Mesoscopic Effects in Adiabatic Spin Pumping

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We show that temporal shape modulations (pumping) of a quantum dot in the presence of spin-orbital coupling lead to a finite dc spin current. Depending on the strength of the spin-orbit coupling, the spin current is polarized perpendicular to the plane of the two-dimensional electron gas, or has an arbitrary direction subject to mesoscopic fluctuations. We analyze the statistics of the spin and charge currents in the adiabatic limit for the full crossover from weak to strong spin-orbit coupling.

There is a growing interest in the physics of spin transport through low-dimensional quantum structures [1] with the aim of controlling and manipulating spin in microelectronic devices. A fascinating tool for spin manipulation is a "spin battery" or "spin pump," a device that generates a spin current without an accompanying charge current [2]. Following a proposal of Mucciolo *et al.* [3], a spin battery was realized recently [4] using a quantum dot in a two-dimensional electron gas (2DEG). In this geometry, the direction of spin polarization is set by an external magnetic field parallel to the plane of the 2DEG. Current is generated by periodic variation of gate voltages, an eventual charge current being suppressed by fine tuning the dot shape. Other proposals for spin pumps have used the idea of locally breaking spin-rotation symmetry to pump a spin-polarized current by a magnetic field [2], magnetic impurities [5–7], and by periodic modulation of the spin-orbit coupling in the 2DEG [8].

Spin current is different from charge current because of the vector nature of spin. In fact, it is the "vector" nature of spin — different directions of spin corresponding to different quantum-mechanical superpositions of "spin up" and "spin down" — that makes it a promising candidate for the practical realization of a quantum computer [9]. Control of magnitude as well as direction of spin current is of paramount importance if the full benefits of a spintronic circuit are to be reaped, or for accumulating spin in a solid-state device that is used as the building block in a scheme that makes essential use of quantum coherence. The spin battery of Refs. [3,4] satisfies this requirement in part, since magnitude and sign of the current can be controlled, but its direction is determined by the external magnetic field, allowing for spin polarization in the 2DEG plane only.

In this Letter, we investigate a quantum-dot based spin pump for which Bychkov-Rashba [10] and Dresselhaus [11] contributions to spin-orbit coupling are the sources for spin-rotation symmetry breaking. We show that, depending on the strength of the spin-orbit coupling, a quantum dot with a variable shape can pump either a spin current polarized *perpendicular* to the 2DEG plane, with the sign and magnitude of the current subject to control via mesoscopic fluctuations, or a spin current with an arbitrary direction of polarization.

The possibility to generate a spin current polarized perpendicular to the plane of the 2DEG allows for an interesting realization of a "spin Hall effect" [12] if the spin current is injected into a 2DEG which shows the anomalous Hall effect [13]. The 2DEG then has skew spin-orbit scattering with spin up and spin down (measured perpendicular to the 2DEG plane) being scattered preferentially in opposite directions, perpendicular to the flow of current. While a current of unpolarized electrons leads to a spin imbalance perpendicular to the current flow [12] and a spin current polarized in the plane of the 2DEG is not affected, a spin current polarized perpendicular to the 2DEG should lead to an anomalous Hall voltage across the sample.

The system under consideration consists of a ballistic quantum dot connected to two electron reservoirs through ballistic point contacts with N_1 and N_2 channels each; see Fig. 1, inset. Gate voltages x_1 and x_2 of two shape-distorting gates allow for a time-dependent variation of the dot shape. The same geometry was used in the spin battery of Ref. [4], with a quantum dot without notable spin-orbit coupling. Electron motion in the dot is characterized by the transit time $\tau_{tr} = L/v_F$, where L is the dot size and v_F the Fermi velocity, and the time $\tau_{esc} = h/N\Delta$ for escape to the reservoirs, where $N = N_1 + N_2$ and Δ is the mean spacing between single-electron levels in the quantum dot. The escape time, which is determined by the point contacts, is typically much larger than τ_{tr}. In the absence of spin-orbit coupling, periodic variation of the gate voltages x_1 and x_2 leads to a dc charge current through the dot [15–17]. Below, we show that a spin current is generated in the presence of spin-orbit scattering.

The Dresselhaus and Bychkov-Rashba contributions to the spin-orbit coupling arise from the spin splitting of the conduction band in bulk GaAs [18] and the asymmetry of the potential well forming the 2DEG, respectively. The corresponding term in the Hamiltonian has the form

$$H_{SO} = \varrho[p_x\sigma_x - p_y\sigma_y] + \alpha[\vec{p} \times \vec{\sigma}]_z, \quad (1)$$
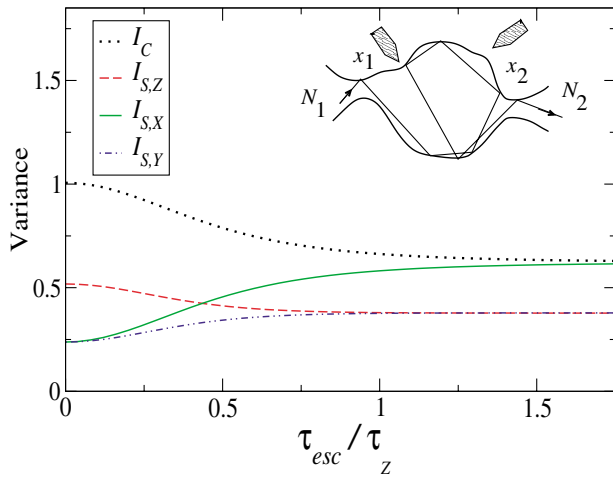
FIG. 1 (color online). Current variance (in units of $I_0^2/2$) as a function of Zeeman coupling strength ($b = \tau_{esc}/\tau_Z$) for a fixed value of SO coupling strengths $c_\perp = 0.47$, and $c_\parallel = 0.12$ taken from Ref. [14]. The difference between $\langle I_{s,x}^2 \rangle$ and $\langle I_{s,z}^2 \rangle$ for large b vanishes for $c_\perp \gg 1$. The inset shows a quantum dot with out of phase shape modulation, and a typical electron trajectory which gathers a Aharonov-Bohm flux depending on its perpendicular spin projection.

where p_x and p_y are the in-plane electron momentum and σ_x, σ_y, and σ_z are the Pauli matrices. The coefficients ϱ and α define a length scale

$$\lambda = \frac{\hbar}{m|\varrho^2 - \alpha^2|^{1/2}},$$

where m is the electron mass. For quantum wires, only the spin projection perpendicular to the wire and in the plane of the 2DEG is conserved. As a result, pumping through such a system yields a spin current with the polarization direction in the plane of the 2DEG [8]. For quantum dots of size $L \ll \lambda$, the role of spin-orbit scattering is qualitatively different from one-dimensional or bulk systems. The difference originates from the fact that, although spin-orbit scattering has a small effect in the transit time τ_{tr} if $L \ll \lambda$, it still may have a large effect within the time scale τ_{esc} [19]. As shown in Ref. [19], a unitary transformation casts H into the form

$$H = \frac{1}{2m}(\vec{p} - \vec{a}_\perp - \vec{a}_\parallel)^2, \qquad (2)$$

where

$$a_\perp = \frac{\hbar \sigma_z [\vec{r} \times \hat{z}]}{2\lambda^2}, \qquad \vec{a}_\parallel = \frac{\vec{a}_\perp}{\hbar}(\alpha \vec{r} \cdot \vec{\sigma} + \varrho[\vec{r} \times \vec{\sigma}]_z) \quad (3)$$

are spin-dependent vector potentials representing the effects of spin-orbit scattering. They are characterized by scattering times $\tau_\perp$ and $\tau_\parallel$,

$$\tau_\perp = \kappa^{-1}\tau_{tr}(2\lambda/L)^4, \qquad \tau_\parallel = (\kappa')^{-1}\tau_\perp(2\lambda/L)^2 \gg \tau_\perp, \tag{4}$$

where κ and κ' are geometry-dependent coefficients of

order unity [20]. Whether or not the spin-orbit term $a_\parallel$ is important depends on the relative size of $\tau_\parallel$ and τ_{esc}. We now discuss the cases $\tau_\parallel \gg \tau_{esc}$ and $\tau_\parallel \lesssim \tau_{esc}$ separately.

For $\tau_\parallel \gg \tau_{esc}$ the only relevant spin-orbit term in the Hamiltonian is the spin-dependent vector potential $a_\perp$. This term has the same form as the vector potential arising from a magnetic field of size $B_{so} = \hbar/2e\lambda^2$ with opposite directions for spin up ($\uparrow$) and spin down ($\downarrow$), measured perpendicular to the plane of the 2DEG. Hence, the perpendicular spin projections are conserved, and separate dc currents $I_\uparrow$ and $I_\downarrow$ will be pumped through the quantum dot for spin up and spin down. Generically, one has a spin current $I_{s,z} = I_\uparrow - I_\downarrow \neq 0$ because electron trajectories that enclose the same geometrical area (see inset of Fig. 1) gather an Aharonov-Bohm flux with opposite signs for electrons with opposite spin projections, thereby giving the scattering phase shift a spin dependence. In a chaotic cavity, both the sign and magnitude of the currents $I_{\uparrow,\downarrow}$ are essentially random, dependent on the detailed shape and electron density of the quantum dot. Following the protocol of Ref. [3], a spin battery with spin current polarized perpendicular to the 2DEG plane results if the shape of the dot is fine-tuned so that the charge current $I_c = I_\uparrow + I_\downarrow = 0$. In order to compare typical (rms) sizes of spin and charge currents, we note that the spin current is formally equal to the magnetic-field antisymmetric component of the charge current for a quantum pump with spinless electrons in a perpendicular magnetic field of size B_{so}. The latter quantity was calculated in Ref. [21] using random matrix theory, which is valid if the dot has an irregular shape and $\tau_{esc} \gg \tau_{tr}$,

$$\mathrm{rms}\, I_{s,z} = \left(1 - \frac{1}{(1 + \tau_{esc}/\tau_\perp)^3}\right)^{1/2} \mathrm{rms}\, I_c. \qquad (5)$$

We now turn to the general case $\tau_\parallel \lesssim \tau_{esc}$. We consider a sinusoidal time dependence of the gate voltages, $x_1(t) = \delta x_1 \cos(\omega t)$ and $x_2(t) = \delta x_2 \cos(\omega t + \phi)$ and consider the contribution to the charge and spin current that is bilinear in the gate voltage amplitudes δx_1 and δx_2. The starting point of our calculation is the relation between the charge and spin currents and the $2N \times 2N$ scattering matrix S of the quantum dot [15],

$$I_c = \frac{\omega \delta x_1 \delta x_2 \sin\phi}{2\pi} \mathrm{tr\, Im}\left[(\Lambda \otimes \mathbb{1}) \frac{\partial S}{\partial x_2} \frac{\partial S^\dagger}{\partial x_1} \right],$$
$$\vec{I}_s = \frac{\omega \delta x_1 \delta x_2 \sin\phi}{2\pi} \mathrm{tr\, Im}\left[(\Lambda \otimes \vec{\sigma}) \frac{\partial S}{\partial x_2} \frac{\partial S^\dagger}{\partial x_1} \right],$$
$$(6)$$

where Λ is an $N \times N$ diagonal matrix with elements $\Lambda_{jj} = N_2/N$ for $j \leq N_1$, and $\Lambda_{jj} = -N_1/N$ for $N_1 < j \leq N$ and $\mathbb{1}$ is the 2×2 unit matrix in spin space.

In order to calculate the average and rms of the charge and spin current for an ensemble of quantum dots, we need to know the average of a product of up to four scattering matrix elements taken at different values of the parameters x_1 and x_2. We calculate this correlator

using random matrix theory (RMT) [22]. In RMT, the spin-orbit part of the Hamiltonian (2) is replaced by a $2M \times 2M$ random Hermitian matrix [20],

$$H_{so} = i\sqrt{\frac{\Delta}{4\pi}}\left[\frac{A_3 \otimes \sigma_3}{\sqrt{\tau_\perp}} + \frac{A_1 \otimes \sigma_1 + A_2 \otimes \sigma_2}{\sqrt{\tau_\parallel}}\right], \quad (7)$$

whereas the dependence on the shape-distorting gate voltages x_1 and x_2 is represented through the random Hermitian matrix

$$H_{shape} = \frac{\Delta}{\pi}\sum_{j=1}^{2} x_j X_j \otimes \mathbb{1}. \quad (8)$$

Here, X_j, $j = 1, 2$ are real symmetric random $M \times M$ matrices and A_j, $j = 1, 2, 3$ are real antisymmetric random $M \times M$ matrices with

$$\mathrm{tr}X_i X_j = \mathrm{tr}A_i A_j^T = M^2 \delta_{ij}. \quad (9)$$

At the end of the calculation, the limit $M \to \infty$ needs to be taken. Performing the random matrix average using standard methods [20,23], we find that the relevant average of a product of four scattering matrices is

$$\langle S(1)_{kl} \otimes S^\dagger(2')_{nk} \otimes S(1')_{mn} \otimes S^\dagger(2)_{lm} \rangle$$
$$\equiv W^{(1)}(12')W^{(1)}(1'2)\delta_{ln} + W^{(2)}(12'1'2), \quad (10)$$

where 1, 2, 1′, and 2′ are shorthand notations for values of the gate voltages x_1 and x_2 and the Fermi energy ε, and we have used tensor notation for the spin degrees of freedom. The first contribution on the right-hand side of Eq. (10) is the product of pair averages [20],

$$W^{(1)}(12') = \frac{1}{M\mathbb{1}_2 - \mathrm{tr}R(1) \otimes R^\dagger(2')} \otimes \mathbb{1}_2,$$

with a similar definition of $W^{(1)}(1'2)$. Here $\mathbb{1}_2 \equiv \mathbb{1} \otimes \mathbb{1}$, the auxiliary matrix R is defined as

$$R(\varepsilon, x_1, x_2) = \exp[2\pi i(\varepsilon - H_{so} - H_{shape}(x_1, x_2))], \quad (11)$$

and the trace "tr" is not taken over the spin degrees of freedom. In taking the inverse in Eq. (11), the rule for multiplication of the tensor products is $(\sigma_j \otimes \sigma_{k'}) \times (\sigma_k \otimes \sigma_{j'}) = (\sigma_j \sigma_k) \otimes (\sigma_{j'} \sigma_{k'})$. The second contribution in Eq. (10) involves the random matrix equivalent of the Hikami box from diagrammatic perturbation theory and can be calculated using the method of Ref. [23]. Using tensor notation with the same product rules as before, the result can be written as

$$W^{(2)}(12'1'2) = W^{(1)}(12')W^{(1)}(1'2)\mathcal{D}(12'1'2)$$
$$\times W^{(1)}(12)W^{(1)}(1'2'), \quad (12)$$

where

$$\mathcal{D}(12'1'2) = M\mathbb{1}_4 - \mathrm{tr}R(1) \otimes R^\dagger(2') \otimes R(1') \otimes R^\dagger(2).$$

The traces are calculated using Eq. (9) after expanding R to second order in H_{so} and H_{shape} and taking $M \to \infty$.

Substituting these results into Eq. (6) one then finds the average and variances of the spin and charge currents. All ensemble averages are zero, as well as the covariances for different components of the spin current or of spin and charge currents. The full results for the average square spin and charge currents can be written by introducing $I_0 = (\omega/2\pi)4\delta x_1 \delta x_2 \sin\phi \sqrt{N_1 N_2/N^4}$, $c_\perp = \tau_{esc}/\tau_\perp$, and $c_\parallel = \tau_{esc}/\tau_\parallel$:

$$\langle I_c^2 \rangle = I_0^2\left(1 + \frac{1}{(1 + 2c_\parallel)^3} + \frac{2}{(1 + c_\parallel + c_\perp)^3}\right),$$

$$\langle I_{s,z}^2 \rangle = I_0^2\left(1 + \frac{1}{(1 + 2c_\parallel)^3} - \frac{2}{(1 + c_\parallel + c_\perp)^3}\right), \quad (13)$$

$$\langle I_{s,x}^2 \rangle = \langle I_{s,y}^2 \rangle = I_0^2\left(1 - \frac{1}{(1 + 2c_\parallel)^3}\right).$$

These general results confirm that spin current is polarized perpendicular to the plane of the 2DEG if $\tau_\parallel \gg \tau_{esc}$, while its direction is arbitrary if $\tau_\parallel \ll \tau_{esc}$. In practice, the ratio $\tau_\parallel/\tau_{esc}$ can be tuned rather straightforwardly by changing the conductances of the point contacts connecting the quantum dot to the outside world or the dot size. Recent experiments by Zumbühl et al. show that both limits can be obtained experimentally [14]: Depending on the size of the quantum dots, Zumbühl et al. find $c_\perp N$ ranging from 0.94 to 20 and $c_\parallel N$ ranging from 0.25 to 20. For $N = 2$ and at a temperature $T \ll \tau_{esc}^{-1}$, we obtain rms $I_{s,z}$/rms $I_c = 0.72$. At finite temperature, both the effect of thermal smearing and a finite dephasing time τ_ϕ need to be taken into account. Dephasing is accounted for by the substitution $1/\tau_{esc} \to 1/\tau_{esc} + 1/\tau_\phi$, whereas thermal smearing requires integration over ε. For large temperatures $T \gg \tau_{esc}^{-1}$, and in the absence of any Zeeman coupling, we can borrow the results of Ref. [21] and find the polarization ratio rms $I_{s,z}$/rms$I_c = [(c_\perp^2 + 2c_\perp)/(c_\perp^2 + 2c_\perp + 2)]^{1/2} = 0.61$.

Before concluding, we discuss the dependence of the pumped current on an applied magnetic field $\vec{B}$. This question is relevant if a spin pump is used in conjunction with spin manipulation by means of in-plane or perpendicular fields. Within RMT, the effect of a magnetic field is modeled by a third random matrix [19,20],

$$H_B = \sqrt{\frac{\Delta}{4\pi}}\left(i\frac{A_0 \otimes \mathbb{1}}{\sqrt{\tau_H}} + \frac{X \otimes \sigma_z}{\sqrt{\tau_{H,\perp}}}\right) - \frac{1}{2\tau_Z}\hat{B} \cdot \vec{\sigma}, \quad (14)$$

where $1/\tau_Z = \mu_B gB$, B is the magnitude of the applied magnetic field, $\hat{B}$ its direction, μ_B is the Bohr magneton, g is the g factor of GaAs, $\tau_H = \kappa^{-1}\tau_{tr}(2/eB_z L^2)^2$, $\tau_{H,\perp} = (4\lambda/\mu_B gLB)^2/\kappa''\tau_{tr}$, κ and κ'' being geometry-dependent coefficients of order unity, A is an $M \times M$ real antisymmetric matrix with trace $\mathrm{tr}AA^T = M^2$, and X is a real symmetric matrix with trace $\mathrm{tr}X^2 = M^2$. The time τ_H describes the orbital effect of the magnetic-field

component perpendicular to the 2DEG; it is the time needed for picking up a quantum of magnetic flux. The time $\tau_{H,\perp}$ describes spin-flip processes arising from the interplay of the component of the magnetic field parallel to the 2DEG and the spin-orbit scattering [24].

Adding the random matrix H_B to the exponent in Eq. (11) and repeating the previous calculations, we find (i) inclusion of τ_H has no effect on the current statistics and (ii) inclusion of $\tau_{H,\perp}$ amounts to the substitution $1/\tau_\perp \rightarrow 1/\tau_\perp + 1/\tau_{H,\perp}$. In general, the effect of Zeeman coupling to the parallel field (time scale τ_Z) is to decrease correlations between $I_\uparrow$ and $I_\downarrow$, so that both rms $I_{s,z}$ and rms I_c are reduced. On the other hand, the variance of the spin current polarized along the direction of the in-plane magnetic field is enhanced. Both of these effects are shown in Fig. 1. The full results for the current variance in the presence of Zeeman coupling are rather lengthy and will be reported elsewhere. Here, we confine ourselves to the limiting cases of Ref. [19] which are distinguished by a parameter $\Sigma = 1, 2$ characterizing the mixing of states with different spins for strong Zeeman splitting. Choosing the magnetic field along the x direction, these are (i) $c_\parallel = \tau_{\mathrm{esc}}/\tau_\parallel \ll 1$, and $b \equiv \tau_{\mathrm{esc}}/\tau_Z$ large or small in comparison to $c_\perp = \tau_{\mathrm{esc}}/\tau_\perp$, for which $\Sigma = 1$, and (ii) $c_\perp \gg 1$, b large or small in comparison to $c_\parallel$, for which $\Sigma = 2$. For case (i), when $b^2/c_\perp \ll 1 \ll c_\perp$, we obtain to the lowest order:

$$\langle I_c^2 \rangle = \langle I_{s,z}^2 \rangle = I_0^2(2 - 12b^2/c_\perp + 96b^4/c_\perp^2),$$
$$\langle I_{s,x}^2 \rangle = \langle I_{s,y}^2 \rangle = I_0^2(12b^2/c_\perp - 96b^4/c_\perp^2), \tag{15}$$

and when $c_\perp \ll 1 \ll b$ we find:

$$\langle I_c^2 \rangle = \langle I_{s,x}^2 \rangle = I_0^2(2 - 3c_\perp + 6c_\perp^2),$$
$$\langle I_{s,z}^2 \rangle = \langle I_{s,y}^2 \rangle = I_0^2(3c_\perp - 6c_\perp^2), \tag{16}$$

which is "dual" to expression (15) under the exchange $I_{s,x} \leftrightarrow I_{s,z}$ and $c_\perp \rightarrow 4b^2/c_\perp$. For case (ii), the lowest order expansion in $b^2/c_\perp \ll 1 \ll c_\perp, c_\parallel$,

$$\langle I_{c(s,x)}^2 \rangle = I_0^2[1 \pm \tfrac{3}{8}c_\perp^{-3} \mp \tfrac{1}{2}(b^2/c_\perp)c_\perp^{-4}],$$
$$\langle I_{s,z}^2 \rangle = \langle I_{s,y}^2 \rangle = I_0^2(1 - \tfrac{1}{8}c_\perp^{-3}), \tag{17}$$

is dual under the transformation $4b^2/c_\perp \rightarrow c_\parallel$ to the variance in the other limit $c_\parallel \ll 1 \ll c_\perp, b^2/c_\perp$. The nonvanishing current variances $\langle I^2 \rangle$ can be written in a unified way as $\langle I^2 \rangle = 4(s/\beta\Sigma)I_0^2$, where $\beta = 2$ defines the time reversal symmetry of orbital motion, and $s = 1$ is the Kramers degeneracy parameter.

In conclusion, we have shown that a quantum pump consisting of a quantum dot with spin-orbit coupling allows for the generation of both a dc charge current and a dc spin current. Depending on the strength of the spin-orbit scattering or on the conductances of the point contacts between the dot and the reservoirs, the pumped

spin current is polarized perpendicular to the 2DEG or has an arbitrary direction. Measurement of the spin current is possible via the spin Hall effect or by connection of the spin pump to a (semiconducting) ferromagnet with known magnetization direction. Furthermore, the method of adiabatic spin pumping, as opposed to other mechanisms such as rectification [4,25], allows the direction of spin polarization to be continuously changed.

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