Optical Telecom Networks as Weak Quantum Measurements with Postselection

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(Received 17 June 2003; published 28 October 2003)

We show that weak measurements with postselection, proposed in the context of the quantum theory of measurement, naturally appear in the everyday physics of fiber optics telecom networks through polarization-mode dispersion (PMD) and polarization-dependent losses (PDL). Specifically, the PMD leads to a time-resolved discrimination of polarization; the postselection is done in the most natural way: one postselects those photons that have not been lost because of the PDL. The quantum formalism is shown to simplify the calculation of optical networks in the telecom limit of weak PMD.

DOI: 10.1103/PhysRevLett.91.180402 PACS numbers: 03.65.Ta, 42.81.Gs

Several times in the history of science, different people working on different fields and with different motivations happened to discover the same thing, or to introduce the same concepts. Think of the connection between differential geometry and general relativity: physics received a convenient mathematical tool for its predictions, mathematics gained in popularity and interest because, apart from its intrinsic beauty, it proved useful. In this paper, we point out a connection which should help to bring together two very different communities: quantum theorists and telecom engineers. The physical degree of freedom that supports this connection is the *polarization* of light; we show that the quantum formalism of *weak measurements and postselection* [1–3] applies to the description of polarization effects in optical networks [4]. The structure of this Letter is as follows: we give first a qualitative description of the announced connection. Then, we introduce the mathematical formalism, and show that the connection does indeed hold down to the detailed formulas; in particular, the knowledge of the "quantum" formalism can simplify some "telecom" calculations.

A modern optical network is composed of different devices connected through optical fibers. With respect to polarization, two main physical effects are present. The first one is *polarization-mode dispersion* (PMD): due to birefringency, different polarization modes (*P modes* in the following) propagate with different velocities; in particular, the fastest and the slowest polarization modes are orthogonal. PMD is the most important polarization effect in the fibers. The second effect is *polarization-dependent loss* (PDL), that is, different P modes are differently attenuated. PDL is negligible in fibers, but is important in devices such as amplifiers, wavelength-division multiplexing couplers, isolators, circulators, etc. In particular, a perfect polarizer is an element with infinite PDL, since it attenuates completely a P mode. Thus, an optical network can be described by a concatenation of trunks, alternating PMD and PDL elements. Combined effects of PMD and PDL elements have been studied in Refs. [5,6]; in particular, interesting phenomena such as anomalous dispersion have been shown to arise even in simple concatenations; namely, a PDL element sandwiched between two PMD elements.

The first piece of the connection we want to point out is the following: *a PMD element performs a measurement of polarization on light pulses* (Fig. 1). In fact, PMD leads to the separation of two orthogonal P modes in time; this separation is called *differential group delay* (DGD), noted $\delta\tau$. If $\delta\tau$ is larger than the pulse width, the measurement of the time of arrival is equivalent to the measurement of polarization—PMD acts then as a "temporal polarizing beam splitter." However, in the usual telecom regime $\delta\tau$ is much *smaller* than the pulse width. In this case, the time of arrival does not achieve a complete discrimination between two orthogonal P modes anymore; but still, some information about the polarization of the input pulse is encoded in the modified temporal shape of the output pulse. We are in a regime of *weak measurement* of the polarization; we are going to show later that we recover indeed the notion of weak measurement of the quantum theorists, by measuring the *mean time of arrival* (that is, the "center of mass" of the output pulse).

The second piece of the connection defines the role of PDL: *a PDL element performs a postselection of some polarization modes.* Far from being an artificial ingredient, postselection of some modes is the most natural situation in the presence of losses: one does always postselect those photons that have not been lost. This would be trivial physics if the losses were independent of any degree of freedom, just like random scattering; but in the

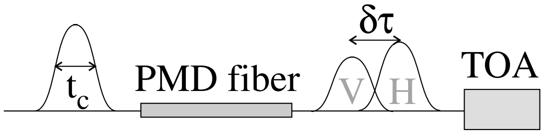

FIG. 1. When a polarized pulse passing through a PMD fiber, the P mode H (parallel to the birefringency axis in the Poincaré sphere) and its orthogonal V are separated in time. A measurement of the time of arrival (TOA) is a measurement, strong or weak, of the polarization.

case of PDL, the amount of losses depends on the meaningful degree of freedom, polarization. An infinite PDL, as we said above, would correspond to the postselection of a precise P mode (a pure state, in the quantum language); a finite PDL corresponds to postselecting different P modes with different probabilities (a mixed quantum state).

In summary: by tuning the PMD, we can move from weak to strong measurements of polarization; by tuning the PDL, we can study the postselection of a pure or of a mixed state of polarization. This is the main result of this Letter, that we are now going to present in mathematical terms.

It is convenient to use the formalism of *two-dimensional Jones vectors*, in which the description of classical polarization is identical to the quantum description of the spin $\frac{1}{2}$ [7]. Thus, e.g., the three typical pairs of orthogonal polarizations — horizontal-vertical linear, diagonal linear, left-right circular — are described, respectively, by the eigenvectors of the Pauli matrices σ_z, σ_x, and σ_y. In this Letter, we shall only need to define the eigenvectors of σ_z: $\sigma_z|H\rangle = |H\rangle$, $\sigma_z|V\rangle = |V\rangle$. Any pure polarization state can be described as a superposition of these vectors, with complex coefficients, the state corresponding to the point $\hat{n} = (\theta, \varphi)$ on the Poincaré sphere being $| + \hat{n}\rangle = \cos\frac{\theta}{2}|H\rangle + \sin\frac{\theta}{2}e^{i\varphi}|V\rangle$.

On a monochromatic wave of frequency ω, a PMD that separates the eigenvectors of σ_z for a birefringency b is represented by the operator [5]

$$\mathrm{PMD}{:}U(b\omega, \hat{z}) = e^{ib\omega\,\sigma_z/2} = \cos\frac{b\omega}{2}\mathbb{1} + i\sin\frac{b\omega}{2}\sigma_z. \tag{1}$$

This is a unitary operation that describes a global rotation of the state of polarization around the z axis of the Poincaré sphere. As for PDL: since the most and least attenuated states are always orthogonal, they can be written as the eigenstates of $\sigma_n = \hat{n}\cdot\vec{\sigma}$, where the direction $\hat{n}$ has *a priori* no link with the direction $\hat{z}$ of the birefringency axis. Neglecting a global attenuation, the PDL is represented by the operator [5]

$$\mathrm{PDL}{:}F(\mu, \hat{n}) = e^{\mu\,\sigma_n/2} = \cosh\frac{\mu}{2}\mathbb{1} + \sinh\frac{\mu}{2}\sigma_n. \tag{2}$$

This is a nonunitary operator, sometimes called a *filter*; in the quantum theory, it appears also in the unambiguous discrimination of nonorthogonal quantum states [8]. It has been shown in Ref. [5] that any optical network can be modeled by an effective PMD followed by an effective PDL, that is, by an operator of the form $F(\mu, \hat{n})U(b, \hat{m})$. However, the study of the general case is involved because the effective parameters μ, $\hat{n}$, b, and $\hat{m}$ depend of the optical frequency ω in a nontrivial way, leading to deformations in the shape of the light pulse. Thus, we focus initially on the simplest optical network, namely *a PMD fiber followed by a PDL element.*

The input state is a Gaussian (Fourier-transform limited) light pulse of coherence time t_c, of central frequency ω_0, prepared in a pure polarization state $|\psi_0\rangle$:

$$\begin{aligned}|\Psi_{\mathrm{in}}\rangle &= \mathcal{A}\, e^{-(1/4)(t/t_c)^2} e^{-i\omega_0 t} \otimes (\alpha|H\rangle + \beta|V\rangle) \\ &= g(t) \otimes |\psi_0\rangle,\end{aligned} \tag{3}$$

with $\mathcal{A} = (\sqrt{2\pi}\, t_c)^{-1/2}$ so that $G(t) \equiv |g(t)|^2$ is a probability distribution [9]. To compute the state of the light at the output of the PMD fiber, we must Fourier transform $|\Psi_{\mathrm{in}}\rangle$ into the frequency domain, apply (1) to any monochromatic component, and integrate back to the time domain. This gives [10]

$$\begin{aligned}|\Psi_{\mathrm{PMD}}\rangle &= \int d\omega\, e^{-i\omega t}\tilde{g}(\omega - \omega_0)\, U(b\omega, \hat{z})|\psi_0\rangle \\ &= \tilde{\alpha}\, g_-(t)|H\rangle + \tilde{\beta}\, g_+(t)|V\rangle,\end{aligned} \tag{4}$$

where $g_\pm(t) \equiv g[t \pm (\delta\tau/2)]$ with $\delta\tau = b$, $\tilde{\alpha} = \alpha e^{ib\omega_0/2}$ and $\tilde{\beta} = \beta e^{-ib\omega_0/2}$. We see that, in addition to the global rotation around the birefringency axis at the frequency ω_0, the PMD has delayed the V polarization with respect to the H polarization, as announced. According to whether the delay $\delta\tau$ is much larger or much smaller than the width t_c of the input pulse, the recording of the time of arrival will provide us with a strong or a weak measurement [11]. For further reference, let us define the polarization state

$$|\psi\rangle = U(b\omega_0, \hat{z})|\psi_0\rangle = \tilde{\alpha}|H\rangle + \tilde{\beta}|H\rangle \tag{5}$$

obtained by retaining only the global rotation, that is, in the limit of continuous light $\delta\tau/t_c \approx 0$.

Now, we should apply the PDL operator (2) to $|\Psi_{\mathrm{PMD}}\rangle$. Before presenting the general case, to become familiar with the concepts, we study the case of *postselection of a pure state*: the PDL element is then a polarizer that projects onto a polarization state $|\psi_1\rangle = \mu|H\rangle + \nu|V\rangle$. Thus, at the output of the optical network we have

$$|\Psi_{\mathrm{out}}\rangle = [\tilde{\alpha}\,\bar{\mu}\, g_-(t) + \tilde{\beta}\,\bar{\nu}\, g_+(t)]|\psi_1\rangle \equiv f(t)|\psi_1\rangle, \tag{6}$$

where $\bar{z}$ is the conjugate of a complex number z. Clearly $f(t)$ is the temporal shape of the selected component of the field. Now we measure the intensity $I(t) = |f(t)|^2$; with $A = \tilde{\alpha}\,\bar{\mu}$ and $B = \tilde{\beta}\,\bar{\nu}$, we have

$$I(t) = |A|^2 G_-(t) + |B|^2 G_+(t) + 2\mathrm{Re}(\bar{A}B)\,\bar{g}_-(t)g_+(t). \tag{7}$$

In the limit of strong measurement, $\delta\tau \gg t_c$, the overlap $\bar{g}_- g_+$ is essentially 0, so the detected intensity corresponds to two well-separated Gaussians: $I(t) = |\alpha\,\mu|^2\, G_-(t) + |\beta\,\nu|^2\, G_+(t)$. A detection in G_- corresponds to the H polarization, so the probability that the polarization was $|H\rangle$ given the preparation and postselection is simply the integral of the Gaussian G_-, normalized to the total intensity: $P(H) = (\int_0^\infty I(t)dt)/(\int_{-\infty}^\infty I(t)dt) = [|\alpha\,\mu|^2/(|\alpha\,\mu|^2 + |\beta\,\nu|^2)]$ But $|\alpha|^2$ is the probability $P(H|\psi_0)$ of finding a photon polarized along $|H\rangle$ given that the state is $|\psi_0\rangle$; using similar notations for $|\beta|^2$, $|\mu|^2$, and $|\nu|^2$, we have found

$$P(H) = \frac{P(\psi_1|H)P(H|\psi_0)}{\sum\limits_{K=H,V} P(\psi_1|K)P(K|\psi_0)}. \tag{8}$$

This is the Aharonov-Bergmann-Lebowitz (ABL) rule [12], which corresponds to the classical rule for the probability of sequential events.

Since we have access to both $P(H)$ and $P(V)$, we can compute $\langle\sigma_z\rangle = P(H) - P(V)$. Moreover, the *mean time of arrival*, defined as usual by $\langle t\rangle = (\int tI(t)dt)/(\int I(t)dt)$, is here $P(H)(\delta\tau/2) + P(V)[-(\delta\tau/2)]$. So, for the case of strong measurement, we have derived the relation

$$\langle t\rangle = \frac{\delta\tau}{2}\langle\sigma_z\rangle. \tag{9}$$

This is the relation that appears in any measurement theory between the *pointer* or *meter* (here, the mean time of arrival) and physical quantity to be measured (here, σ_z). Even though it has been derived from more intuitive grounds in the regime of strong measurements, the relation (9) is the fundamental relation of a measurement process in which the coupling between the pointer and the observable quantity is made by the PMD [11]. In particular, contrary to $P(H)$ and $P(V)$, $\langle t\rangle$ can be defined and measured for any $I(t)$. We shall then take (9) as *the definition of the mean value of σ_z when measured by the PMD.* With this, we can remove the assumption of strong measurement.

Starting with $I(t)$ given by (7), $\langle t\rangle$ can be calculated analytically in a straightforward way, and the relation (9) yield

$$\langle\sigma_z\rangle = \frac{|A|^2 - |B|^2}{|A|^2 + |B|^2 + 2\mathrm{Re}(\bar{A}B)\, e^{-(1/2)(\delta\tau/2t_c)^2}}. \tag{10}$$

Note that the dependence in the strength of the measurement (i.e., in $\delta\tau/2t_c$) is very explicit in (10). In the limit of strong measurement, $\delta\tau/2t_c \to \infty$, we recover the above results. In the opposite limit, $e^{-(1/2)(\delta\tau/2t_c)^2} = 1 - O(\delta\tau/2t_c)$, corresponding to a weak measurement, we have $\langle\sigma_z\rangle_w = \mathrm{Re}[(A - B)(A + B)]$. Noticing that

$$A \pm B = \tilde{\alpha}\,\bar{\mu} \pm \tilde{\beta}\,\bar{\nu} = \begin{cases} \langle\psi_1|\psi\rangle, \\ \langle\psi_1|\sigma_z|\psi\rangle, \end{cases} \tag{11}$$

with $|\psi\rangle$ given in (5), we find

$$\langle\sigma_z\rangle_w = \mathrm{Re}\left(\frac{\langle\psi_1|\sigma_z|\psi\rangle}{\langle\psi_1|\psi\rangle}\right). \tag{12}$$

This is exactly the formula for the *weak value* of σ_z when the postselection is done on a pure state $|\psi_1\rangle$ as given by the quantum theorists [1,2]. Note, in particular, that $\langle\sigma_z\rangle_w$ can reach arbitrarily large values, leading to an apparently paradoxical situation since the eigenvalues of σ_z are ± 1. But there is no paradox at all: $\langle\sigma_z\rangle_w > 1$ simply means $\langle t\rangle > (\delta\tau/2)$, and this situation is reached by postselecting a state $|\psi_1\rangle$ that is almost orthogonal to $|\psi\rangle$; these are very rare events, the shape $f(t)$ of the pulse is strongly distorted, and it is not astonishing that its center of mass could be found far away from its expected position in the absence of postselection.

We can now examine the case of a finite value of the PDL after the PMD fiber. For conciseness, we write $F(\mu, \hat{n}) \equiv F$ for the PDL operator (2). At the output of the PMD-PDL trunk, the state is

$$|\Psi_{\mathrm{out}}\rangle = F|\Psi_{\mathrm{PMD}}\rangle = A(t)|H\rangle + B(t)|V\rangle, \tag{13}$$

where

$$\begin{aligned} A(t) &= \langle H|F|H\rangle\,\tilde{\alpha} g_-(t) + \langle H|F|V\rangle\,\tilde{\beta} g_+(t) \\ &= (C + n_z S)\,\tilde{\alpha} g_-(t) + S n_-\,\tilde{\beta} g_+(t), \end{aligned} \tag{14}$$

$$\begin{aligned} B(t) &= \langle V|F|V\rangle\,\tilde{\beta} g_+(t) + \langle V|F|H\rangle\,\tilde{\alpha} g_-(t) \\ &= (C - n_z S)\,\tilde{\beta} g_+(t) + S n_+\,\tilde{\alpha} g_-(t), \end{aligned} \tag{15}$$

with $C \equiv \cosh\frac{\mu}{2}$, $S \equiv \sinh\frac{\mu}{2}$, and $n_\pm = n_x \pm i n_y$. We can then calculate the detected intensity $I(t) = |A(t)|^2 + |B(t)|^2 = |\alpha|^2(\cosh\mu + n_z \sinh\mu)G_- + |\beta|^2(\cosh\mu - n_z \sinh\mu)G_+ + 2\sinh\mu\,\mathcal{R}\,\bar{g}_+ g_-$ with $\mathcal{R} = \mathrm{Re}(\alpha\bar{\beta} n_+ e^{ib\omega_0})$. The mean time of arrival is then calculated; with $\gamma = \tanh\mu$, the result is

$$\langle t\rangle = \frac{\delta\tau}{2}\,\frac{|\alpha|^2 - |\beta|^2 + \gamma n_z}{1 + \gamma[n_z(|\alpha|^2 - |\beta|^2) + 2\mathrm{R}e^{-(1/2)(\delta\tau/2t_c)^2}]}. \tag{16}$$

Again, in the limit of weak measurement and using (9), we find

$$\langle\sigma_z\rangle_w = \frac{\langle\sigma_z\rangle_\psi + \gamma n_z}{1 + \gamma\vec{n}\cdot\langle\vec{\sigma}\rangle_\psi} = \mathrm{Re}\left(\frac{\langle F^\dagger F\,\sigma_z\rangle_\psi}{\langle F^\dagger F\rangle_\psi}\right), \tag{17}$$

with $|\psi\rangle$ given by (5) as before. The right-hand side is the weak value obtained by postselection on the mixed state $\rho = [1/\mathrm{Tr}(F^\dagger F)]F^\dagger F$ [2,3]. The limiting case $\gamma = 0$ means $\mu = 0$, thence $\rho = \frac{1}{2}\mathbb{1}$: if there is no PDL, $\langle\sigma_z\rangle_w = \langle\sigma_z\rangle_\psi$ as it should. At the other extreme, $\gamma = 1$ means $\mu \to \infty$ thence $\rho = \frac{1}{2}(\mathbb{1} + \sigma_n)$, and we recover the formula (12) for the postselection of the pure state $|\psi_1\rangle = |+\hat{n}\rangle$. Finally, we stress that the *principal states of polarization* of the PMD-PDL network, as defined, e.g., in Ref. [5], are $F|H\rangle$ and $F|V\rangle$ [13].

We have then demonstrated our claims: an optical PMD-PDL network is an everyday realization of the abstract notions of weak measurement and postselection introduced in the theory of quantum measurement. We had also said that telecom engineers would benefit by learning some quantum formalism, were it only because it could simplify their calculations. Indeed, consider a more complicated optical network, composed of three trunks: PMD-PDL-PMD, represented by the operator $T = U(b_2\omega, \hat{m})F(\mu, \hat{n})U(b_1\omega, \hat{z})$. As we noticed above, this simple network is sufficiently complex to yield anomalous dispersion. The calculation can of course be done following the same steps as above, but it is heavy and not really instructive. Another approach, that is moreover scalable to any network consisting of $2N + 1$ trunks alternating PMD and PDL, is possible if the two PMD's are weak, that is, in the telecom limit where the DGD's $\delta\tau_k = b_k$ are much smaller than the width t_c of the pulse; for conciseness, we write $\varepsilon = \tau_k/t_c$. This means that $\tilde{g}(\omega) = \tilde{g}(\omega_0 + x)$ is significantly different from zero only for $|x| \leq \frac{1}{t_c}$, that is, $b_k x = O(\varepsilon)$. So we can expand all the PMD operators (1) as [10]

$$U(b\,\omega, \hat{m}) = [\mathbb{1} + i(bx/2)\sigma_m + O(\varepsilon^2)]U(b\,\omega_0, \hat{m}). \quad (18)$$

Let us then calculate the three-trunk network:

$$T(x) \simeq \mathcal{F} + ix\left(\frac{b_1}{2}\mathcal{F}\sigma_z + \frac{b_2}{2}\sigma_m\mathcal{F}\right) + O(\varepsilon^2), \quad (19)$$

with $\mathcal{F} = U(b_2\,\omega_0, \hat{m})FU(b_1\,\omega_0, \hat{z})$. In what follows, we define the two orthogonal states of polarization $|\psi_F\rangle = \mathcal{F}|\psi\rangle/\sqrt{\langle\mathcal{F}^\dagger\mathcal{F}\rangle_{\psi_0}}$ and $|\psi_F^\perp\rangle$, and we systematically omit global attenuations. We have:

$$\begin{aligned} T(x)|\psi_0\rangle &= \langle\psi_F|T|\psi_0\rangle|\psi_F\rangle + \langle\psi_F^\perp|T|\psi_0\rangle|\psi_F^\perp\rangle \\ &\propto (1 + ixW)|\psi_F\rangle + xD\,|\psi_F^\perp\rangle + O(\varepsilon^2), \end{aligned} \quad (20)$$

where $W = \langle\psi_0|\mathcal{F}^\dagger(\frac{b_1}{2}\mathcal{F}\sigma_z + \frac{b_2}{2}\sigma_m\mathcal{F})|\psi_0\rangle/\langle\mathcal{F}^\dagger\mathcal{F}\rangle_{\psi_0}$ and $xD \sim O(\varepsilon)$. The passage from the Fourier to the time domain yields

$$\begin{aligned} |\Psi_3\rangle &= \int dx e^{-i(x+\omega_0)t}\tilde{g}(x) \otimes T(x)|\psi_0\rangle \\ &\propto g[t - \mathrm{Re}(W)]e^{-i\omega_0 t} \otimes |\psi_F\rangle + h(t) \otimes |\psi_F^\perp\rangle, \end{aligned} \quad (21)$$

where we used $1 + ixW = e^{ixW} + O(\varepsilon^2)$ and where $h(t) \sim O(\varepsilon)$. The measurement of the intensity of the light pulse $|\Psi_3\rangle$ gives $I(t) \propto G(t - \mathrm{Re}(W)) + O(\varepsilon^2)$: the center of the pulse is now in

$$\langle t\rangle = \mathrm{Re}(W) = \frac{b_1}{2}w_1 + \frac{b_2}{2}w_2, \quad (22)$$

with w_1 given by (17) and $w_2 = \langle\psi_F|\sigma_m|\psi_F\rangle$. This result is intuitively clear: the first term is the weak value obtained by forgetting the second PMD element; the second term is just the mean value of σ_m on the filtered state obtained by forgetting the first PMD element. For the case of any network composed of $2N + 1$ trunks alternating PMD and PDL elements, the result generalizes immediately as $\langle t\rangle = \sum_k(\delta\tau_k/2)w_k$, with w_k the suitable weak values [13]. This example shows how the formalism of weak measurements simplifies some calculations of networks combining PMD and PDL, adding an intuitive meaning to the formulas.

In conclusion, we have shown that the quantum theoretical formalism of weak measurements and postselection, often thought of as a weirdness of theorists, describes important effects in the physics of telecom fibers. In particular, the notion of postselection appears naturally, since the telecom engineers select only those photons that are not lost in the fiber.

Just a final remark, to say that, with this investigation, we close a loop of analogies. On the one hand, in Ref. [14], Gisin and Go stressed the analogy between the PMD-PDL effects in optical networks and the mixing and decay of kaons. On the other hand, in Ref. [15] it was shown that adiabatic measurements in metastable systems are a kind of weak measurement, and point out that kaons provide experimental examples of this. By showing the link between PMD-PDL and weak measurements with postselection, this work closes the loop.

Note added in proof.—For a related independent work, see [16].

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[9] The "time" t in our equations is not the evolution parameter alone, but should rather be $t - nz/c$, where z is the position in the fiber and c/n is the average light speed in the fiber. That is why quantum theorists can well consider t as a "position" and ω as its conjugate "momentum."

[10] Note that $U(b(\omega_0 + \delta\omega), \hat{z}) = U(b\omega_0, \hat{z})U(b\delta\omega, \hat{z})$.

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