Fast and Accurate Single-Island Charge Pump: Implementation of a Cooper Pair Pump

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We introduce a Cooper pair "sluice" for the implementation of a frequency-locked current source. The device consists of two mesoscopic SQUIDs and of a single superconducting island with a gate. We demonstrate theoretically that it is possible to obtain a current as high as 0.1 nA at better than ppm accuracy via periodically modulating both the gate charge and the effective Josephson coupling. We find that the device is tolerant against background charge noise and operates well even in a dissipative environment. The effect of the imperfect suppression of the Josephson coupling and the finite operating frequency are also investigated.

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Single-electron and Cooper pair devices have attracted considerable attention recently. Applications such as the single-electron pump [1] and the Cooper pair box for quantum computing [2] have demonstrated that at sufficiently low temperatures and high charging energies the quantization of charge leads to some very interesting effects. Especially, it has been shown that single electrons can be pumped extremely accurately at frequencies f of a few MHz with a relative uncertainty of 10^{-8} in normal metal devices according to the relation $I = ef$ [3]. This has resulted in a standard of capacitance. However, the pump frequencies, and thus current levels, have been too low for the realization of a practical accurate current source for nanoelectronic applications or for realizing the quantum measurement triangle [4]. The attempts to generalize the single-electron pump to a superconducting Cooper pair pump [5,6] that, in theory, would allow for higher-frequency pumping have been unsuccessful so far due to a variety of reasons. In particular, Landau-Zener tunneling between energy levels induces pumping errors. In addition, there is always a considerable amount of supercurrent leaking through the pump. Also, the interplay of the two conjugate variables, the phase and the number of Cooper pairs, results in a coherent correction such that the current is no longer given by the relation $I = 2ef$ [7]. Further, the coherent correction is proportional to $\cos\varphi$, where φ is the phase difference over the whole pump, whereas the supercurrent is proportional to $\sin\varphi$ rendering it impossible to choose φ to eliminate both of these simultaneously. The effect of nonidealities can be reduced by adding more junctions, but this will complicate the practical implementation due to the increasing number of control parameters and cross capacitances. Furthermore, one has to take into account the effect of the fluctuating background charges responsible for the $1/f$ noise and the phase fluctuations caused by the electromagnetic environment. The latter, however, may help in achieving $\langle\cos\varphi\rangle = \langle\sin\varphi\rangle = 0$ if desired.

In this Letter we propose and critically analyze a simplified scenario for implementing a Cooper pair sluice that ideally has no dynamical supercurrent leaking through the junctions and is governed by the relation $I = 2ef$ or more generally $I = 2nef$, where n is the number of pairs carried per cycle. First, we present the general idea of the device. We also study the viability of implementing the device by considering different sources of error and show that the sluice is tolerant against several kinds of nonidealities. We demonstrate that it is possible to construct a frequency-locked current source that has, with realistic assumptions, a yield of 0.1–0.2 nA with better than 1 ppm error.

The device consists of just one superconducting island that works as the sluice chamber and of two mesoscopic SQUIDs; see Fig. 1. The role of the SQUID loops is to serve as the sluice doors for the flow of Cooper pairs. The control parameters which are varied periodically and adiabatically include the gate voltage V_g and the magnetic fluxes Φ_a (a = l, r) through the SQUID loops. The idea of controlling the effective Josephson coupling is used throughout in the Josephson qubit literature; see, e.g., Ref. [2]. Utilizing flux pulses in Cooper pair shuttles [8] has also been suggested in Ref. [9] but in a nonadiabatic context. Here we work in the adiabatic limit. Note that the device is particularly simple; there is only one voltage

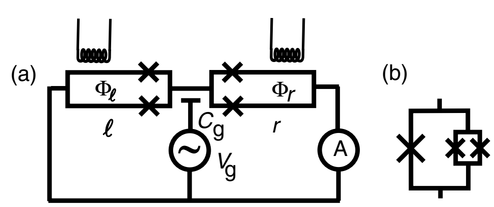

FIG. 1. (a) Schematic illustration of the device, the "sluice." The role of the coils is to apply controlled flux pulses through the SQUID loops, and they are synchronized with the periodic gate voltage. (b) An improved three-junction SQUID.

gate to adjust. The current through the sluice is given by the time integral of the expectation value of the current operator of either of the two SQUIDs. The dynamics is governed by the Schrödinger equation and the Hamiltonian of the device is (in the case of identical junctions)

$$\hat{H} = \frac{2e^2}{2C_J + C_g}(\hat{n} - n_g)^2 - E_J^r\left(\pi\frac{\Phi_r}{\Phi_0}\right)\cos(\phi + \varphi/2) - E_J^l\left(\pi\frac{\Phi_l}{\Phi_0}\right)\cos(\varphi/2 - \phi). \tag{1}$$

Here $C_J/2$ is the capacitance of a single junction, C_g is the capacitance of the gate, $n_g = C_g V_g/2e$ is the gate charge in $2e$ units, $\Phi_0 = h/2e$, and φ is the phase difference over the sluice. Furthermore, $E_J^a(\pi\frac{\Phi_a}{\Phi_0}) = E_J^{max}\cos(\pi\frac{\Phi_a}{\Phi_0})$ ($a = l, r$) denotes the effective flux-dependent signed Josephson energy of the left and the right SQUID, respectively. The Josephson energy of a single junction is thus $E_J^{max}/2$. The factor $E_C \equiv (2e^2)/(2C_J + C_g)$ is the charging energy. The quantum mechanical conjugate variables are the number of Cooper pairs on the island $\hat{n}$ and the superconducting phase ϕ. They obey the canonical commutation relation $[\hat{n}, \phi] = i$. The case of nonidentical junctions is modeled below by not allowing the Josephson energy to vanish during the cycle. We note that it is possible to use more complicated SQUIDs [see Fig. 1(b)] for which one of the junctions is replaced by a SQUID biased with a static field to match the E_J of the other half when $\Phi_0/2$ threads the primary loop. Self-inductance may be ignored for two junctions (other sources of error dominate) but for the three-junction design the self-inductance sets a limit for suppression at $\pi L I_C/\Phi_0$ where $I_C = 2\pi E_J/\Phi_0$. An achievable value for this could be 10^{-3}. The current operator of the, say, right SQUID is

$$I^r = \frac{2e}{\hbar}E_J^r\left(\pi\frac{\Phi_r}{\Phi_0}\right)\sin(\phi + \varphi/2). \tag{2}$$

The total charge flowing through the system over one cycle has two components in the adiabatic limit [7], namely, the contribution from the dynamical supercurrent

$$Q_s = \int_0^{t_{cycle}} \langle 0; \mathbf{q}(t)|I^r|0; \mathbf{q}(t)\rangle dt, \tag{3}$$

and the pumped charge (γ is the loop in parameter space)

$$Q_p = 2\hbar\,\mathrm{Im}\left[\sum_{n\neq 0}\oint_\gamma \frac{\langle 0; \mathbf{q}|I^r|n; \mathbf{q}\rangle}{E_0(\mathbf{q}) - E_n(\mathbf{q})}\langle n; \mathbf{q}|\nabla_{\mathbf{q}}|0; \mathbf{q}\rangle \cdot d\mathbf{q}\right]. \tag{4}$$

We have denoted above the control parameters collectively by the vector $\mathbf{q}$ which is varied in time. In the present context $\mathbf{q} = (n_g, E_J^r, E_J^l)^T$. Because of the adiabaticity criterion, the sluice stays at all times in the ground state with negligible Zener tunneling. The nth eigenstate at the point $\mathbf{q}$ is denoted by $|n; \mathbf{q}\rangle$ and the energy eigenvalue by $E_n(\mathbf{q})$.

Figure 2 illustrates a model control-parameter sequence. Note that the SQUIDs are biased in such a manner that one door is always closed, such that the dynamical contribution of Eq. (3) vanishes. Moreover, the signal is designed such that the system Hamiltonian (1) is always nondegenerate. This validates the use of Eq. (4). Varying just the gate voltage would lead to a degeneracy at $n_g = 0.5$, but because just one of the doors is open at this point, the problem is resolved. The sluice is ideally a switchable Cooper pair box. During the first half of the sequence one of the SQUIDs works as a Josephson junction while the other is effectively a capacitor. Then the roles are exchanged. It is easy to see that this sequence leads to the transport of exactly one Cooper pair through the sluice per cycle. In the beginning of the sequence the system is in the eigenstate of charge (zero pairs) due to the fact that the effective Josephson couplings are set to zero. In the middle of the sequence when both doors are again closed, the island is in the eigenstate of charge but now with one extra Cooper pair. The Cooper pair has tunneled through the right SQUID since the left one was closed. Finally, in the end of pulse the system is again at the eigenstate of charge with zero Cooper pairs and the charge must have gone through the left SQUID. Repeating this sequence results in $I = 2ef$, where $f = 1/t_{cycle}$. The form of the pulse may also be generalized for the purpose of allowing n Cooper pairs to flow through the sluice over t_{cycle}, thus increasing the current to $I = 2nef$, simply by operating between $n_g = 0$ and $n_g = n$.

Assuming that the SQUIDs can be closed to a high degree renders the system almost entirely insensitive to the actual operating point of voltage. Instead of operating

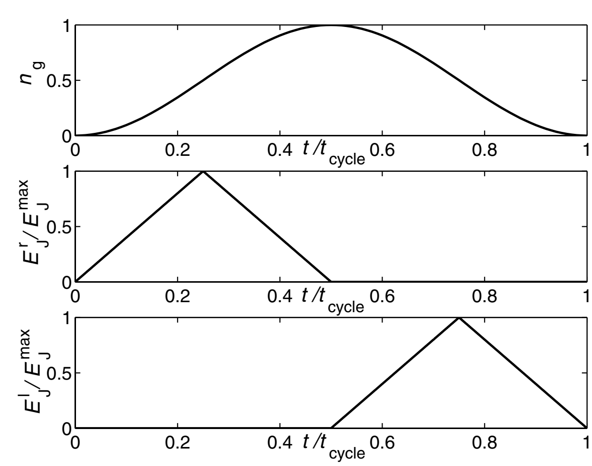

FIG. 2. Pulse sequence for pumping a single Cooper pair through the sluice. The exact form of the pulses is not crucial as long as the synchronization is maintained. The gate charge (or voltage) pulse, which is a shifted harmonic one here, may be generalized to have a larger amplitude and thus a larger number of pairs could be pumped.

between $n_g = 0$ and $n_g = 1$ (or $n_g = 0$ and $n_g = n$) we may just as well operate between $n_g = \delta$ and $n_g = \delta + 1$ (or $n_g = \delta$ and $n_g = n + \delta$) as long as $\delta \neq \frac{1}{2}$. However, the adiabaticity criterion becomes harder to fulfill if we start close to the degeneracy point. Considering that a typical measured power spectrum of the background $1/f$ charge noise is $S(f) = 10^{-8}e^2/f$ [2,10], there will be a need to reconfigure the sluice only after time scales of hours. This is a definite strength of the present approach and it is attributable to the use of the controllable SQUIDs. It should be emphasized that the exact shape of the pulses is not crucially important as long as the maxima and minima are synchronized as in Fig. 2. Even though we consider imperfections in suppressing E_J below, the effect of flux noise still needs to be studied in an experiment.

Let us comment on the maximum operating frequency of the device. Because of imperfections in the flux control and nonidentical Josephson junctions, there is always some residual E_J^{res}. This implies that one should have $E_J^{max} < E_C$ to avoid excess leakage and to make the sluice insensitive to background charge fluctuations. Furthermore, since the minimum gap in the energy spectrum of the sluice is roughly E_J^{max} whenever $E_J^{max} \leq E_C$ holds, one should have $hf \ll E_J^{max}$. It is often asserted that one should also have $E_C \ll \Delta_{BCS}$ in order to avoid quasiparticle effects. It follows that there would be an inequality chain $hf \ll E_J^{max} < E_C \ll \Delta_{BCS}$ which seriously limits the operation frequency of the device. However, it suffices to have

$$hf \ll E_J^{max} \approx E_C \lesssim \Delta_{BCS} \tag{5}$$

in the present context. Namely, the criterion $E_C \ll \Delta_{BCS}$ is now superfluous because, assuming adiabaticity, the sluice is never in its excited state. That is, it is sufficient to have Δ_{BCS} such that the second band [11] is just slightly below the lowest quasiparticle state which cannot be excited due to adiabaticity. In the case of nonadiabatic evolution $E_C \ll \Delta_{BCS}$ is, of course, necessary whenever we consider exciting the system, as in the case of the Josephson charge qubit [2]. We can also set $E_J^{max} \approx E_C$ in Eq. (5) and still get satisfactory performance as we show below.

We proceed to present numerical results obtained by integrating the Schrödinger equation corresponding to the Hamiltonian Eq. (1) over discrete time steps. The pumped charge was then obtained by numerically integrating the time-dependent expectation value of the current operator in Eq. (2). This nonadiabatic method reveals the effect of the finite operating frequency. We also estimate the effect of several kinds of nonidealities. We choose for the rest of the paper the typical parameters $C_J = C$, $C_g = 0.1C$, and $E_J^{max} = e^2/C$ such that $E_J^{max} \approx E_C$. Integrating the system at varying frequencies results in the pumped charge illustrated in Fig. 3. The path of

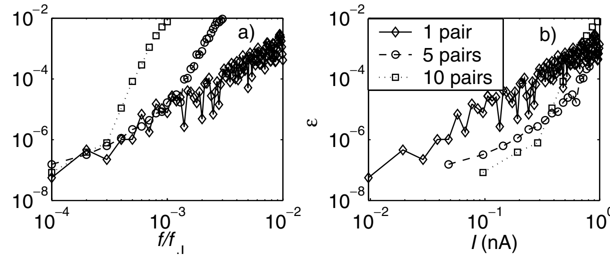

FIG. 3. Error (a) in the pumped charge over a single period and (b) in the current as a function of (a) frequency and (b) current. Here $C_J = C$, $C_g = 0.1C$, $E_J^{max} = e^2/C$, and $f_J = E_J^{max}/\hbar$. The error is $\varepsilon \equiv 1 - Q_P/2ne \equiv \Delta I/I$. The line marked by diamonds represents pumping a single Cooper pair, the line marked by circles represents pumping five Cooper pairs, whereas the squared line represents pumping ten Cooper pairs per cycle. In (b) we assume $f_J = 300 \times 10^9$ s^{-1}.

integration is the ideal sequence of Fig. 2. In light of Fig. 3, it seems that we could quite safely pump single Cooper pairs at the frequency $f = E_J^{max}/\hbar \times 10^{-3}$ and still have an accuracy of 7 ppm. Fabricating the island and the leads out of aluminum is the most viable option for the present, and by standard lithography one obtains $C < 10^{-15}$ fF. The well known BCS gap would be roughly $\Delta_{BCS}/h \approx 50$ GHz. Choosing the charging energy optimally, that is, $E_C \lesssim \Delta_{BCS}$, results in an operating frequency of some 300 MHz and a current of about 0.1 nA. However, Fig. 3 also illustrates the adiabaticity error for pumping five Cooper pairs; that is, the gate charge pulse has an amplitude of $C_g\Delta V_g/2e = 5$. When this is converted to current, we conclude that it may be possible to pump 0.2 nA with better than 1 ppm error. The result of pumping altogether ten Cooper pairs per cycle is also shown, and it turns out that a current of about 0.1 nA at 0.1 ppm error is possible. Ramps of the Josephson energy cause adiabaticity errors and, in comparison, varying the gate voltage does not contribute as much at least when pumping only a few Cooper pairs. The optimum number of pairs per cycle is yet an open question which we have not solved due to numerical difficulties.

The quantitative effect of background charge and the residual value of E_J, E_J^{res}, is illustrated in Fig. 4. We calculated the actual pumped charge, in the case of a single attempted Cooper pair in Fig. 4(a), over one cycle as a function of the gate charge deviation δ and E_J^{res}. The result has been averaged over different evenly spaced phase bias values, namely, $\varphi = \pi/2$, π, $3\pi/2$, and 2π (for justification see below). The frequency was $f = E_J^{max}/\hbar \times 10^{-4}$ which corresponds roughly to 0.1 nA. The performance of the sluice degrades rapidly with increasing E_J^{res} at fixed phase bias values. However, a physical sample would always be subject to some phase fluctuations. Keeping the phase constant over one cycle, as done above, is a realistic assumption if the dephasing time is long compared to t_{cycle}. We see that the error

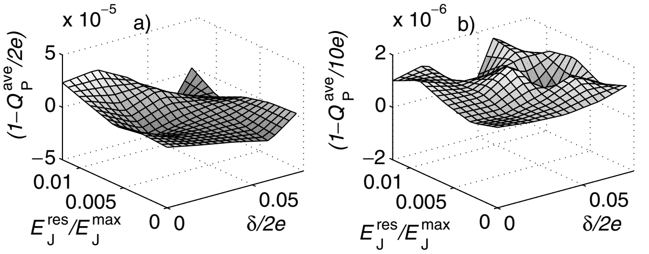

FIG. 4. (a) Averaged pumping error over the phase bias values $\varphi = \pi/2$, π, $3\pi/2$, and 2π as a function of δ and E_J^{res} at $f = E_J^{max}/\hbar \times 10^{-3} (\approx 300$ MHz). (b) The same as (a) but for pumping five Cooper pairs at $f = 4E_J^{max}/\hbar \times 10^{-4} \approx$ 120 MHz which corresponds to $I \approx 0.2$ nA.

averages out to a great accuracy even though the deviation from the ideal point (i.e., $E_J^{res}/E_J^{max} = 0$ and $\delta = 0$) is quite large. Note that the span of gate charge is some 10% and the span of the residual E_J^{res} some 1%. Achieving even $E_J^{res}/E_J^{max} = 10^{-3}$ should be possible with the design of Fig. 1(b). A similar calculation for pumping five Cooper pairs was also performed at a frequency corresponding to $I \approx 0.2$ nA, and the averaged result is shown in Fig. 4(b). A yield of at least 0.2 nA is possible even in the presence of nonidealities with a relative error of some 10^{-6}.

It is easy to see why phase averaging suppresses the errors when $E_J^{res} \neq 0$ as suggested by Fig. 4. Namely, the supercurrent is proportional to $E_J^r E_J^l \sin\varphi$ [12] and the average of this is clearly zero. It is identically zero whenever one of the sluice doors is closed. We obtain in a two-state adiabatic approximation a perturbative formula in E_J^{res} (for pumping a single Cooper pair)

$$\frac{Q_P}{2e} \approx 1 - \frac{2\sqrt{(E_J^{max})^2 + E_C^2}}{E_J^{max} E_C} E_J^{res} \cos\varphi \qquad (6)$$

such that we may confirm that the error is proportional to $\cos\varphi$ as in the conventional pump [7]. We have utilized the fact that $Q_P = -2e \frac{d}{d\varphi} \gamma$, where γ is the Berry phase associated with the adiabatic loop [13]. The effect of δ on the performance of the sluice is negligible compared to the effect of nonzero E_J^{res} with fixed φ. Phase averaging, i.e., placing the sluice in a dissipative environment, may be used to cancel the effect of small nonidealities. Figure 4 clearly indicates that the sluice is quite insensitive to background charge fluctuations.

We assumed that choosing the phases evenly is a representative sample of the whole. Over time scales of seconds one may consider the phase to be evenly distributed between 0 and 2π due to dissipation. The even distribution is asymptotically identical to a wide Gaussian distribution on the whole real axis. The Gaussian nature can be justified by assuming a thermal bath of harmonic oscillators coupled to the phase with a sufficiently high effective impedance. The variance of the phase increases with the real part of the impedance seen by the device due to the fluctuation-dissipation theorem. Thus $\langle \exp(\pm i\varphi)\rangle = \exp(\pm i\langle\varphi\rangle - \langle\Delta\varphi^2\rangle/2)$ decays exponentially as do the pumping errors. Phase averaging has been used in the R-pump scenarios [6] by inserting large series resistors. At high currents this leads inevitably to overheating. In the present context the phase averaging is needed only as a second order mechanism since most of the errors are suppressed by the controlled modulation of the Josephson coupling. Finally, we comment on the effect of the ammeter. An ammeter with high R can cause a significant voltage over the sluice. A good choice would be a cryogenic current comparator modeled by L and C in parallel. With, e.g., $L = 10$ H, $C = 1$ nF, and $E_J^{max} \approx E_C$ we would get $V(t) \approx V_0 \sin(2\pi f t)$ with $V_0 \approx e/C\pi \approx 50$ pV which is negligible.

To conclude, we have introduced and analyzed an idea of a Cooper pair sluice with just three control parameters. Compared to other Cooper pair pumping scenarios, we have suppressed undesired cotunneling, supercurrent leakage, and, most importantly, the need to have a long error-prone array of junctions with numerous gates. The idea for the control of the sluice is similar to the control of Josephson junction qubits. The sluice is much simpler, though, since superpositions and entanglement are not pursued and relatively slow pulses are sufficient.

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