Teleportation with a Uniformly Accelerated Partner

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In this work, we give a description of the process of teleportation between Alice in an inertial frame, and Rob who is in uniform acceleration with respect to Alice. The fidelity of the teleportation is reduced due to Davies-Unruh radiation in Rob's frame. In so far as teleportation is a measure of entanglement, our results suggest that quantum entanglement is degraded in noninertial frames.

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The new field of quantum information science is a vindication of Landauer's insistence that we recognize the physical basis of information storage, processing, and communication [1]. Recognizing that information science must be grounded in our understanding of the physical world, one is prompted to ask how relativistic considerations might impact tasks, such as teleportation [2], that rely on quantum entangled states. Recently, a number of authors have studied quantum entanglement in the context of Lorentz transformations. While Lorentz transformations cannot change the overall quantum entanglement of a bipartite state [3,4], they can change which observables of the local systems are entangled [5,6].

In this paper, we consider quantum entanglement in noninertial frames. We concentrate on a particular quantum information task, quantum teleportation, and show that the fidelity of teleportation is reduced when the receiver is making observations in a uniformly accelerated frame. This is distinct from reduction in fidelity through the Lorentz mixing of degrees of freedom noted by Gingrich and Adami [5]. Rather it is a direct consequence of the existence of Unruh-Hawking radiation for accelerated observers.

Let Alice be an inertial Minkowski observer with zero velocity, located on the world line passing through event P as shown in Fig. 1. A noninertial observer Rob is traveling with positive constant acceleration in the z direction with respect to Alice. Rob is initially moving in the negative z direction, but comes to rest at event P, before moving off in the positive z direction.

We now discuss how Alice and Rob can come to share an entangled resource for teleportation. Suppose that Alice and Rob each hold an optical cavity, at rest in their local frame. At event P their two frames coincide when Rob's frame is instantaneously at rest. At this event, we suppose that the two cavities overlap and simultaneously a four photon source excites a two photon state in each cavity, as depicted in Fig. 2. We will also assume that, prior to event P, Alice and Rob ensure that all photons are removed from their cavities.

Suppose that each cavity supports two orthogonal modes (spatial modes, we ignore polarization, and model the photons by the massless modes of a scalar field), with the same frequency, labeled A_i, R_i with $i = 1, 2$, which are each excited to a single photon Fock state at event P. At P, the total state held by Alice and Rob is then the entangled state

$$|0\rangle_M \rightarrow |1\rangle_{A_1}|0\rangle_{A_2}|1\rangle_{R_1}|0\rangle_{R_2} + |0\rangle_{A_1}|1\rangle_{A_2}|0\rangle_{R_1}|1\rangle_{R_2}, \quad (1)$$

where $|1\rangle_{A_i}, |1\rangle_{R_i}$ are single photon excitations of the Minkowski vacuum states in each of the cavities. Treating these states as single particle excitations of the *Minkowski* vacuum is an approximation. We expect this to be valid so long as the cavities do not move appreciably over the time taken for the source to excite the modes. As

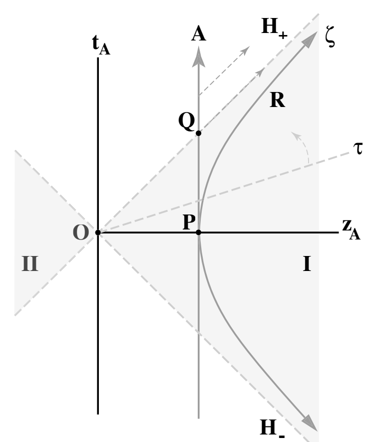

FIG. 1. Minkowski diagram for the case of Alice stationary while Rob (gray hyperbola) undergoes constant acceleration. Alice and Rob share an entangled Bell state at the event P and can communicate as long as Alice stays within Rob's past $\mathcal{H}_-$ and future $\mathcal{H}_+$ particle horizons, corresponding to Rob's proper times $\tau = -\infty$ and $\tau = +\infty$, respectively.

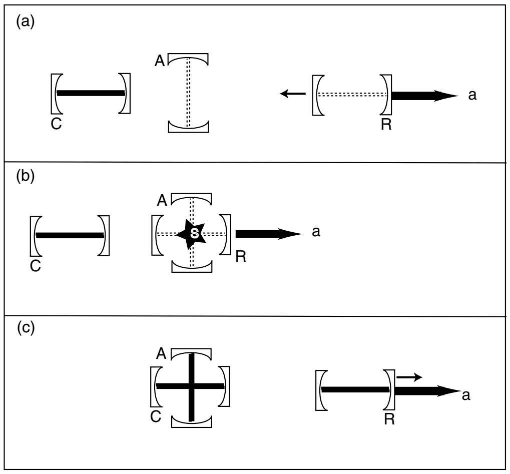

FIG. 2. Cavities A (Alice, Minkowski), R (Rob, Rindler), and C (client cavity, Minkowski). (a) For $\tau < 0$, Rob's cavity moving with constant acceleration a in the negative z direction. (b) At $\tau = 0$, A and R overlap and a four-photon entangled state is shared between the two cavities. (c) At some time $\tau > 0$, Alice makes a Bell measurement with the unknown state in C and her half of the Bell state in A. R moves with constant acceleration in positive z direction.

this time cannot be smaller than the round-trip time in the cavity, we are implicitly assuming the cavities are very small. The state in Eq. (1) encodes the two qubit entangled state

$$|\mathbf{0}\rangle_M|\mathbf{0}\rangle_M + |\mathbf{1}\rangle_M|\mathbf{1}\rangle_M, \qquad (2)$$

where the first qubit in each term refers to cavity A, the second qubit refers to cavity R, and the logical states $|\mathbf{0}\rangle_M$, $|\mathbf{1}\rangle_M$ are defined in terms of the physical Fock states for A's cavity by the *dual rail* basis states

$$|\mathbf{0}\rangle_M = |1\rangle_{A_1}|0\rangle_{A_2}, \qquad |\mathbf{1}\rangle_M = |0\rangle_{A_1}|1\rangle_{A_2}, \qquad (3)$$

with similar expressions for R's cavity.

The previous construction implicitly assumes that we have chosen a modal decomposition of the Minkowski vacuum based on intracavity and extracavity modes. This is a legitimate alternative to the usual way of quantizing the vacuum in terms of plane wave modes [7]. Once the cavities are loaded with a photon, we assume the cavity is perfect and cannot emit the photon. The quasimodes of Ref. [7] then become genuine orthogonal modes.

In order to set up a teleportation protocol [8], we now suppose that Alice has an additional cavity, which we will call the client cavity (C), again containing a single qubit with dual rail encoding by a single photon excitation of a two mode Minkowski vacuum state. This qubit is in an unknown state

$$|\boldsymbol{\psi}\rangle_M = \alpha|\mathbf{0}\rangle_M + \beta|\mathbf{1}\rangle_M. \qquad (4)$$

As Rob's cavity accelerates away, the client cavity is brought near to A's cavity so that a joint measurement can be made on the two orthogonal modes of each cavity. The joint measurement should correspond to an effective measurement of the two qubit system in the Bell basis for A and C.

The results of this measurement are then sent to Rob, and can be received by him as long as Alice transmits them before she moves across Rob's horizon (see Fig. 1). Rob now uses these measurements to make transformations, and possibly measurements, to verify the protocol in his local *accelerating* frame. However, we now must confront the possibility that, as Rob is accelerating, his cavity will become populated by thermally excited photons through the Davies-Unruh mechanism [9]. As we will show, this reduces the fidelity of a teleportation protocol between accelerated partners.

In Minkowski coordinates Rob's world line is

$$t_R(\tau) = a^{-1}\sinh a\tau, \qquad z_R(\tau) = a^{-1}\cosh a\tau, \qquad (5)$$

where τ is the proper time along the world line. Rob's trajectory is a hyperbola in the right Ridler wedge (RRW) labeled region I, of constant positive Rindler position $\zeta(\tau) = 1/a$ (Fig. 1). It is bounded by the lightlike asymptotes $\mathcal{H}_-$ and $\mathcal{H}_+$ which represent Rob's past and future horizons; $\zeta = 0$ with $\tau = -\infty$ and $\tau = \infty$, respectively. The time reversed, mirror image left Rindler wedge (LRW), labeled region II, corresponds to trajectories with $\zeta < 0$. The two regions are causally disconnected from each other.

It is well appreciated now [9–11] that the quantization of fields in Minkowski and Rindler coordinates are inequivalent, implying that the vacuum $|0\rangle_I$ seen by Rob in the RRW is different than the Minkowski vacuum seen by Alice $|0\rangle_M$ [12]. The main consequence of this fact is that Rob must expand the single particle Minkowski states $|0\rangle_M$ and $|1\rangle_M$ in his corresponding logical states $|\mathbf{0}\rangle_M$ and $|\mathbf{1}\rangle_M$ in terms of the Rindler region I and II Fock states as

$$|0\rangle_M = \frac{1}{\cosh r}\sum_{n=0}^{\infty}\tanh^n r|n\rangle_I \otimes |n\rangle_{II}, \qquad (6)$$

$$|1\rangle_M = \frac{1}{\cosh^2 r}\sum_{n=0}^{\infty}\tanh^n r\sqrt{n+1}|n+1\rangle_I \otimes |n\rangle_{II}, \qquad (7)$$

where r is the dimensionless acceleration parameter defined by $\tanh r \equiv e^{-2\pi\Omega}$ with $\Omega \equiv \omega/(a/c)$, and ω is the common frequency of both Alice's and Rob's cavity [13]. The exponential factor can be rewritten as $e^{-\hbar\omega_R/k_bT}$ in terms of the Davies-Unruh temperature $T \equiv \hbar a/2\pi c$. The state $|n\rangle_I$ corresponds to n excitations in one of R's Rindler region I spatial cavity modes.

Second, the state Rob observes for $\tau > 0$ must be adapted to the right Rindler wedge, region I in which his motion is now confined. Since he is causally disconnected from the LRW, the state he observes in region I will be the Minkowski state traced over region II:

$$\rho^{(I)} = Tr_{II}(\rho^{(M)}). \qquad (8)$$

To establish notation, let us consider for the moment the teleportation protocol between Alice and Bob, both Minkowski observers. Upon making a joint projective measurement on her two logical qubits with the result $|\boldsymbol{i}\rangle_M \otimes |\boldsymbol{j}\rangle_M$ with $i, j \in \{0, 1\}$, the full state is projected into $|\boldsymbol{i}\rangle_M \otimes |\boldsymbol{j}\rangle_M \otimes |\boldsymbol{\phi}_{i,j}\rangle_M$, where Bob's state is given by $|\boldsymbol{\phi}_{ij}\rangle_M \equiv x_{ij}|\mathbf{0}\rangle_M + y_{ij}|\mathbf{1}\rangle_M$. Here we have defined the four possible conditional state amplitudes as $(x_{00}, y_{00}) = (\alpha, \beta)$, $(x_{01}, y_{01}) = (\beta, \alpha)$, $(x_{10}, y_{10}) = (\alpha, -\beta)$, and $(x_{11}, y_{11}) = (-\beta, \alpha)$. After receiving the classical information $\{i, j\}$ of the result of Alice's measurement, Bob can rotate his qubit of the entangled state into $|\boldsymbol{\psi}\rangle_M$ by applying the operations $Z_M^i X_M^j$ to $|\boldsymbol{\phi}_{ij}\rangle_M$, where Z and X are single qubit rotations on the logical states. The fidelity of the teleported state is unity in this idealized situation.

We now turn to the case of teleportation between Alice and Rob. When Alice sends the result of her measurement $\{i, j\}$, which can be received by Rob, if Alice has not yet crossed Rob's future horizon $\mathcal{H}_+$, Rob's state will be projected into (written in the Fock basis)

$$\rho_{ij}^{(I)} \equiv \sum_{k=0}^{\infty}\sum_{l=0}^{\infty} {}_{II}\langle k, l|\boldsymbol{\phi}_{ij}\rangle_M\langle\boldsymbol{\phi}_{ij}|k, l\rangle_{II} = \frac{1}{\cosh^6 r}\sum_{n=0}^{\infty}\sum_{m=0}^{n}[(\tanh^2 r)^{n-1}[(n-m)|x_{ij}|^2 + m|y_{ij}|^2] \times |m, n-m\rangle_I\langle m, n-m| + (x_{ij}\, y_{ij}^* \tanh^{2n} r\sqrt{(m+1)(n-m+1)} \times |m, n-m+1\rangle_I\langle m+1, n-m| + \text{H.c.})]. \quad (9)$$

In Eq. (9), $|m, n-m\rangle_I$ is a state of n total excitations in the region I product state, with $0 \le m \le n$ excitations in the leftmost mode. Rob's premeasurement state can be written as

$$\rho_{ij}^{(I)} = \sum_{n=0}^{\infty} p_n \rho_{ij,n}^{(I)} \quad \text{in particular with}$$

$$\rho_{ij,1}^{(I)} = |\boldsymbol{\phi}_{ij}\rangle_I\langle\boldsymbol{\phi}_{ij}|, \quad p_0 = 0, \quad p_1 = 1/\cosh^6 r. \quad (10)$$

Upon receiving the result (i, j) of Alice's measurement, Rob can apply the rotation operators $Z_I^i X_I^j|_{n=1}$ restricted to the one-excitation sector of his state spanned by $\{|\mathbf{0}\rangle_I, |\mathbf{1}\rangle_I\} = \{|0, 1\rangle_I, |1, 0\rangle_I\}$ to turn this portion of his density matrix into the region I analogue of the state Alice attempted to teleport to him, namely,

$$|\boldsymbol{\psi}\rangle_I = \alpha|\mathbf{0}\rangle_I + \beta|\mathbf{1}\rangle_I. \quad (11)$$

The fidelity of Rob's final state with $|\boldsymbol{\psi}\rangle_I$ is then

$$F^{(I)} \equiv \text{Tr}_I(|\boldsymbol{\psi}\rangle_I\langle\boldsymbol{\psi}|\rho^{(I)}) = {}_I\langle\boldsymbol{\psi}|\rho^{(I)}|\boldsymbol{\psi}\rangle_I = \cosh^{-6} r. \quad (12)$$

Rob can check to see whether any Davies-Unruh photons have been excited in his local cavity using a nonabsorbing detector. The probability of obtaining the answer NO is $\cosh^{-6} r$. In that case, he has restored the required entanglement, and teleportation can be completed without error. However, if Rob obtains the more likely result YES, then he cannot complete the protocol without error. In principle, Rob may be able to implement an error correction protocol that simply looks for more than one photon in each of his modes and makes the appropriate correction.

It is of some interest to consider the reduction of fidelity in terms of entropy. Consider the von Neumann entropy $S = -\text{Tr}(\rho \log\rho)$ of Rob's premeasurement state, postmeasurement state upon learning the result of Alice's measurement, and the vacuum state, as a function of r. The premeasurement state is obtained from Eq. (9) by averaging (i, j) over all four possible input states, which reduces it to a diagonal density matrix. The postmeasurement state is given by Eq. (9) with the input state to the teleportation protocol chosen to be $|\boldsymbol{\psi}\rangle_M = 1/\sqrt{2}(|\mathbf{0}\rangle_M + |\mathbf{1}\rangle_M)$, without loss of generality.

For any acceleration $r(a)$, the one-excitation sector of $\rho^{(I)}$ is always $(|\boldsymbol{\phi}_{ij}\rangle_I\langle\boldsymbol{\phi}_{ij}|)/\cosh^6 r$. For the particular choice of $x_{ij} = y_{ij} = 1/\sqrt{2}$ for the teleported state, the probabilities of Rob's diagonal premeasurement state are given by $p_{n,m}^{\text{pre}} = n/2(1-\xi)^3\xi^{n-1} \equiv p_n^{\text{pre}}$, independent of m for $n \ge 0$ and $0 \le m \le n$, with $\xi \equiv \tanh^2 r$. The eigenvalues of the postmeasurement state are given by $p_{n,m}^{\text{post}} = m(1-\xi)^3\,\xi^{n-1}$ for $n \ge 1$ and $0 \le m \le n$, with $p_{0,0}^{\text{post}} \equiv 0$. As the acceleration increases to infinity (i.e., $r \to \infty$; $\xi \to 1$), the higher n-excitation density matrices $\rho_{ij,n}^{(I)}$ of Eq. (10) make their presence known with probability proportional to $(1-\xi)^3\xi^{n-1}$. The relationship between eigenvalues of Rob's premeasurement state, before he receives the result of Alice's measurement, and his postmeasurement state is $p_n^{\text{pre}} = (n+1)^{-1}\sum_{m=0}^{n} p_{n,m}^{\text{post}}$, where $n+1$ is the number of states of the form $|m, n-m\rangle_I$ for fixed n, spanning $\rho_{ij,n}^{(I)}$.

It is worthwhile to note that the Minkowski vacuum state Rob moves through is perceived by him as the thermal Rindler state $\rho_{|0\rangle_M}^{(I)} \equiv \text{Tr}_{II}(|0\rangle_M\langle 0|)$ with diagonal entries $p_{n,m}^{\text{vac}} = (1-\xi)^2\xi^n \equiv p_n^{\text{vac}}$ for $n \ge 0$ and $0 \le m \le n$. In a sense, each normalized n-excitation density matrix of the premeasurement state is individually thermalized with equal entries proportional to ξ^{n-1} as opposed to ξ^n as in $\rho_{|0\rangle_M}^{(I)}$. Within the same n-excitation subspace, the postmeasurement state retains a character distinct from a thermalized state, with probabilities proportional to $m\xi^{n-1}$ for each of its $n+1$ diagonalized component states.

The difference of the von Neumann entropies between $\rho_{\text{pre}}^{(I)}$ and $\rho_{\text{post}}^{(I)}$ is plotted in Fig. 3 along with a normalized five-state model incorporating the $n = \{1, 2\}$ sectors of both density matrices. Individually, the components of each density matrix are approaching zero due to the factors of $(1-\xi)^3\xi^{n-1}$. However, the observation that both the complete and approximate model show that $\Delta S \equiv S_{\text{pre}} - S_{\text{post}}$ approaches zero very slowly (note that $r = 3$ in Fig. 3 implies $\xi \sim 0.58$) indicates that $\rho_{\text{post}}^{(I)}$

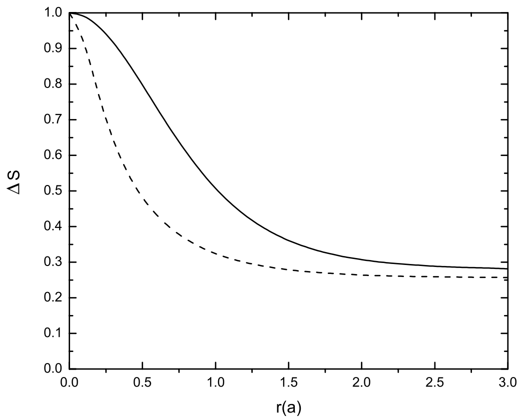

FIG. 3. Rob's entropy information gain (in bits) $\Delta S_{\rm gain} = S_{\rm pre} - S_{\rm post}$ upon receiving Alice's measurement results: numerical (solid) and $\Delta S^{n=2}_{\rm gain}$ (dashed) for a five-state, $n = \{1, 2\}$ excitation model using the $\{|0,1\rangle_I, |1,0\rangle_I\}$ and $\{|0,2\rangle_I, |1,1\rangle_I, |0,2\rangle_I\}$ sector from Rob's postmeasurement state, with $x_{ij} = y_{ij} = 1/\sqrt{2}$.

retains a nonthermalized nature in each n-excitation subspace for finite r. As a whole, Rob's state $\rho^{(I)}_{\rm post}$ is being thermalized by his acceleration through the Minkowski vacuum, which he perceives as a thermal state, and asymptotically $\lim_{r\to\infty(\xi\to 1)}\Delta S = 0$.

In an attempt to teleport a state $|\boldsymbol{\psi}\rangle_M = \alpha|\mathbf{0}\rangle_M + \beta|\mathbf{1}\rangle_M$ to Rob, the best we can expect this uniformly accelerated observer to recover at the end of the protocol is $|\boldsymbol{\psi}\rangle_I = \alpha|\mathbf{0}\rangle_I + \beta|\mathbf{1}\rangle_I$. We have shown that the fidelity of Rob's postmeasurement state with this best possible result $|\boldsymbol{\psi}\rangle_I$ is $\cosh^{-6} r$. In addition, we have demonstrated that the information gain obtained by Rob (defined as the difference in the von Neumann entropies of his pre- and postmeasurement states) decreases with increasing acceleration through the Minkowski vacuum, which Rob perceives as a Rindler thermal state. At high acceleration (high Davies-Unruh temperatures), all information is lost and Rob perceives only the thermalized vacuum state.

Recently, Anderson *et al.* [14] have discussed teleportation and the Unruh vacuum, using Rindler observers Alice and Bob traveling on the *mirror modes* trajectories of [15], one of which exists in region I and the other in region II. Our protocol is physically quite different. Kok and Yurtsever [16] have recently considered the interaction of a uniformly accelerated qubit with a massless scalar field (in a similar fashion to the classic "particle detector" calculation of Unruh and Wald [17]) and show that the qubit decoheres. For long interaction times and slow enough accelerations, the decoherence can be made arbitrarily small.

We have given an explanation of the reduction of teleportation fidelity in terms of the Unruh radiation seen by Rob in his frame. Note that this is an operationally meaningful statement as Rob can attempt to verify that he has received the desired state $(x_{lm}|\mathbf{0}\rangle_I + y_{lm}|\mathbf{1}\rangle_I)$ by local verification measurements (e.g., a single photon interference experiment), and then send the results to Alice. From an operational point of view, Alice would conclude that the shared entangled resource has become decohered. It is well known that entanglement is a fragile resource in the presence of environmental decoherence. It appears also to be a fragile resource when one of the entangled parties undergoes acceleration. While the degree of decoherence is exceedingly small for practical accelerations, the apparent connection between space time geometry and quantum entanglement is intriguing.

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[1] R. Landauer, Phys. Today **44**, 23 (1991).
[2] C. H. Bennett *et al.*, Phys. Rev. Lett. **70**, 1895 (1993).
[3] A. Peres, P. F. Scudo, and D. R. Terno, Phys. Rev. Lett., **88**, 230402 (2002); A. Peres and D. R. Terno, quant-ph/0208128.
[4] P. M. Alsing and G. J. Milburn, Q. Inform. Comput. **2**, 487 (2002).
[5] R. M. Gingrich and C. Adami, Phys. Rev. Lett. **89**, 270402 (2002).
[6] R. M. Gingrich, A. J. Bergou, and C. Adami, Phys. Rev. A **68**, 042102 (2003).
[7] B. J. Dalton and P. L. Knight, J. Mod. Opt. **46**, 1817 (1999); **46**, 1839 (1999).
[8] M. A. Nielsen and I. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, England, 2000), p. 25–28.
[9] P. C. W. Davies, J. Phys. A **8**, 609 (1975); W. G. Unruh, Phys. Rev. D **14**, 870 (1976).
[10] S. Takagi, Prog. Theor. Phys. Suppl. **88**, 1 (1986).
[11] L. N. Pringle, Phys. Rev. D **39**, 2178 (1989).
[12] The Minkowski vacuum $|0\rangle_M$ is unitarily related to the Rindler vacuum $|0\rangle_R \equiv |0\rangle_I \otimes |0\rangle_{II}$. All states in the RRW described by Rob are formed solely by region I operators acting on $|0\rangle_R$, since he is causally disconnected from the LRW, and are of the form $|\psi\rangle_I \otimes |0\rangle_{II}$. Thus, we can speak of $|0\rangle_I$ as the "vacuum seen by Rob."
[13] A given Minkowski mode of frequency ω_M is actually spread over an infinite number of Rindler modes with frequencies ω_R [10], with a distribution peaked about ω_M. Equations (6) and (7) result from a single mode approximation that keeps only the $\omega_R = \omega_M$ Rindler frequency. This can be accomplished by an appropriate configuration of Rob's cavity.
[14] X. Anderson, S. J. van Enk, and T. Rudolph, Q. Inform. Comput. **3**, 423 (2003).
[15] J. Audretsch and R. Müller, Phys. Rev. D **49**, 4056 (1994).
[16] P. Kok and U. Yurtsever, Phys. Rev. D **68**, 085006 (2003).
[17] W. G. Unruh and R. M. Wald, Phys. Rev. D **29**, 1047 (1984).