Comment on "Ultraviolet Modifications of Dispersion Relations in Effective Field Theory"

In a recent Letter Myers and Pospelov [1] considered Lorentz violating effects arising from quantum gravity in the language of effective field theory describing such effects in terms of dimension 5 operators. In fact, some of these terms are obtained in heuristic models suggested by loop quantum gravity (LQG). We touch upon three aspects of the discussion. First, we bring out that the mechanism suggested by the authors to avoid the appearance of new unsuppressed terms through radiative corrections does not work beyond the linear order in the dimensionless parameters ξ, η_1, and η_2 below. Second, we argue against the idea that the natural cutoff for such terms might be low, say, the SUSY (supersymmetric) scale ($\approx$TeV). Third, we point out that, although these results could be seen as essentially ruling out some heuristic models [2], they are not, as of now, saying anything about LQG besides limiting the degree to which its relevant semiclassical states might break Lorentz invariance over a macroscopic regime.

In [1] the Lorentz violating terms for the gauge field A_μ and the fermion field Ψ are given by $\mathcal{L}_{LV} = \frac{\xi}{M_p} C^{abc} F_{ad} \partial_b^* F_c^d + \bar{\Psi} C^{abc} (\frac{\eta_1}{M_p}\gamma_a + \frac{\eta_2}{M_p}\gamma_a\gamma_5)\partial_b\partial_c\Psi$, where $C^{abc} = W^a W^b W^c$ with W^a the 4-velocity of the preferential frame.

The question now is, what is the effect of using these modifications in the self-energy of the fermion? Myers and Pospelov [1] noted that these would lead to a generation of large Lorentz violating effects represented by dimension-2 or -3 operators. They then suggest modifying the scheme by replacing the tensor C^{abc} by the tensor $\tilde{C}^{abc} = W^a W^b W^c - (1/6)(W^a \eta^{bc} + W^b \eta^{ac} + W^c \eta^{ab})$ which has the property that it vanishes on contraction with the flat Minkowski tensor η_{ab} in any pair of indices. This feature then ensures that integrals, such as $\int dk^4 \frac{k_\mu k_\nu}{k^4} \propto \eta_{\mu\nu}\Lambda^2$ (where Λ is the cutoff of the effective theory), appearing in the calculation of the self-energies, will not produce large Lorentz violating terms. Our first point is that when one goes beyond the lowest order in ξ, η_1, and η_2 (in particular, beyond second order) one finds integrals such as $\int dk^4 \frac{k_\mu k_\nu k_\rho k_\sigma k_\tau k_\alpha k_\beta k_\gamma}{k^8} \propto \Lambda^4(\eta_{\mu\nu}\eta_{\rho\sigma}\eta_{\tau\alpha}\eta_{\beta\gamma} + \text{perm.})$, which when contracted with $C_{\mu\nu\rho}C_{\sigma\tau\alpha}C_{\beta\gamma\delta}$ result in a nonvanishing term proportional to W_δ.

These then generate the dangerous low dimension operators that one was trying to avoid. In particular, the one loop self-energy of a charged fermion generates the following Lorentz violating effective term: $e^2 \frac{\Lambda^4}{M_p^3} P^{(3)}(\xi, \eta_1, \eta_2)\bar{\Psi} W^a \gamma_a \gamma_5 \Psi$, where e is the electromagnetic coupling and $P^{(3)}$ is a polynomial of degree 3 with coefficients of order 1. This is in agreement with the well-known expectation from effective field theories — that all operators allowed by the remaining symmetries would be generated. Similarly, considering the vacuum polarization one generates a Chern-Simon (CS) term in the photon propagator $e^2 \frac{\Lambda^4}{M_p^3} P'^{(3)}(\eta_1, \eta_2) W_a A_b^* F^{ab}$, where $P'^{(3)}(\eta_1, \eta_2)$ is also a polynomial of degree 3.

Next we consider the idea mentioned in [1] to set the cutoff scale for the effective theory at a very low value such as Λ_{SUSY}. This would mean that the physics of the intermediate scale is dramatically affecting the way in which the Lorentz violation, originating at M_P, manifests itself at low energy, and would require a reanalysis in terms of a concrete proposal. Supersymmetry, by itself, does not seem to be sufficient.

For consistency with the underlying rational, one should set the cutoff Λ at around M_p. The resulting fermion corrections are tightly bounded experimentally. Direct comparison with Eq. (9) in [3] results in the remarkable bound $P^{(3)}(\xi, \eta_1, \eta_2) < 10^{-45}$. Thus ξ, η_1, and η_2 must be at most of order 10^{-15}. Similarly, a comparison of the CS term with the effect studied in [4] leads to $P'^{(3)}(\eta_1, \eta_2) < 10^{-57}$, indicating that η_1 and η_2 must be at most of order 10^{-19}. We should point out that while the last bound could, in fact, be dramatically lowered by introduction of the hypothesis of supersymmetry, the former (and weaker) is very robust. These bounds dwarf all existing ones.

Finally, we must emphasize that at this point one is not testing LQG, as nothing in this framework prevents the construction of semiclassical states that do not select a (global) vector field. In such a scenario the theory would be immune to the constraints being set by the current explorations of quantum gravity phenomenology.

We thank M. Gunaydin and A. Ashtekar for helpful discussions, and we acknowledge support by NSF Grant No. PHY00-90091, by the Eberly Research Funds of Penn State, by DGAPA–UNAM Project No. IN 112401, and by CONACYT.

Alejandro Perez[1] and Daniel Sudarsky[1,2]
[1]CGPG
The Pennsylvania State University
University Park, Pennsylvania 16802, USA
[2]ICN
UNAM
México D.F., México

Received 24 June 2003; published 20 October 2003
DOI: 10.1103/PhysRevLett.91.179101
PACS numbers: 11.30.Cp

[1] R. C. Myers and M. Pospelov, Phys. Rev. Lett. **90**, 211601 (2003).
[2] R. Gambini *et al.*, Phys. Rev. D **59**, 124021 (1999); J. Alfaro *et al.*, Phys. Rev. D **65**, 103509 (2002); Phys. Rev. Lett. **84**, 2318 (2000).
[3] D. Sudarsky *et al.*, Phys. Rev. Lett. **89**, 231301 (2002).
[4] R. J. Gleiser *et al.*, Phys. Rev. D **64**, 083007 (2001).

