Quantum Measurement of the Degree of Polarization of a Light Beam

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We demonstrate a coherent quantum measurement for the determination of the degree of polarization (DOP). This method allows us to measure the DOP in the presence of fast polarization state fluctuations, difficult to achieve with the typically used polarimetric technique. A good precision of the DOP measurements is obtained using eight type II nonlinear crystals assembled for spatial walk-off compensation.

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The history of the concept of polarization of light is fascinating and very instructive of the way science progresses; see, e.g., [1]. Today, there is a renewed interest because of the fast developments in optics, both on the applied side for optical communication and on the more academic side for quantum optics. In this Letter we concentrate on the *degree of polarization* (DOP) which is often desired to reach its maximum value of 1, as well for close-to-ideal classical as for quantum communication [2]. We analyze this problem from a quantum perspective, and then apply the gained insight to an experimental measurement of the DOP using classical nonlinear optics.

It is well known that depolarization is due to decoherence. A light beam can be (partially) depolarized (DOP < 1) for any combination of three basic causes: mixture of spatial modes with different polarization, mixture of temporal modes with different polarization, and mixture of spectral modes with different polarization.

Clearly, light propagating in a single-mode fiber cannot suffer from depolarization due to the first cause. Moreover, one is often not interested in depolarization due to time fluctuations (see, e.g., the discussion below about polarization mode dispersion). Consequently, one would like a measurement technique providing information on the *instantaneous* DOP of a single-mode light beam. Note that "instantaneous" does not refer to an infinitesimal time interval — for which polarization is not even defined — but to the coherence time of the signal. Measuring the instantaneous DOP is a nontrivial task, since classical polarimeters measure the four Stokes parameters and then compute the DOP. In other words, the usual measurement technique is an indirect one, necessarily requiring some time to average the intensities on the four detectors providing the Stokes parameters. Let us look at this problem from a fundamental point of view, considering the quantum nature of light. If one has only a single photon at disposal and measures its polarization along any (linear or elliptical) direction, one obtains one out of two possible results. It is easy to convince oneself (and this can be made rigorous [3]) that this single result provides absolutely no information on the DOP (not even probabilistic information, i.e., it does not help at all to guess the correct DOP) of the beam from which this photon was extracted. It is only by accumulating several results on photons from the same beam that one can gain some information. But accumulating results necessarily takes some time, hence possibly the DOP measurement gets spoiled by time fluctuations of the state of polarization. Note that classical linear optics does nothing else than accumulating measurement results on individual photons, thus measuring the DOP in an indirect way. Consequently, the only possibility to improve DOP measurements consists in processing the photons in pairs (or triplets, etc.), i.e., accessing directly the DOP.

From quantum information theory we learned in recent years that *coherent measurements*, that is measurements represented by self-adjoint operators whose eigenstates are entangled, do indeed generally provide more information than successive individual measurements [4]. This came as a surprise, since it applies also to the case where the measured systems are not entangled, as for the case under investigation: the photons of a classical light beam are not entangled, but coherent measurements do provide more information. For DOP measurement [5], the optimal coherent quantum measurement is represented by the operator *projection on the singlet state*:

$$P_{\text{singlet}} = \frac{1}{2}(|H, V\rangle - |V, H\rangle)(\langle H, V| - \langle V, H|). \qquad (1)$$

This can be understood intuitively. If light is perfectly polarized, DOP $= 1$, then all photons are in the same polarization state. Consequently, the projection of any pair of photons on the singlet state is zero (recall that the singlet state is rotationally invariant). But if the DOP is less than unity, then there is a finite probability that a pair of photon projects during a measurement process onto the singlet state. Let us make this quantitative. Let $\{S_j\}_{j=0,1,2,3}$ denote the Stokes parameters. The polarization vector $\vec{M}$ on the Poincaré sphere is then $M_j = (S_j/S_0)$, $j = 1, 2, 3$, and the quantum state of polarization is represented by the density matrix $\rho = (\mathbb{1} + \vec{M}\,\vec{\sigma})/2$, where $\vec{\sigma}$ are the Pauli matrices. The DOP is related to the

Poincaré vector by $DOP = |\vec{M}|$. Accordingly, the probability that a pair of photons from a classical light beam of polarization $\vec{M}$ gets projected onto the singlet state reads:

$$\text{Prob(singlet)} = \text{Tr}(\rho \otimes \rho \cdot P_{\text{singlet}}) \tag{2}$$

$$= \frac{1 - \vec{M}^2}{4} = \frac{1 - \text{DOP}^2}{4}. \tag{3}$$

The coherent quantum measurement "projection onto the singlet state" provides thus a *direct access* to the DOP. In section II we present a measurement setup, inspired by quantum optics experiments (projection onto the singlet state is useful, among others, for the fascinating demonstration of quantum teleportation [6]), but extended into the classical domain using nonlinear optics. However, before this we would like to present an example where a direct and fast DOP measurement is of great practical value.

Polarization mode dispersion (PMD) is presently one of the main limitations to high bit-rate fiber optics communication [7]. Consequently, the telecom industry aims at developing compensators. This road has been taken successfully to fight against chromatic dispersion. However, contrary to chromatic dispersion, PMD is a statistical quantity which fluctuates on various time scales, down to microseconds in the worst case. Hence, any PMD compensator needs a fast feedback parameter. Ideally, this parameter should be the bit error rate (BER). However, today's BER specifications of 10^{-9}, or even 10^{-12}, impose much too long measurement times, even at bit rates of tens of gigabits per second. An often proposed alternative to the BER as feedback parameter is the DOP [8]. Indeed, when PMD affects the transmission of light pulses, then, in first order, one part of the pulse travels slightly faster than the other, though they do still overlap. Hence, the DOP during this overlap is the desired feedback parameter. Clearly, in this case the depolarization is never due to mixtures of spatial modes and the time fluctuations, e.g., from one pulse to another, do not represent the physical quantity of interest. This is a clear example where a direct and fast measurement of the DOP is needed. In the frequency domain PMD can be understood as follows. The light fields contains three dominant optical frequencies, the carrier and the carrier $\pm$ the modulation frequency. Each of these wavelengths undergo slightly different polarization evolutions, hence the depolarization of interest is clearly due to the third cause listed in the introduction. For frequency modulations from giga- to terabits per second, the wavelengths differences range from 8 pm to 8 nm.

Experimental setup.—The experimental implementation of the projection onto the singlet state measurement is presented in Fig. 1. The idea is to coherently combine two stages of parametric up-conversion, using χ^2 type II nonlinear crystals. In the first stage, the phase matching is

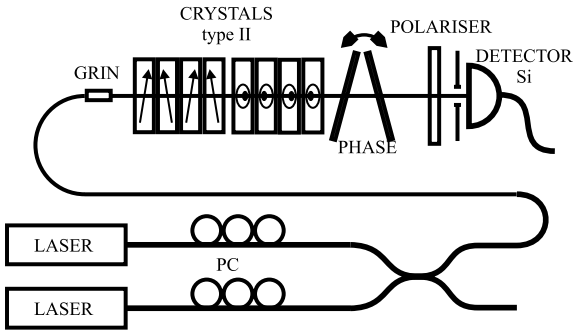

FIG. 1. Diagram of the setup. The two walk-off compensated stages of four nonlinear crystals are turned by 90° with respect to each other. PC: polarization controller; GRIN: graded-index lens.

such that a photon from the shorter range of the spectrum and one from the longer range are upconverted to a photon in a horizontal polarization state. The second stage is rotated by 90°, and consequently, the upconverted photon is vertically polarized. The upconverted photons then pass a linear polarizer at 45°, which erases the information where they were created. Depending on the phase between the two stages, controlled by tilting two birefringent plates, the overall intensity of the upconverted signal corresponds to the desired "singlet fraction," and is consequently a measure for the DOP [Eq. (3)]. Note that the probability for up-conversion is important during a time interval given by the coherence time of the pump photons (position uncertainty). This means that the signal amplitude at a given moment comes from pump fields averaged over their coherence time. According to this "response time" of the nonlinear interaction, the outcome of our DOP-meter is the instantaneous DOP as defined in the introduction.

A preliminary investigation using only two, orthogonally orientated crystals [9] showed that an undesired phase-matching condition coexists for photons with little wavelength difference. For example, in the same crystal, the two nonlinear interactions $[(H_1, V_2) \rightarrow H_3]$ and $[(V_1, H_2) \rightarrow H_3]$ are possible. This poses a serious limitation to the scheme. The wavelength separation under which this detrimental phenomenon appears is determined by the phasematching acceptance of the crystal. Hence, the narrower the wavelength acceptance of the nonlinear crystals, the better, contrary to the typical use of such crystals. To reduce the wavelength acceptance, we can use longer crystals or choose materials having better characteristics. Promising candidates as GaSe, HgS (Cinnabar), or Banana are, however, hard to fabricate or difficult to manipulate. We therefore decided to stay with KTP, but to increase the crystal length. This leads to a spatial walk-off problem, limiting the effective length for SFG to well below the physical crystal length. Usually this is dealt with by adding linear birefringent crystals for compensation. Here, we compensate the walk-off using a

second nonlinear birefringent crystal. As is described in [10], two identical nonlinear crystals are combined so that their walk-off angles are opposite and the waves generated in both are in-phase. To realize the desired effective length, we use stages consisting of four KTP crystals each, hence our setup contained eventually eight nonlinear crystals in series. This is an interesting result in itself, since recently many experiments presented configurations using just pairs of nonlinear crystals [11].

A structure of four 3 mm KTP elements gives an effective length of almost 12 mm, thereby reducing the wavelength acceptance by four compared to a 3 mm crystal as used in [9]. The expected wavelength phase-matching acceptance becomes 4.5 nm, making it possible to realize a projection onto the singlet state for wavelengths separated by ~1.5 nm only. Notice that the spatial walk-off is totally compensated for, so contrary to normal crystals, the spatial modes of λ_1 and λ_2 are as well overlapped before the second stage as before the first one. This favorizes both identical conversion efficiencies in both stages and a better spatial overlap of the created waves.

Results.—Here we demonstrate the performance of our projection on the singlet state with the eight KTP crystals. To test the setup, we use a source composed of two lasers, one at the wavelength λ_1 and the other at λ_2 [Fig. 1]. Mimicking PMD, the polarization of each wavelength is adjusted separately with polarization controllers. The DOP of such a source is given by $[(I_1 + I_2)^2 - 4I_1I_2\sin^2\varphi]^{1/2}/(I_1 + I_2)$ where 2φ is the angle between the states of polarization of $\vec{M}(\lambda_1)$ and $\vec{M}(\lambda_2)$ (Poincaré sphere). With this source, it is very simple to study the response of our system for many configurations. In the following, we concentrate on the case $\lambda_1 = 1552$ nm and $\lambda_2 = 1554$ nm. Similar results were obtained for larger wavelength separations.

First, we characterize the quality of our projection onto the singlet state. For any input polarization combination, the output of our device has to be proportional to $1 - \mathrm{DOP}^2$ [Eq. (3)]. To well cover the possible inputs with a reasonable number of measurements, we choose polarization states on three orthogonal great circles of the Poincaré sphere. For each great circle, $\vec{M}(\lambda_1)$ is set to five polarization states separated by 40°. For each of those states, $\vec{M}(\lambda_2)$ is chosen on the same circle so that $2\varphi = 0, 10, \ldots, 90°$, corresponding to ten different values for the DOP. The measured data are shown in Fig. 2, where the values obtained from the different circles are represented by different symbols (squares, circles, and triangles). Because of the choice of polarization states, for each circle we have five points for a given DOP (corresponding to the five different absolute input polarization directions). As expected, the detected intensity reflects the DOP of our source, and is quasi-independent of the absolute polarization states of λ_1 and λ_2. The residual fluctuations observed for a given DOP value

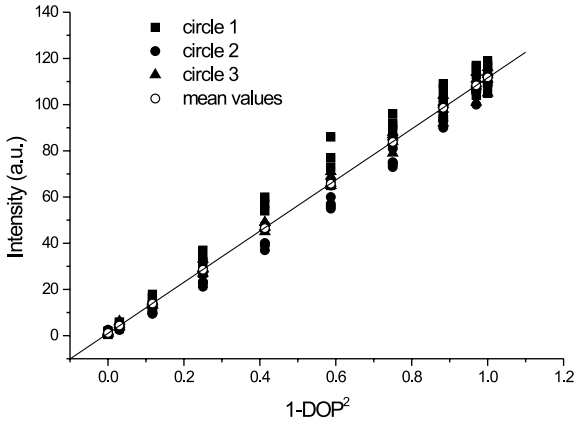

FIG. 2. Measured intensity of the projection onto the singlet state as a function of $1 - \mathrm{DOP}^2$ for $\lambda_1 = 1552$ nm and $\lambda_2 = 1554$ nm.

are due to misalignments of the setup. Specifically, the small variations for a DOP of 1 are essentially due to a slightly reduced visibility of the interferences between the two waves from the two stages (see [9] for more details). We can estimate a visibility of more than 96%. This is achieved thanks to a proper spatial overlap of the modes created in the two stages due to walk-off compensation in the crystals. If we estimate the precision of our measurement with the standard deviation of the fluctuations, the error of our device on the determination of the DOP is a few percent for a DOP close to 1 and about 15% for a totally depolarized source. Figure 2 also shows the mean values for a given DOP (open circles). They follow very well the linear law predicted by the theory (solid line).

So far the analyzed signal was constant in time. In order to demonstrate that we really measure the instantaneous DOP, a source with constant DOP but rapidly fluctuating state of polarization is required. We realize this by shaking the fiber linking the source to the DOP-meter (fiber after the coupler in Fig. 1). This leads to variations in the birefringence axis direction and Berry's phase in this fiber, and consequently the polarization states $\vec{M}(\lambda_1)$ and $\vec{M}(\lambda_2)$ will strongly fluctuate in time. If the amount of birefringence is small enough compared to the wavelength difference $\lambda_1 - \lambda_2$, the relative polarization angle φ between $\vec{M}(\lambda_1)$ and $\vec{M}(\lambda_2)$ (i.e., the DOP) is conserved even when agitating the fiber. In our experiment, we are manually moving the fiber leading to a time scale of the polarization fluctuations of ~100 ms. Accordingly, an integration time of a few seconds is chosen in order to be sure that the polarization state strongly fluctuates during this time interval. Figure 3 shows corresponding results for three different values of the DOP (open symbols, integration time 10 s). The fiber was not shaken for the first and last measurement points to have two reference values. As can be seen, the same values for the DOP are obtained when shaking the fiber.

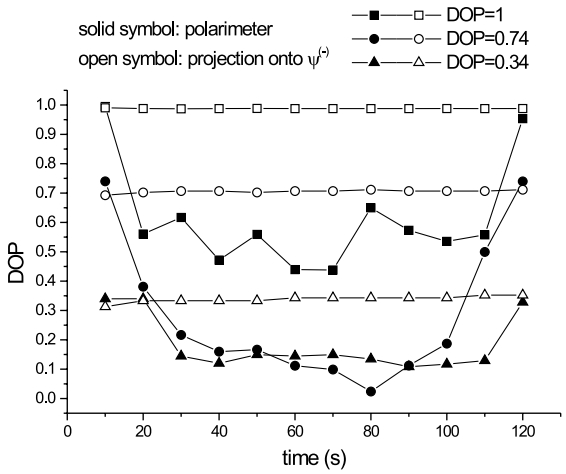

FIG. 3. DOP measured with our device (open symbols) and with a polarimeter (solid symbols) as a function of time. The DOP of the source is constant but its polarization state fluctuates with time, except for the first and last measurement points of each curve where it was fixed.

This clearly demonstrates the projection onto the singlet state does indeed give the instantaneous DOP.

To illustrate that this is not the case for the standard measurement techniques, we repeated the measurement using a polarimeter with 10 s integration time (PAT-9000, Profile). On the first and last point, we measure the same value as with the singlet state projection. But when the fiber is shaken the measured value of the DOP strongly decreases and also fluctuates somewhat. This behavior is observed both for 10 s (Fig. 3) and 1 s integration times. Clearly, the DOP is no longer measured correctly. Note that although a polarimeter can integrate much faster than 1 s (e.g., 33 ms for the PAT-9000), the same problem will be observed for fluctuations of the order of milliseconds as they can occur for PMD.

In conclusion, a concrete application of a coherent quantum measurement has been realized: a DOP-meter. It is based on the projection onto the singlet state, and allows to measure the instantaneous DOP in a direct way. This is different from the standard, indirect method of DOP evaluation (polarimetric technique) where the DOP is averaged over the integration time of the detection, which is typically longer than the coherence time of the signal to be measured. Consequently, for a signal with temporally fluctuating polarization only the first method gives the correct DOP.

Experimentally the projection onto the singlet state is realized exploiting up-conversion in two type II nonlinear crystals. In order to increase the efficiency of the process and to be able to measure signals with narrow spectra, the effective crystal length should be large. We achieved this by stacking 2×4 KTP crystals of 3 mm length in a walk-off compensation arrangement, giving an effective length of almost 12 mm for each of the two stages. With this compensation technique, we obtained a high quality DOP measurement for wavelengths separated by 2 nm. Further, we demonstrated that the projection onto the singlet state gives indeed the instantaneous DOP. For a signal with temporally fluctuating polarization we still obtained the correct value, whereas this was not the case for a standard polarimetric measurement.

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