality flipping events, due to fermion-loop exchange in the ILM. Circles are RILM (quenched) results; squares are IILM (unquenched) results. In the unquenched model the unitarity requirement, $R^{\rm NS}(\tau) \leq 1$ is restored.

interact through fermion exchange. Such an interaction generates an attraction between instantons and anti-instantons leading to a screening of the topological charge [10] and providing a realization of the Leutwyler-Smilga [11] relation (topological susceptibility vanishing linearly with quark mass, at small quark mass). As a result of such a screening, quarks crossing the field of an instanton are very likely to find, in the immediate vicinity, an anti-instanton which restores their initial chirality configuration (see Fig. 2). (From this discussion it follows that, in a correlated instanton vacuum, the topological screening length coincides with the typical time interval $\bar{\tau}$ between two consecutive chirality flipping interactions. Indeed, in [10] such a screening length was estimated from numerical simulations in the ILM and was found to be 0.2–0.3 fm, consistent with the value $\bar{\tau} = 1/m_{\eta'} \simeq 0.2$ fm, obtained by one of the authors in [6])

In Fig. 3 we compare the chirality flipping ratio $R^{\rm NS}(\tau)$ obtained from a quenched (RILM) and unquenched [IILM (interacting instanton liquid model)] version of the ILM. We observe that, with the inclusion of the fermionic determinant, the condition $R^{\rm NS}(\tau) < 1$ is restored. We stress that, although such a restoration must necessarily take place in QCD, it represents a remarkable success of the ILM, which is not a unitary field theory.

In conclusion, we have presented a study of the microscopic structure of the nonperturbative interaction in QCD, based on the analysis of the results of some recent lattice simulations with chiral fermions. We have used these data to test model descriptions of the microscopic quark dynamics. We have found evidence for large scalar and pseudoscalar components of the effective quark-quark vertex. We have argued that this result rules out models in which the quarks couple nonperturbatively though a purely vector quark-gluon vertex. On the contrary, we have observed impressive quantitative agreement between lattice and ILM results. In addition, the ILM can also explain how quark loops cause a drastic suppression of quark chirality flips.

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*Electronic address: faccioli@ect.it
†Electronic address: thomas.degrand@colorado.edu

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