Connection between the Elastic G_{Ep}/G_{Mp} and $P \to \Delta$ Form Factors

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It is suggested that the falloff in Q^2 of the $P \to \Delta$ magnetic form factor G_M^* is related to the recently observed falloff of the elastic electric form factor G_{Ep}/G_{Mp}. Calculation is carried out in the framework of a generalized parton distribution model whose parameters are determined by fitting the elastic form factors F_{1p} and F_{2p} and isospin symmetry. When applied to the $P \to \Delta$ transition with no additional parameters, the shape of G_M^* is found to exhibit the requisite falloff with Q^2.

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The $P \to \Delta(1232)$ form factor G_M^* exhibits a more rapid decrease with respect to Q^2 than is typically observed in other baryons [1,2] such as G_{Mp} in elastic scattering from a proton, or $A_{1/2}$ in the transition $P \to S_{11}(1535)$. A recent Jefferson Laboratory (JLab) measurement [3] finds that the ratio G_{Ep}/G_{Mp} for elastic scattering falls with Q^2 more rapidly than previously expected. This has given rise to much theoretical activity [4,5] to attempt to understand the underlying physics. In this Letter, it is suggested that this behavior in G_{Ep}/G_{Mp} is related to that of G_M^*.

As a basis, it is assumed that the form factor is dominated by soft mechanisms, and a generalized parton distribution (GPD)-handbag approach [6–8] is utilized. Form factors are the zeroth moments of the GPDs with skewedness $\xi = 0$. For elastic scattering from a proton, with $t = -Q^2$, the Dirac and Pauli form factors are written

$$F_{1p}(t) = \int_{-1}^{1} \sum_q e_q H_p^q(x, \xi, t)\, dx, \tag{1}$$

$$F_{2p}(t) = \int_{-1}^{1} \sum_q e_q E_p^q(x, \xi, t)\, dx, \tag{2}$$

where q signifies quark flavors, and for brevity the GPDs are denoted $H_p^q(x, t) \equiv H_p^q(x, 0, t)$ and $E_p^q(x, t) \equiv E_p^q(x, 0, t)$.

A similar relation holds for neutrons.

Resonance transition form factors access components of the GPDs which are not accessed in elastic scattering. The $N \to \Delta$ form factors, in the large N_c limit, are related to isovector components of the GPDs [9,10]:

$$2G_M^* = \int_{-1}^{1} H_M(x, t)\, dx, \tag{3}$$

$$2G_E^* = \int_{-1}^{1} H_E(x, t)\, dx, \tag{4}$$

$$2G_C^* = \int_{-1}^{1} H_C(x, t)\, dx, \tag{5}$$

where G_M^*, G_E^*, and G_C^* are magnetic, electric, and Coulomb transition form factors [11], and H_M, H_E, and H_C are the respective isovector GPDs. Analogous relationships can be obtained for the $N \to S_{11}$ and other transitions. Here, the connection between GPDs involved in the elastic and $N \to \Delta$ form factors is explored to obtain the connection between the t dependence of the G_{Ep} and G_M^*.

In Refs. [9,10], it is noted that, in the large N_c limit, assuming chiral and isospin symmetry the GPDs for the $P \to \Delta(1232)$ transition are expected to be isovector components of the elastic GPDs, given by

$$H_{Mp} = \frac{2}{\sqrt{3}} E_M^{(\mathrm{IV})} = \frac{2}{\sqrt{3}} (E_p^u - E_p^d). \tag{6}$$

where IV denotes isovector, and E_p^u and E_p^d are the GPDs for the proton elastic u and d quarks, respectively. Thus, the $P \to \Delta$ form factor should be obtainable by analysis of the Pauli form factor F_{2p} [Eq. (2)]. The Dirac and Pauli form factors, F_{1p} and F_{2p}, are related to the measured Sachs form factors G_{Mp} and G_{Ep} by

$$F_{1P} = \frac{1}{\tau + 1} (\tau G_{Mp} + G_{Ep}), \tag{7}$$

$$F_{2P} = \frac{1}{\kappa(\tau + 1)} (G_{Mp} - G_{Ep}), \tag{8}$$

with $\tau = -t/4M_p$. To obtain E_p^u and E_p^d needed for Eq. (6), the available data for G_{Mp} and the recent JLab data [3] on G_{Ep}/G_{Mp} were fit, as reported in Ref. [12], using a parametrization of the GPDs such as in [13–16].

The specific functional form for $H_p^q(x, t)$ and $E_p^q(x, t)$ is a Gaussian plus small power law shape in $-t$ to account for the high measured form factors at very high $-t(\equiv Q^2)$.

$$H_P^q(x, t) = f_p^q(x) \exp(\bar{x}t/4x\lambda_H^2) + \cdots \tag{9}$$

$$E_P^q(x, t) = k_p^q(x) \exp(\bar{x}t/4x\lambda_E^2) + \cdots, \tag{10}$$

in which $\bar{x} \equiv 1 - x$ and $\cdots$ indicates the addition of small power components added in Ref. [12]. The conditions at $t = 0$ are $H_p(x, 0) = e_u f_p^u(x) + e_d f_p^d(x)$ and $E_p(x, 0) =$

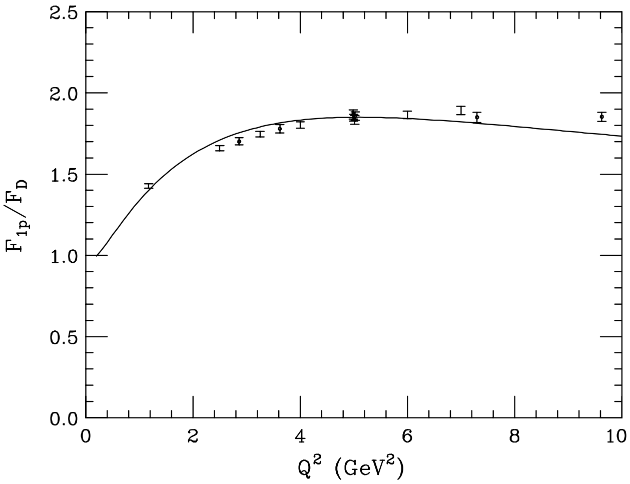

FIG. 1. The Dirac form factor $F_{1p}(Q^2)$ relative to the dipole $G_D = 1/(1 + Q^2/0.71)^2$. The data are extracted using the recent JLab data [3] for G_{Ep}/G_{Mp}, and a recent reevaluation [19] of SLAC data of G_{Mp} [20,21]. The curve is the result of the fit as discussed in the text.

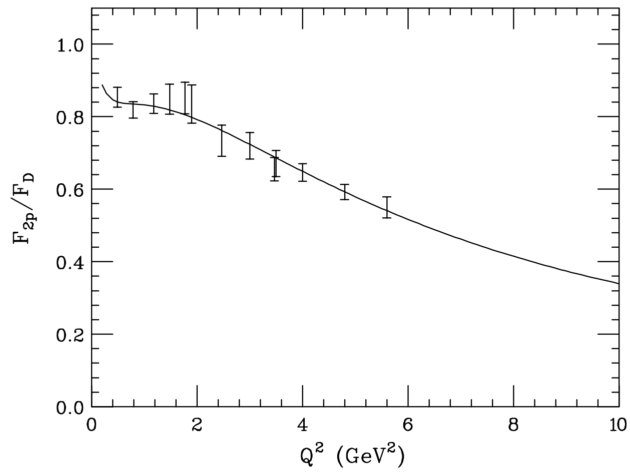

FIG. 2. The Pauli form factor $F_2/1.79F_D$ relative to the dipole $F_D = 1/(1 + Q^2/0.71)^2$. The data are extracted using the recent JLab data [3] for F_{2p}/F_{1p}, multiplied by the fit curve for F_{1p}/F_D shown in Fig. 1. The curve is the result of the simultaneous fit to the G_{Ep}/G_{Mp} and G_{Mp} data as discussed in the text and Fig. 1.

$e_u k_p^u(x) + e_d k_p^d(x)$. Here, $f_p^u(x)$ and $f_p^d(x)$ are proton u and d valence quark distribution functions evaluated from deep inelastic scattering (DIS) [13,17]. The functions $k_p^u(x)$ and $k_p^d(x)$ are not obtainable from evaluations of DIS. In Refs. [15,18], the assumptions were made that $k_p^q(x) = c(x, k_\perp)f_p^q(x)$, where $c(x, k_\perp)$ are phenomenologically chosen to yield the t dependence of the ratios F_{2p}/F_{1p} constrained by recent experimental data from JLab [3]. For the present purposes $k_p^q(x) \propto \sqrt{1-x} f_p^q(x)$ was used. This results in a satisfactory ratio of F_{2p}/F_{1p}, since for large $-t$ the quantity $\sqrt{1-x} \to 1/\sqrt{-t} = 1/Q$. The normalizations were obtained by requiring the proton and neutron form factors to have their known values near $Q^2 = 0$, that is, $F_{1p}(0) = 1$, $F_{1n}(0) = 0$, $F_{2p}(0) = 1.79$, $F_{2n}(0) = -1.91$, and to obey isospin symmetry. Thus,

$$F_{1p}(0) = 2e_u + 1e_d = 1, \qquad F_{1n}(0) = 1e_u + 2e_d = 0,$$

with

$$\int f_u(x)\,dx = 2, \qquad \int f_d(x)\,dx = 1,$$

and

$$F_{2p}(0) = e_u\kappa^u + e_d\kappa^d = 1.79,$$

$$F_{2n}(0) = e_u\kappa^d + e_d\kappa^u = -1.91,$$

with

$$\kappa^u = \int k^u(x)\,dx = 1.67, \qquad \kappa^d = \int k^d(x)\,dx = -2.03.$$

Adequate fits to G_{Mp} and G_{Ep}/G_{Mp}, or equivalently F_{1p} and F_{2p}/F_{1p}, were obtained with $\lambda_H = 0.76$ GeV/c and $\lambda_E = 0.67$ GeV/c. The results are shown in Figs. 1 and 2.

The resulting E_p^u and E_p^d were inserted into Eq. (6) to obtain an estimate for G_M^*. At $Q^2 = 0$, one gets $G_M^*(0) = 2.14$, which is somewhat lower than the experimental value of $G_M^*(0) \sim 3$. Such a disagreement is not surprising [9,10] given the very approximate nature of Eq. (6). The obtained G_M^* was overall renormalized to take this ratio into account, and the result is shown in Fig. 3.

The similar shapes of the curves in Figs. 2 and 3 can be ascribed to their connection via Eq. (6). This can be understood by the observation that F_2 is nearly all isovector spin flip, as is the G_M^*. However, the inherent approximate nature of the $1/N_c$ expansion, and the fact that F_1 also has an isovector component would make the

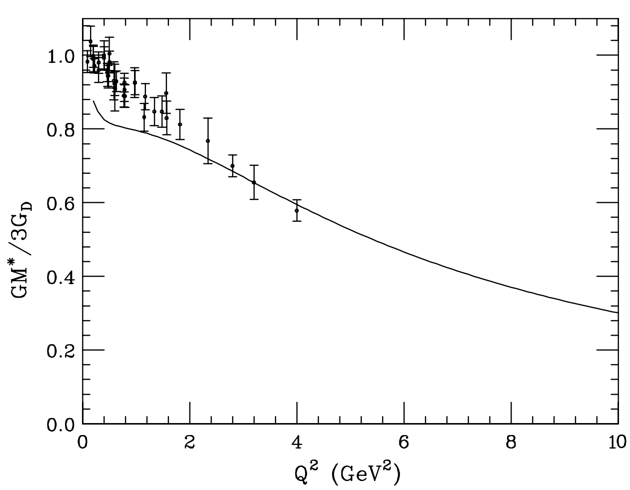

FIG. 3. The $N \to \Delta$ magnetic form factor $G_M^*(Q^2)/3G_D$ relative to the dipole $G_D = 1/(1 + Q^2/0.71)^2$. The data are a compendium of world data by Ref. [22]. The curve is the result of the procedures discussed in the text.

observed non-negligible differences in the normalization not surprising.

Although this note suggests a common physical origin in the Q^2 behavior of G_{Ep}/G_{Mp} and G_M^*, a complete understanding will require theoretical treatments based on rigorous and consistent relativistic treatment which are beyond the scope of this communication.

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