Electromagnetic Wave Collapse in a Radiation Background

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The nonlinear interaction, due to quantum electrodynamical (QED) effects between an electromagnetic pulse and a radiation background, is investigated by combining the methods of radiation hydrodynamics with the QED theory for photon-photon scattering. For the case of a single coherent electromagnetic pulse, we obtain a Zakharov-like system, where the radiation pressure of the pulse acts as a driver of acoustic waves in the photon gas. For a sufficiently intense pulse and/or background energy density, there is focusing and the subsequent collapse of the pulse. The relevance of our results for various astrophysical applications are discussed.

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The theory for electromagnetism in vacuum can be roughly divided into that for coherent and that for incoherent systems. In the former case, Maxwell's equations are the explicit starting point, and that theory leads to most of the well-known applications. In the latter case, the ensemble average of the electromagnetic field is zero everywhere, and the system can then essentially be treated as a gas of ultrarelativistic particles [1]. Most of the theory for radiation gases cannot be considered as pure "electromagnetism in vacuum," since the interaction of photons with other particles is one of the main features of the theory. In the present Letter, we will consider the *self-interaction* between photons, due to quantum electrodynamical (QED) effects. One of the many interesting aspects of such an analysis is the possibility of photon-photon scattering, due to the interaction of light quanta with *virtual* electron-positron pairs [2]. By integrating out the virtual pairs, one obtains an effective field theory for the photon-photon interaction, in terms of the electromagnetic field variables ($F_{\mu\nu}$). The lowest order correction to Maxwell's vacuum equations is then conveniently expressed by means of the Heisenberg-Euler Lagrangian [see Eq. (1) below]. In that way, many papers on QED photon-photon scattering have been written (e.g., [3–9] and references therein). References [5,6,8] concern techniques for laboratory detection of QED effects, involving second harmonic generation [5], self-focusing [6], and nonlinear wave mixing in cavities [8], respectively. Other aspects of general theoretical interest that have been dealt with are, e.g., effects of static magnetic background fields on higher harmonic generation, wave propagation velocities in the QED vacuum and one-loop corrections to the Heisenberg-Euler Lagrangian [4], two-dimensional collapsing scenarios [7], and the refractive properties of the QED vacuum [9]. However, the above studies involve strictly coherent fields, or one-photon phenomena. For instance, the nontrivial propagation of photons in strong background electromagnetic fields, due to effects of nonlinear electrodynamics, has been considered in a number of papers (see, e.g., Refs. [3,9] and references therein). The main focus in these papers was on the interesting effects of photon splitting and birefringence in vacuum. However, Thoma [10] investigated the interaction of photons with a photon gas, using the real-time formalism, and calculated the corresponding change in the speed of light due to the cosmic microwave background (CMB).

In the present Letter, we are going to combine methods from radiation hydrodynamics [1] with QED theory for photon-photon scattering, in order to give a framework for the interaction of coherent electromagnetic fields with a radiation gas background. For the case of a single coherent electromagnetic pulse, we obtain a Zakharov-like system [11], where the radiation pressure of the pulse acts as a driver of acoustic waves in the photon gas. Similar to ordinary acoustic waves, these waves are longitudinal and characterized by variations in pressure, and thereby in energy density. The index of refraction depends on the photon gas energy density, and thus the excitation of the acoustic waves leads to a backreaction on the pulse. For a sufficiently intense pulse and/or background energy density, we may have focusing and subsequent collapse of the pulse. Applications to coherent pulses propagating in the present as well as in the early cosmic radiation background are discussed, together with other astrophysical phenomena. While there are possible explanations for the small CMB structures recently detected [12], we think that our mechanism below for CMB structure formation involving photon-photon interaction should be of interest.

The nonlinear self-interaction of photons can be formulated in terms of the Heisenberg-Euler Lagrangian [2]

$$L = \varepsilon_0 F + \kappa\varepsilon_0^2[4F^2 + 7G^2], \qquad (1)$$

where $F = (E^2 - c^2B^2)/2$ and $G = c\boldsymbol{E}\cdot\boldsymbol{B}$. Here $\kappa \equiv 2\alpha^2\hbar^3/45m_e^4c^5 \approx 1.63\times10^{-30}$ ms^2/kg, α is the fine-structure constant, $\hbar$ is the Planck constant, m_e the electron mass, and c the velocity of light in vacuum (for higher order corrections, see, e.g., expression (26) of

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Ref. [3]). The Lagrangian (1) is valid as long as there is no pair creation and the field strength is smaller than the critical field, i.e.,

$$\omega \ll m_e c^2/\hbar \quad \text{and} \quad |\boldsymbol{E}| \ll E_{\rm crit} \equiv m_e c^2/e\lambda_c, \tag{2}$$

respectively. Here e is the elementary charge, λ_c is the Compton wavelength, and $E_{\rm crit} \simeq 10^{18}$ V/m.

According to Ref. [3], the dispersion relation for a low energy photon in a background electromagnetic field can be derived from the Lagrangian (1), with the result (see also Ref. [9] and references therein)

$$\omega(\boldsymbol{k}, \boldsymbol{E}, \boldsymbol{B}) = c|\boldsymbol{k}|(1 - \tfrac{1}{2}\lambda|\boldsymbol{Q}|^2), \tag{3}$$

where

$$|\boldsymbol{Q}|^2 = \varepsilon_0[E^2 + c^2B^2 - (\hat{\boldsymbol{k}}\cdot\boldsymbol{E})^2 - c^2(\hat{\boldsymbol{k}}\cdot\boldsymbol{B})^2 - 2c\hat{\boldsymbol{k}}\cdot(\boldsymbol{E}\times\boldsymbol{B})], \tag{4}$$

and $\lambda = \lambda_\pm$, where $\lambda_+ = 14\kappa$ and $\lambda_- = 8\kappa$ for the two different polarization states of the photon. Furthermore, $\hat{\boldsymbol{k}} \equiv \boldsymbol{k}/k$. The approximation $\lambda|\boldsymbol{Q}|^2 \ll 1$ has been used. The background electric and magnetic fields are denoted by $\boldsymbol{E}$ and $\boldsymbol{B}$, respectively.

We will below study two scenarios: (i) a plane wave pulse propagating on a background consisting of a radiation gas in equilibrium, and (ii) a radiation gas affected by an electromagnetic (EM) pulse propagating through the gas. For case (i), the relations $(\hat{\boldsymbol{k}}_p\cdot\boldsymbol{E})^2 = \frac{1}{3}E^2$, $(\hat{\boldsymbol{k}}_p\cdot\boldsymbol{B})^2 = \frac{1}{3}B^2$, and $\boldsymbol{E}\cdot\boldsymbol{B} = 0$, hold, where $\boldsymbol{k}_p$ is the pulse wave vector. Hence, from (4) we obtain

$$|\boldsymbol{Q}_{\rm gas}|^2 = \tfrac{4}{3}\mathcal{E}, \tag{5}$$

where $\mathcal{E} = \varepsilon_0(E^2 + c^2B^2)/2$ is the energy density of the radiation gas. For case (ii), we argue in a similar manner. To lowest order, the directions $\hat{\boldsymbol{k}}$ of the photons in the gas are approximately random, and the EM pulse is a superposition of unidirectional plane waves, such that $\boldsymbol{E} = E_p\hat{\boldsymbol{e}}$, and $\boldsymbol{B} = E_p\hat{\boldsymbol{k}}_p\times\hat{\boldsymbol{e}}/c$, where $\hat{\boldsymbol{e}}$ is the unit electric vector. Then (4) yields

$$|\boldsymbol{Q}_{\rm pulse}|^2 = \tfrac{4}{3}\varepsilon_0|E_p|^2. \tag{6}$$

With the relation (5), and using standard methods for slowly varying envelopes [13], we then derive the dynamical equation for a pulse on a photon gas background:

$$i\left(\frac{\partial}{\partial t} + c\hat{\boldsymbol{k}}_p\cdot\nabla\right)E_p + \frac{c}{2k_p}\nabla_\perp^2 E_p + \frac{2}{3}\lambda c k_p \mathcal{E} E_p = 0, \tag{7}$$

where $\nabla_\perp^2 = \nabla^2 - (\hat{\boldsymbol{k}}_p\cdot\nabla)^2$, noting that the *self-interaction* of the pulse vanishes.

For a dispersion relation $\omega = ck/R(\boldsymbol{r}, \boldsymbol{t})$, where R is the refractive index, we have the Hamiltonian ray equations

$$\dot{\boldsymbol{r}} = \frac{\partial\omega}{\partial\boldsymbol{k}} = \frac{c}{R}\hat{\boldsymbol{k}}, \tag{8a}$$

$$\dot{\boldsymbol{k}} = -\nabla\omega = \frac{\omega}{R}\nabla R, \tag{8b}$$

where $\dot{\boldsymbol{r}}$ denotes the group velocity of the photon, $\dot{\boldsymbol{k}}$ the force on a photon, and the dot denotes time derivative.

The equation for the collective interaction of photons can then be formulated as [14]

$$\frac{\partial N(\boldsymbol{k}, \boldsymbol{r}, t)}{\partial t} + \nabla\cdot[\dot{\boldsymbol{r}}N(\boldsymbol{k}, \boldsymbol{r}, t)] + \frac{\partial}{\partial\boldsymbol{k}}\cdot[\dot{\boldsymbol{k}}N(\boldsymbol{k}, \boldsymbol{r}, t)] = 0, \tag{9}$$

where the distribution function has been normalized such that the number density is $n(\boldsymbol{r}, t) = \int N(\boldsymbol{k}, \boldsymbol{r}, t)\,d\boldsymbol{k}$.

For a general function $f(\boldsymbol{k}, \boldsymbol{r}, t)$, the moment equation is

$$\frac{\partial}{\partial t}(n\langle f\rangle) + \nabla\cdot(n\langle\dot{\boldsymbol{r}}f\rangle) = n\left[\left\langle\frac{\partial f}{\partial t}\right\rangle + \langle\dot{\boldsymbol{r}}\cdot\nabla f\rangle + \left\langle\dot{\boldsymbol{k}}\cdot\frac{\partial f}{\partial\boldsymbol{k}}\right\rangle\right], \tag{10}$$

where $\langle f\rangle \equiv n(\boldsymbol{r}, t)^{-1}\int fN\,d\boldsymbol{k}$. Choosing $f = \hbar\omega$, we obtain the energy conservation equation

$$\frac{\partial\mathcal{E}}{\partial t} + \nabla\cdot(\mathcal{E}\boldsymbol{u} + \boldsymbol{q}) = -\frac{\mathcal{E}}{R}\frac{\partial R}{\partial t}, \tag{11a}$$

where $\mathcal{E}(\boldsymbol{r}, t) = n\langle\hbar\omega\rangle$ is the energy density, and $\boldsymbol{q}(\boldsymbol{r}, t) = n\langle\hbar\omega\boldsymbol{w}\rangle$ the energy flux. Here we have made the split $\dot{\boldsymbol{r}} = \boldsymbol{u} + \boldsymbol{w}$, where $\langle\boldsymbol{w}\rangle = 0$. Thus, $\boldsymbol{w}$ represents the random velocity of the photons. With $f = \hbar\boldsymbol{k}$, we obtain the momentum conservation equation

$$\frac{\partial\boldsymbol{\Pi}}{\partial t} + \nabla\cdot[\boldsymbol{u}\otimes\boldsymbol{\Pi} + \mathsf{P}] = \frac{\mathcal{E}}{R}\nabla R, \tag{11b}$$

where $\boldsymbol{\Pi} = n\langle\hbar\boldsymbol{k}\rangle$ is the momentum density, and $\mathsf{P} = n\langle\boldsymbol{w}\otimes(\hbar\boldsymbol{k})\rangle$ is the pressure tensor. It follows immediately from the definition of the pressure tensor that the trace satisfies $\mathrm{Tr}\,\mathsf{P} = n\langle\hbar k\boldsymbol{w}\cdot\hat{\boldsymbol{k}}\rangle = nR\langle\hbar\omega\boldsymbol{w}\cdot\hat{\boldsymbol{k}}\rangle/c$. For an observer comoving with the fluid, i.e., a system in which $(\boldsymbol{u})_0 = 0$ (the 0 denoting the comoving system), Eq. (8a) shows that $(\boldsymbol{w}\cdot\hat{\boldsymbol{k}})_0 = (R)_0/c$, so that the trace of the pressure tensor in the comoving system becomes $(\mathrm{Tr}\mathsf{P})_0 = (\mathcal{E})_0$. For an isotropic distribution function, the pressure can be written as $P = \mathrm{Tr}\,\mathsf{P}/3$, satisfying the equation of state $P = \mathcal{E}/3$.

The system of Eqs. (11a) and (11b) needs closure, and this can be obtained by choosing an equation of state. Furthermore, we still have some gauge freedom since we have not specified the frame. Two choices prevail in the literature: (i) the energy frame, in which $\boldsymbol{q} = 0$, and (ii) the particle frame, where $\boldsymbol{u} = 0$. We will here adopt a particle frame, and furthermore choose the equation of state to be $\mathsf{P}_{ij} = \delta_{ij}\mathcal{E}/3$. The relation between the heat flow and the momentum density is straightforward to derive. The result is $\boldsymbol{q} = c^2\boldsymbol{\Pi}/R^2$. From the dispersion relation (3) and Eq. (6), we have $R \approx 1 + \frac{2}{3}\lambda\varepsilon_0|E_p|^2$, from which, through Eqs. (11a) and (11b), we obtain

$$\frac{\partial\mathcal{E}}{\partial t} + c^2\nabla\cdot\boldsymbol{\Pi} = \frac{2}{3}\lambda\varepsilon_0\left[-\mathcal{E}\frac{\partial|E_p|^2}{\partial t} + c^2\nabla\cdot(\boldsymbol{\Pi}|E_p|^2)\right], \tag{12a}$$

and

$$\frac{\partial \boldsymbol{\Pi}}{\partial t} + \frac{1}{3}\nabla\mathcal{E} = \frac{2}{3}\lambda\varepsilon_0\mathcal{E}\nabla|E_p|^2. \tag{12b}$$

We now introduce $\mathcal{E} = \mathcal{E}_0 + \delta\mathcal{E}$, where $\mathcal{E}_0$ ($\gg \delta\mathcal{E}$) is the unperturbed energy density in the absence of the electromagnetic pulse, noting that for the unperturbed state $\boldsymbol{\Pi} = 0$. Taking the divergence of (12b) and eliminating $\boldsymbol{\Pi}$ using (12a), we obtain

$$\frac{\partial^2\delta\mathcal{E}}{\partial t^2} - \frac{c^2}{3}\nabla^2\delta\mathcal{E} = -\frac{2\lambda\varepsilon_0\mathcal{E}_0}{3}\left[c^2\nabla^2|E_p|^2 + \frac{\partial^2|E_p|^2}{\partial t^2}\right], \tag{13a}$$

describing acoustic waves in a photon gas driven by a coherent electromagnetic pulse. We note that the set of Eqs. (13a) and (7) resembles the well-known Zakharov system, describing coupled Langmuir and ion-acoustic waves in plasmas [11]. Transforming to a system moving with the pulse $\tau = t$, $\xi = x - ct$ (letting the pulse wave vector be $\boldsymbol{k}_p = k_p\hat{\boldsymbol{x}}$), we have

$$\frac{\partial^2\delta\mathcal{E}}{\partial \xi^2} - \frac{1}{2}\nabla_\perp^2\delta\mathcal{E} = -\lambda\varepsilon_0\mathcal{E}_0\left[\nabla_\perp^2|E_p|^2 + 2\frac{\partial^2|E_p|^2}{\partial \xi^2}\right], \tag{13b}$$

dropping the slow derivatives proportional to $\partial/\partial\tau$. For a general pulse geometry Eq. (13b) requires extensive analysis, but for specific forms of the pulse we can significantly simplify the expression for $\delta\mathcal{E}$. For a broad electromagnetic pulse where $\nabla_\perp \ll \partial/\partial\xi$, we thus integrate (13a) to obtain $\delta\mathcal{E} = -2\lambda\varepsilon_0\mathcal{E}_0|E_p|^2$, whereas for a long needle-shaped pulse with $\partial/\partial\xi \ll \nabla_\perp$ we instead have $\delta\mathcal{E} = 2\lambda\varepsilon_0\mathcal{E}_0|E_p|^2$. The unperturbed photon gas energy density $\mathcal{E}_0$ will give a frequency shift for the solution of Eq. (7), but this shift can be transformed away. The resulting equation is

$$i\frac{\partial E_p}{\partial\tau} + \frac{c}{2k_p}\nabla_\perp^2 E_p \mp \frac{4}{3}\lambda^2 ck_p\varepsilon_0\mathcal{E}_0|E_p|^2E_p = 0, \tag{14}$$

where the minus (plus) sign refers to a broad (needle-shaped) pulse. For a broad pulse, wave collapse does not occur, but for a needle-shaped pulse, collapse will occur when the nonlinear term dominates over the diffraction. Comparing the terms in Eq. (14), we can obtain a rough estimate of the collapse criterion, namely,

$$k_p^2r_p^2|E_p|^2|E_{\text{gas}}|^2 > |E_{\text{char}}|^4 \tag{15}$$

which is consistent with Ref. [15]. Here r_p denotes the pulse width, $\mathcal{E}_0 = \varepsilon_0|E_{\text{gas}}|^2$, and $|E_{\text{char}}| \equiv (\varepsilon_0\kappa)^{-1/2} \approx 2.6 \times 10^{20}$ V/m. Although the collapse of electromagnetic pulses is complicated [16], the qualitative features can be described as a divergence in the pulse energy density, while its width decreases to zero, *in a finite time*. A rough estimate of the collapse properties can be obtained assuming cylindrical symmetry of the pulse. Following [15], the trial function $E_T(\tau, r) = A(\tau)\,\text{sech}[r/a(\tau)]\exp[ib(\tau)r^2]$ is used together with Rayleigh-Ritz optimization, in order to reduce the problem to a differential equation for the width $a(\tau)$. The field strength then satisfies $|A(\tau)|/|A(0)| = a(0)/a(\tau)$, and the width behaves as $[a(\tau)/a(0)]^2 - 1 \propto [1 - a(0)^2|A(0)|^2/I_c]\tau^2$, where $I_c \simeq 0.5 \times (k_p^2\lambda^2\varepsilon_0\mathcal{E}_0)^{-1}$ Thus, if $a(0)^2|A(0)|^2 > I_c$ the pulse will collapse to zero width in a finite time. Of course, when the pulse intensity increases, higher order effects will become important, possibly halting the collapse. Furthermore, as the pulse width decreases, derivative corrections to (1) become important [17], and thus change the collapse scenario.

The most intense pulses in our universe are the gamma-ray bursts [18]. Powers of the order 10^{45} W in the gamma range ($k_p \sim 10^{13}$ m^{-1}) mean that we may have $|E_p|k_pr_p \sim 10^{38}$ V/m [19]. In today's universe, the energy density of the CMB is 2.6×10^5 eV/m^3, which corresponds to $|E_{\text{gas}}| \sim 7 \times 10^{-2}$ V/m. Thus, the collapse condition (15) is (far from surprising) not fulfilled for gamma-ray bursts propagating in the present microwave background. However, we note that if there were mechanisms for generating equally intense gamma pulses in the early universe at an age $t \lesssim 6 \times 10^5$ yr, such pulses should indeed have collapsed, as for those times the energy density of the CMB obeyed $|E_{\text{gas}}| \gtrsim 2 \times 10^4$ V/m implying that (15) is fulfilled.

Furthermore, we note that collapse may occur for much less intense pulses than gamma-ray bursts, in case the propagation takes place in a photon gas which is more energetic than the CMB. Highly energetic photon gases should exist today in the vicinity of pulsars and magnetars. For pulsars, the low-frequency dipole radiation is not able to leave the system directly, as it is first reflected by the surrounding plasma. The momentum of the low-frequency photons in the vicinity of pulsars are not necessarily thermally distributed, but still we should be able to think of the pulsar environment as a highly energetic photon gas. For magnetars (i.e., pulsars with surface magnetic field strength 10^{10} T [20]), the high fields create a surface tension leading to star quakes, and subsequent generation of low-frequency photons. This environment could be in an electromagnetically turbulent state [21]. Thus, electromagnetic pulses of a comparatively moderate intensity propagating in such environments may collapse, due to the high energy densities of the photon gases. Rough estimates give energy densities of such photon gases that can be 20–22 orders of magnitude larger than that of today's CMB.

While the conditions for collapse may be fulfilled only during extraordinary circumstances, there is still a possibility that energetic pulses, although not intense enough for collapse, can leave a certain imprint in the CMB. Since we are mainly interested in effects that persist after the EM pulse has passed a given area, we need to abandon the approximation of a broad or needle-shaped pulse and instead investigate solutions to (13b). If the radiation gas energy density is well below that for collapse, the back-reaction on the EM pulse is small, and we do not have to solve the system (7) and (13b) self-consistently, but can take the EM pulse as given. Furthermore, investigating the generation of acoustic waves during times short compared to the diffraction time, we can consider the pulse to be static in the comoving frame. To simplify the analysis by making the r and ξ dependence separable, we assume that the source term obeys $\nabla_\perp^2|E_p|^2 + r_0^{-2}|E_p|^2 = 0$, where r_0 is the characteristic radius of localization. For a pulse with no angular dependence, this means that we can write $|E_p|^2 = J_0(r/r_0)S(\xi)$, where J_0 is the zeroth-order Bessel function, and we leave $S(\xi)$ unspecified. Letting the generated acoustic waves have the same radial dependence as the pulse, i.e., $\delta\mathcal{E}(r,\xi) = J_0(r/r_0)\mathcal{E}_\xi(\xi)$, we then obtain from (13b)

$$\frac{d^2\mathcal{E}_\xi}{d\xi^2} + \frac{1}{2r_0^2}\mathcal{E}_\xi = \lambda\varepsilon_0\mathcal{E}_0\left[r_0^{-2}S - 2\frac{d^2S}{d\xi^2}\right]. \tag{16}$$

With the boundary condition of no acoustic waves before the pulse has passed, the solution of (16) is

$$\mathcal{E}_\xi(\xi) = \frac{\sqrt{8}\lambda\varepsilon_0\mathcal{E}_0}{r_0}\int_\infty^\xi S(\xi')\sin[(\xi-\xi')/\sqrt{2}r_0]\,d\xi'. \tag{17}$$

Apparently, after the pulse passage the field can be written as $\mathcal{E}_\xi = \mathcal{E}_{\xi 0}\sin[(\xi/\sqrt{2}r_0) + \delta]$, where the amplitude $\mathcal{E}_{\xi 0}$ and the phase angle δ depend on the detailed form of the pulse profile $S(\xi')$ (see Ref. [22] for analytical expressions). For a pulse length slightly less than r_0, we have $\mathcal{E}_{\xi 0} \sim \lambda\varepsilon_0\mathcal{E}_0 S_{\max} \sim \lambda\varepsilon_0\mathcal{E}_0|E_p|^2_{\max}$, where $|E_p|^2_{\max}$ is the central value of $|E_p|^2$. Currently, measurements of the CMB can detect relative temperature anisotropies of the order of 10^{-6} [23]. Thus, we see that $|E_p| \gtrsim 10^{17}$ V/m must be fulfilled for possible detection of the resulting background anisotropy. Our estimate for the acoustic wave amplitude applies only relatively close after the pulse passage, before the acoustic wave begins to spread due to diffraction [as associated with the slow τ dependence omitted in Eq. (13b)]. It is clear from the above that intense pulses leave an inprint in the CMB after the pulse has passed a given area, and the results show that anisotropies in the high-frequency electromagnetic spectrum partly transfer to the low-frequency background. However, our calculations apparently do not include all effects that will influence the earth-based measurements. Thus, much work is still needed in order to determine whether the effects induced in the CMB by astrophysical sources can be seen in the detailed experiments that are currently made [23].

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