# Modeling Dynamics of Information Networks

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We propose an information-based model for network dynamics in which imperfect information leads to networks where the different vertices have widely different numbers of edges to other vertices, and where the topology has hierarchical features. The possibility to observe scale-free networks is linked to a minimally connected system where hubs remain dynamic.

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Complex adaptive systems can often be visualized as networks in which each element is represented by a vertex (node), and its interactions by edges (links) to other vertices. Network studies have been inspired by the observation that working networks often have a broad distribution of edges and possibly even scale free as reported for the Internet [1–3] and some molecular networks [4]. Further, real-world networks often exhibit nonrandom topological features. This may be modular [5–7], hierarchical [8], or other features [9], that, for example, may help specificity in signaling [10].

Most networks are the result of a dynamical process. One hypothesis is preferential growth that predicts scale-free networks [11–13]. The preferential growth is, however, questionable in many networks, whereas transmission of information plays a fundamental role in nearly all networks, including neural networks with synaptic rewiring [14], molecular networks, and social networks [15], exemplified by the Internet [2,3,16,17]. In fact, networks may be viewed as the natural embedding of a world with a limited information horizon. Thus, it is interesting to explore a network topology that is dynamically coupled to information transmission and formed in an ongoing competition for edges between a fixed number of vertices. We will suggest that a broad range of vertex degrees could be understood not as an extension of the narrow distributions of the Erdős-Rényi networks [18], but rather as the result of an intrinsic instability of a centralized system illustrated in Fig. 1.

We consider a dynamic network where each vertex attempts to optimize its position, given limited information. A natural quantity to optimize is the participation in the activities on the network. In economic terms, this corresponds to optimization of trading activity [19], or to maximization of access to a variety of different products. One activity-related measure would be the "betweenness" discussed by [20]. Another measure is vertex-vertex distances, and, accordingly, any vertex would attempt to place itself close to all other vertices. The globally optimized network is then the hublike structure [21], shown in the left panel of Fig. 1. The distances between vertices are minimal and can only be minimized further by adding additional edges between vertices on the periphery of the central hub. The addition of such extra edges is not cost free, as any edge puts a cost to the system. We primarily consider a dynamics constrained by having the total number of edges (and vertices) conserved.

In practice, each vertex may have only limited information about the location of other vertices. When changing their neighbors by moving edges from one vertex to another, they may make mistakes due to their limited local information. This will destabilize the optimal topology with a central hub and may lead to a distributed network as shown in the right panel of Fig. 1.

To study the interplay between information exchange and dynamical rewiring of edges in a network, we introduce a simple agent-based model where different agents have different and adjustable memories in a way reminiscent of the trading model in [19]. Every agent, named by a number $i = 1, 2, 3, \dots, n$, is a vertex in a connected network that consists of $N$ vertices and $E$ edges. Agent $i$ has a memory

$$M_i = \begin{cases} D_i(l) \\ P_i(l) \end{cases}, l = 1, 2, \dots, i-1, i+1, \dots, N,$$

with $N - 1$ *distances D* and *pointers P* to the other agents in the network. The distance $D_i(l)$ is agent $i$'s estimated

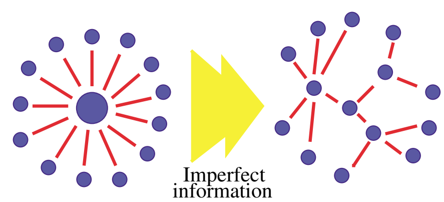



FIG. 1 (color online). In a perfect world, a single vertex that can differentiate all exit edges from each other might distribute all tasks and information efficiently. In real-world networks, no perfect "distributor" exists: Even when every vertex "tries" to minimize its distances to all other vertices, typical vertices tend to connect through more than one intermediate. Imperfections destabilize the central hub, and the vertices in the network obtain a wide range of vertex degrees.

shortest path length to $l$. The pointer $P_i(l)$ is agent $i$'s nearest neighbor on the estimated shortest path to $l$. Thus, $M_i$ may be seen as a simplified version of the gateway protocol used by the autonomous systems to direct transmission of Emails across the hardwired Internet. Here, however, the memory will be used to rewire edges in the network.

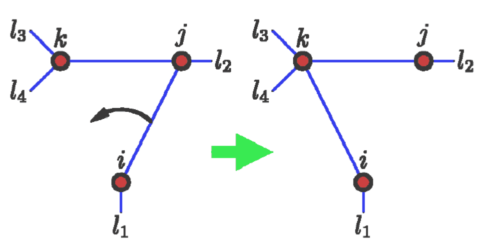
Initially, the network is a hub of the $N - 1$ agents connected to a center agent by $N - 1$ edges (as in Fig. 1, left) plus $E - N + 1$ randomly placed edges on the periphery of the hub. The basic move, illustrated in Fig. 2, consists of a rewiring attempt and, if successful, some information exchange in the local region of the network. In detail, the move consists of three steps.

(i) An agent $i$ and one of its neighbors $j$ is chosen randomly.

(ii) An agent $l \neq i, j$ is chosen randomly and, if $D_i(l) > D_j(l)$, then the edge between $i$ and $j$ is rewired to an edge between $i$ and $k = P_j(l)$. If $l$ did not satisfy the above criteria, a new $l$ is randomly chosen. If no such $l$ exists, the rewiring is aborted.

(iii) The information $i$ has lost by disconnecting $j$ is replaced by information from $k$. Further, there is full exchange of information between $i$ and $k$. If agent $k$ lists a shorter path to some other agents, then $i$ adopts this path with a pointer to $k$. Similarly for $k$, if agent $i$ lists a shorter path, then $k$ adopts this path through $i$. The information $j$ has lost by disconnecting $i$ is replaced by forcing agent $j$ to change all its previous pointers toward $i$ to pointers toward $k$ and add 1 to the corresponding distances.

Notice that above there is no information transfer between $j$ and $k$: $j$ does not read any of $k$'s information, $j$ is only using the information that the rewiring took place.




| | $i$ | $j$ | $k$ | $l_1$ | $l_2$ | $l_3$ | $l_4$ |
|---|---|---|---|---|---|---|---|
| $D_i$ | - | 1 | 2 | 1 | 2 | 3 | 4 |
| $P_i$ | - | $j$ | $j$ | $l_1$ | $j$ | $j$ | $l_1$ |
| $D_j$ | 1 | - | 1 | 2 | 1 | 2 | 3 |
| $P_j$ | $i$ | - | $k$ | $i$ | $l_2$ | $k$ | $k$ |
| $D_k$ | 2 | 1 | - | 3 | 2 | 1 | 1 |
| $P_k$ | $j$ | $j$ | - | $l_4$ | $j$ | $l_3$ | $l_4$ |

| | $i$ | $j$ | $k$ | $l_1$ | $l_2$ | $l_3$ | $l_4$ |
|---|---|---|---|---|---|---|---|
| $D_i$ | - | 2 | 1 | 1 | 3 | 2 | 2 |
| $P_i$ | - | $k$ | $k$ | $l_1$ | $k$ | $k$ | $k$ |
| $D_j$ | 2 | - | 1 | 3 | 1 | 2 | 3 |
| $P_j$ | $k$ | - | $k$ | $k$ | $l_2$ | $k$ | $k$ |
| $D_k$ | 1 | 1 | - | 2 | 2 | 1 | 1 |
| $P_k$ | $i$ | $j$ | - | $i$ | $j$ | $l_3$ | $l_4$ |

FIG. 2 (color online). Dynamics of edge rewiring: The edge between $i$ and $j$ is rewired to an edge between $i$ and $k$, if local information predicts that $k$ provides a shorter path to the random agent $l$ ($l = l_3$ in the figure). The agents' information about the network is subsequently updated as shown by the shift from lower left to lower right panels. Notice that the local information is not necessarily correct.

The model defines an update of both the network (ii) and the information that agents in the network have about each other's locations (iii). The step (ii) represents local optimization where agent $i$ rewires from $j$ to $k$ with a probability given by the fraction of the network which is estimated to be closer to the center. We stress that only a small part of the system is informed about a changed geometry and that decisions on moves may be based on outdated information. When repeated many times, the model leads to a breakdown of the central hub into a steady state ensemble of networks with a broad distribution of vertex degrees.

Figure 3(b) shows that the degree distribution for vertices in the network is broad, in fact close to the Zipf law $1/C^2$ reported for some real-world networks [2,4], as well as for the size distributions of industrial companies [22]. However, there is a correction to scaling at intermediate and large vertex degrees. This limitation of the model can be removed by increasing the information between agents during the rewiring, for example, by adding information exchange between agent $j$ and agent $k$ in Fig. 2.

(iv) $j$ considers a fraction $S$ of the information it has stored with a pointer toward $k$. For this fraction, it is checked whether $k$ lists a shorter path than $j$. For each path where this is the case, the memory of $k$ is used to update the memory of $j$.

Notice again that the update in (iv) takes place no matter which agent had the right data. When $S = 0$, the result is as in the simple model (i)–(iii), whereas $S = 1$ leads to a hublike structure illustrated with the isolated distribution of highly connected vertices in Fig. 3(d). In between there is a critical value of $S = S_{\text{crit}} \sim 0.1$ (for $\langle C \rangle = 3$), where one obtains a scale-free distribution of vertex degree [Fig. 3(c)]. $S_{\text{crit}}$ depends on the overall edge density in the system, and increases as the average degree $\langle C \rangle$ increases. Decreasing $\langle C \rangle$ below 2.9 even $S = 0$ becomes supercritical and the central hub of a big system ($N \gg 100$) will never break down. Oppositely, it is remarkable that an increase in $C$ for fixed $S$ makes it increasingly difficult to obtain vertices with a very high $C$. In any case, at conditions when one hub dominates the topology, the hub becomes frozen and will never break down. Clearly, a scale-free degree distribution requires an instability and the possibility for vertices to change status dynamically. On the other hand, when the instability becomes too large, no large hubs develop and the degree distribution becomes exponential.

For simplicity, in Figs. 4 and 5, we consider the case of $N = 1000$, $E = 1500$, and thus $\langle C \rangle = 2E/N = 3$ with $S = S_{\text{crit}} = 0.1$. We stress that the reported results are similar for other values of $\langle C \rangle$, provided that $S$ is not too far from $S_{\text{crit}}(C)$. For example, $S_{\text{crit}}(\langle C \rangle = 2.5) = 0$ and $S_{\text{crit}}(\langle C \rangle = 5) = 0.45$. Also, it is important to stress that the particular choice of rewiring attempt and information exchange in the above model is somewhat arbitrary. Therefore we have tested robustness of the obtained results against a number of variations, including selection

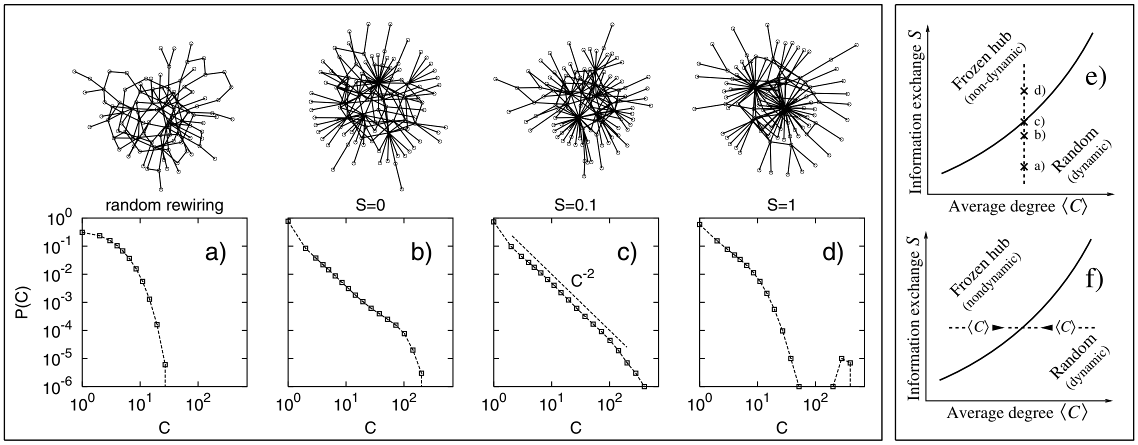



FIG. 3. Left panel: Vertex degree distribution of the evolved network with four levels of information exchange: No exchange, i.e., only rules (i) and (ii), applies in (a), full exchange as in (iii) with exchange of rate $S$ in (iv) with $S = 0$, $S = 0.1$, and $S = 1.0$ in (b)–(d). In all lower cases, we sample dynamics of an $N = 1000$ vertex system with $E = 1500$ edges ($\langle C \rangle = 3$). The plots show averages of many samples. The upper graphs show the corresponding networks of size $N = 100$. Right panel: Schematic phase diagram illustrating the critical line which separates the dynamic and nondynamic regimes. The positions of (a)–(d) are illustrated in (e) and the suggested adaptation of $\langle C \rangle$ that drives the network towards a scale-free degree distribution in (f).

of agent $i$ with weight proportional to its degree, aborting step (ii) after only one attempted $l$, and introducing information exchange between $i$ and $j$. In all cases, we are able to reproduce the qualitative features of Figs. 3–5. In particular, a higher overall edge density always requires a higher information exchange to obtain similar large hubs, as illustrated in Fig. 3(e). For any amount of information exchange, a scale-free network is obtained for the minimal $\langle C \rangle$ where the hubs remain dynamic [Fig. 3(f)].

Figure 4(a) shows the average information content related to agents of vertex degree $C$ and 4(b) the temporal development of one particular agent. In both panels, $I_{\text{of}}(i)$ is the fraction of the information $i$ has about distances and directions to all other agents that is correct. Information $I_{\text{about}}(i)$ is defined as the fraction of other agents that have correct information about their paths to $i$. The upper curve in Fig. 4(a) shows that the system systematically increases the $I_{\text{about}}(i)$ as the vertex degree of $i$ is increased. More surprisingly is the nonmonotonous behavior of $I_{\text{of}}(i)$: Agents with intermediate vertex degree $C$ know the least about the system. They are messed up by false information about directions, whereas the lowly connected agents are better informed through their typically higher connected neighbor.

Figure 4(b) follows a particular agent through a period of success, where it evolves to become one of the major hubs in the system. The figure shows both the degree of

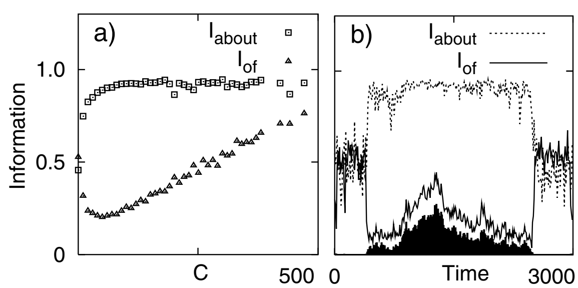



FIG. 4. (a) Average information related to agents with vertex degree $C$ for a simulation with critical information exchange. The upper curve is the fraction of agents with correct information $I_{\text{about}}$ about their paths to the specific agent of degree $C$. The lower curve similarly refers to the information $I_{\text{of}}$ the agent with degree $C$ has about paths to other agents. (b) Trajectory for a specific agent with its vertex degree (dark shaded area), the information the system has about the agent, $I_{\text{about}}$, and the information the agent has about the system, $I_{\text{of}}$. Time is counted as the number of rewiring updates per agent.

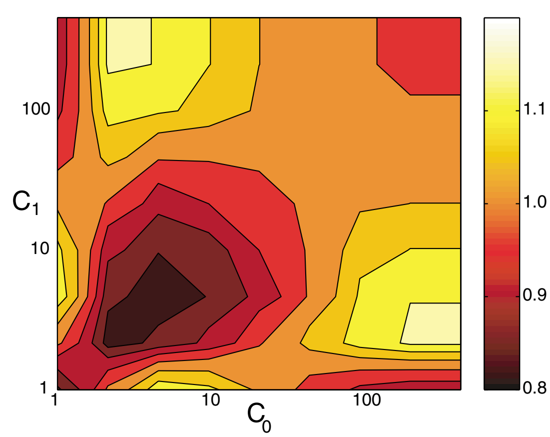



FIG. 5 (color online). Correlation profile for an ensemble of model networks with $\langle C \rangle = 3$. The correlation profile measures the probability for an edge between two vertices of degree $C_0$ and $C_1$ in units of what it would be in a properly randomized network. One notices that agents with $C \sim 1$ often connect to agents with $C \sim 5$ that preferentially connect to agents with high $C$. Thus, the network exhibits hierarchical features.

the agent and the information related to it. Notice that an initially moderate increase in degree $C$ at time $\sim$500 triggers an increase in $I_{\mathrm{about}}$ and a sharp decrease in $I_{\mathrm{of}}$. Subsequent increases in $C$ have little effect on the near perfect information that the system has about the agent, but a roughly proportional effect on the quality of the information $I_{\mathrm{of}}$. Thus, the trajectory of a particular agent again reflects the ease at which one may locate anybody in or above the "middle class," and the exclusiveness of having system-wide correct information.

To explore the connectivity pattern between low and high connected agents, in Fig. 5 we investigate the correlation profile of the evolved network [10]. This quantifies the tendency of agents with different vertex degrees to connect to each other, by normalizing to a randomized network where degrees of all vertices are exactly maintained [8]. We see that all types of connections exist, but also that there is a tendency towards hierarchical organization: Agents with $C \sim 1$ often connect to agents with degree $C \sim 5$, that preferentially connect to agents with very high $C$. This hierarchical pattern is also seen at other values of $\langle C \rangle$, with decreased amplitude as $\langle C \rangle$ is increased. Going in the opposite direction, towards decreasing $\langle C \rangle$, our standard model quickly becomes supercritical even for $S = 0$. This can be adjusted by decreasing the information transfer between $i$ and $k$ in step (iii) such that this transfer is less than complete.

It is interesting to explore the sociological implications of the proposed network dynamics, e.g., the response to increased information associated to a particular agent. If we start with an agent of degree $C = 1$ and from this instant keep it perfectly informed about the position of all other agents, $I_{\mathrm{of}}(i) = 1$, the result is insignificant. Similarly, when an agent constantly broadcasts its correct position to all other agents, that is $I_{\mathrm{about}}(i) = 1$, the agent only performs slightly better than average. However, an agent that allows all of its neighbors to update their information by using his information very quickly becomes a central hub in the system. This happens in spite of the fact that his information may be as bad as that of anybody else. Communication, not correctness, is the key to success.

Finally, we reiterate that the critical line in Fig. 3(f) corresponds to the minimal $\langle C \rangle$ where the major hub remains dynamic. This suggests a principle in which the network could self-organize to become scale free. This idea is investigated by allowing agents, at a low rate, to create and destroy edges with probabilities $P_c$ and $1 - P_c$, dependent on the dominance of the major hub. That is, we set $P_c$ to be an increasing function of the dominance of the largest hub, reflecting a situation where links are created in a persistently centralized system and removed in an unstructured system. For example, $P_c = 1 - C_2/C_1$, where $C_1$ and $C_2$ are the highest and next highest degrees in the network, results in self-organization around the critical line as in Fig. 3(f) with degree distribution $\propto 1/C^2$.

The present work suggests a dynamical model where networks with both small and large hubs emerge from local optimization of activity through guesses based on imperfect information. The frame is formulated in an agent-based model, which is comparable to a sociological setting. For static snapshots, the model predicts a hierarchical organization of vertices with the highly connected vertices in the center. This is a plausible feature of business networks and a quantifiable characteristic of the hardwired Internet [8].

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