Orbital Entanglement and Violation of Bell Inequalities in Mesoscopic Conductors

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We propose a spin-independent scheme to generate and detect two-particle entanglement in a mesoscopic normal-superconductor system. A superconductor, weakly coupled to the normal conductor, generates an orbitally entangled state by injecting pairs of electrons into different leads of the normal conductor. The entanglement is detected via violation of a Bell inequality, formulated in terms of zero-frequency current cross correlators. It is shown that the Bell inequality can be violated for arbitrary strong dephasing in the normal conductor.

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Entanglement is one of the most intriguing features predicted by quantum theory [1]. It leads to correlation between distant particles, which cannot be described by any local, realistic theory [2]. This nonlocal property of entanglement has been demonstrated convincingly in optics [3], where entangled pairs of photons have been studied over several decades. Apart from the fundamental aspects, there is a growing interest in using the properties of entangled particles for quantum cryptography [4] and quantum computation [5].

Recently, much interest has been shown for entanglement of electrons in solid state systems. A controlled generation and manipulation of electronic entanglement is of importance for a large scale implementation of quantum information and computation schemes. Electrons are, however, in contrast to photons, massive and electrically charged particles, which raises new fundamental questions and new experimental challenges. Existing suggestions are based on creating [6,7], manipulating, and detecting [8–10] spin-entangled pairs of electrons. This requires experimental control of individual spins via spin filters or locally directed magnetic fields on a mesoscopic scale. Here we propose a spin-independent scheme for creating and detecting *orbital* entanglement in a mesoscopic normal-superconductor system.

We show that a superconductor, weakly coupled to a normal conductor (see Fig. 1), creates an orbitally entangled state by emitting a coherent superposition of pairs of electrons into different leads of the normal conductor. In the tunneling limit, the zero-frequency correlation between currents flowing into different normal reservoirs is shown to be equivalent to a pair coincidence measurement: only correlations between the electrons from the same entangled pair contribute. As a consequence, a standard Bell inequality (BI) can be directly formulated in terms of the zero-frequency current correlators. We find that a violation of the BI, demonstrating the entanglement of the pair state, can be obtained for arbitrary dephasing in the normal conductor.

We first consider a simplified version of the system [11] (see Fig. 1) (a more detailed discussion is given below). A single [12] superconductor is weakly coupled to a normal conductor, a ballistic two-dimensional electron gas, via two tunnel barriers (1 and 2) with transparency $\Gamma \ll 1$. The normal conductor consists of four arms, $1A$, $1B$, $2A$, and $2B$, with equal lengths L. The arms $1A$ and $2A$ ($1B$ and $2B$) are crossed in a controllable beam splitter $A(B)$, parametrized via the angle $\phi_A(\phi_B)$, and then connected to normal reservoirs $+A$ and $-A$ ($+B$ and $-B$).

The splitters [13] are assumed to support only one propagating mode. The states $|+, \eta\rangle$ and $|-, \eta\rangle$ for electrons going out into the normal reservoirs and the states $|1, \eta\rangle$ and $|2, \eta\rangle$ of the electrons emitted from superconductor are related via a scattering matrix ($\eta = A, B$):

$$\begin{bmatrix} |+, \eta\rangle \\ |-, \eta\rangle \end{bmatrix} = \begin{pmatrix} \cos\phi_\eta & -\sin\phi_\eta \\ \sin\phi_\eta & \cos\phi_\eta \end{pmatrix} \begin{bmatrix} |2, \eta\rangle \\ |1, \eta\rangle \end{bmatrix}. \quad (1)$$

The angles ϕ_A and ϕ_B can be tuned between 0 and $\pi/2$ by turning the splitter from "open," when the electrons are transmitted through from 1(2) to $-(+)$, to "closed," for complete reflection from 1(2) to $+(-)$. We consider the low temperature limit, $kT \ll eV$. A negative voltage $-eV$, smaller than the superconducting gap Δ, is applied to all the normal reservoirs and the superconductor is grounded. The size of the system is smaller than the phase breaking length.

We first present a simple and transparent explanation of how the entanglement is generated and detected, a rigorous derivation follows below. The superconductor emits pairs of particles into the normal arms. Since the

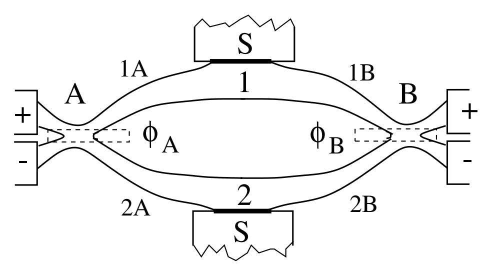

FIG. 1. The system. A single superconductor (S) is connected to four normal arms via two tunnel barriers (1 and 2) (thick black lines). The arms are joined pairwise in beam splitters A and B and end in normal reservoirs + and −.

superconductor is a single, coherent object, the state of an emitted pair is a linear superposition of states corresponding to a pair emitted through barriers 1 and 2. The emitted pair can either split, with one electron going to each splitter A and B, or both electrons can go to the same splitter. However, the latter process does not contribute [14] to the quantity of interest, the current cross correlations, to leading order in pair emission (Andreev reflection) probability, proportional to Γ^2. The relevant part of the state of the emitted pair can thus be written as

$$|\Psi\rangle = |\Psi_{12}\rangle \otimes |\Psi_{AB}\rangle, \qquad |\Psi_{12}\rangle = (|11\rangle + |22\rangle)/\sqrt{2}, \quad (2)$$

a product of a state $|\Psi_{12}\rangle$, orbitally entangled with respect to emission across barriers 1 and 2 (called 12-space below), and a state $|\Psi_{AB}\rangle$, describing one electron going towards A and one towards B, containing all additional information, such as energy and spin dependence. The splitters A and B rotate the state $|\Psi_{12}\rangle$ [see Eq. (1)].

The entanglement, giving rise to a nonlocal correlation of the two electrons, is detected by violation of a BI. We point out that a violation does not, due to the solid state environment, rule out all possible local, realistic theories. Earlier works on BI in conductors started from an entangled two-electron state at a single energy [9], not appropriate for our situation. Reference [10] formulates a BI in terms of correlators of number of particles transferred in a given time τ. In the tunnel limit, for τ shorter than the average emission time of the pairs but longer than the coherence time, the BI is expressed in terms of zero-frequency current correlators. Here instead we consider explicitly the entangled two-particle state in Eq. (2) and a BI is formulated based on the observation that the zero-frequency noise correlations is proportional to the probability of joint detection of the two electrons in the pair. We recall that in the original formulation [2], a source emitting spin-1/2 singlets was considered. The BI, as formulated in Ref. [15],

$$S \equiv |E(\phi_A, \phi_B) - E(\phi_A, \phi_B') + E(\phi_A', \phi_B) + E(\phi_A', \phi_B')| \le 2, \quad (3)$$

is expressed in terms of spin correlation functions [16]

$$E(\phi_A, \phi_B) = P_{++} - P_{+-} - P_{-+} + P_{--}. \quad (4)$$

Here $P_{\alpha\beta}(\phi_A, \phi_B)$ are the joint probabilities to observe one particle in detector A with a spin $\alpha = \pm[+(-)$ denoting up (down)] along the ϕ_A direction, and the other in detector B with a spin $\beta = \pm$ along the ϕ_B direction. The joint probabilities are given by

$$P_{\alpha\beta}(\phi_A, \phi_B) = \{1 + \alpha\beta\cos[2(\phi_A - \phi_B)]\}/4. \quad (5)$$

Inserting $P_{\alpha\beta}$ into Eq. (4), we get $E(\phi_A, \phi_B) = \cos[2(\phi_A - \phi_B)]$. We then find that for angles $\phi_A = \pi/8$, $\phi_B = \pi/4$, $\phi_A' = 3\pi/8$, and $\phi_B' = \pi/2$, the BI in Eq. (3) is maximally violated, i.e., we get $S = 2\sqrt{2}$.

In our orbital setup (see Fig. 1), it is clear from Eq. (2) that the 12-space plays the role of a pseudospin space and the normal reservoirs act as detectors. We can thus, as in Ref. [2], formulate a BI in terms of an observable which is proportional to the corresponding joint probability $P_{\alpha\beta}(\phi_A, \phi_B)$ for our state $|\Psi_{12}\rangle$ [here, $\alpha, \beta = \pm$ denote the reservoirs; see Fig. 1]. We find below that the zero-frequency current cross correlator is given by

$$S_{\alpha\beta} \equiv 2\int_{-\infty}^{\infty} dt \langle \delta\hat{I}_{\alpha A}(t)\delta\hat{I}_{\beta B}(0)\rangle = P_0 P_{\alpha\beta}(\phi_A, \phi_B). \quad (6)$$

Here, $P_0 = 2e^3V\Gamma^2/h$, and $\delta\hat{I}_{\alpha\eta}(t) = \hat{I}_{\alpha\eta}(t) - \langle\hat{I}_{\alpha\eta}\rangle$ is the fluctuating part of the current $\hat{I}_{\alpha\eta}(t)$ in reservoir $\alpha\eta$. This leads to the important result that the Bell inequality, Eq. (3), can be directly formulated in terms of the zero-frequency current correlators in Eq. (6). We note [16] that when substituting $P_{\alpha\beta}$ with $S_{\alpha\beta}$ in Eq. (4), we must divide by the sum of all correlators $S_{++} + S_{+-} + S_{-+} + S_{--} = P_0$, which just eliminates P_0.

The simple result in Eq. (6) can be understood by considering the properties of the time-dependent correlator $\langle\delta\hat{I}_{\alpha A}(t)\delta\hat{I}_{\beta B}(0)\rangle$. It is finite only for times $t \lesssim \tau_c$, where $\tau_c = \hbar/eV$ is the correlation time of the emitted pair (see Fig. 2). In the tunneling limit under consideration, $\Gamma \ll 1$, the correlation time is much smaller than the average time between the arrival of two pairs $e/I \sim \hbar/eV\Gamma^2$. As a result, only the two electrons within a pair are correlated with each other, while electrons in different pairs are completely uncorrelated. Thus, the zero-frequency current correlator in Eq. (6) is just a coincidence counting measurement running over a long time, collecting statistics over a large number of pairs.

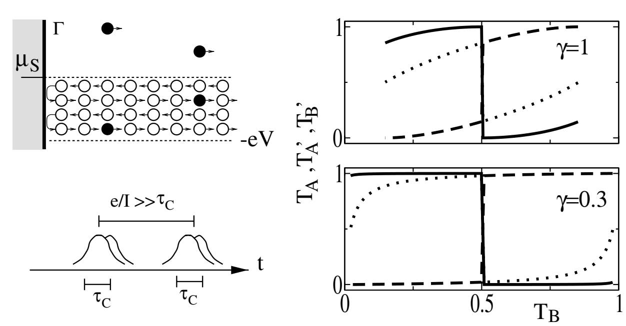

FIG. 2. Upper left: In the filled stream of incoming holes (open circles) from the normal reservoir, occasionally a hole is backreflected as an electron (solid circles). The "missing hole" (i.e., an electron) and the Andreev reflected electron constitute the pair emitted from the superconductor. Lower left: The correlation time $\tau_c = \hbar/eV$ (width of the wave packet) and the average time between emission of two subsequent pairs $e/I = \hbar/eV\Gamma^2 \gg \tau_c$ of the current correlator. The small time difference $\hbar/\Delta \ll \tau_c$ between the emissions of the two electrons in the pair is shown as a split of the wave packet. Right: The transmission probabilities T_A (dashed line), T_A' (dotted line), and T_B' (solid line) as a function of T_B [$T_\eta = \cos^2(\phi_\eta)$], giving optimal violation of the Bell inequalities for dephasing parameters $\gamma = 1$ and $\gamma = 0.3$.

For a rigorous derivation of the above result, we first discuss the role of the superconductor as an emitter of pairs of orbitally entangled electrons. Since the system is phase coherent, we can work within the scattering approach to normal-superconducting systems [17,18]. The starting point is the many-body state of the normal reservoirs, describing injection of hole quasiparticles with spin $\sigma = \uparrow, \downarrow$ at energies $0 < E < eV$,

$$|\Psi_{\rm in}\rangle = \prod_{E,\sigma} \gamma_{1A}^{\sigma\dagger}(E)\gamma_{2A}^{\sigma\dagger}(E)\gamma_{1B}^{\sigma\dagger}(E)\gamma_{2B}^{\sigma\dagger}(E)|0\rangle, \quad (7)$$

where the ground state $|0\rangle$ is the vacuum for quasiparticles in the normal reservoir. The operator $\gamma_{1A}^{\uparrow\dagger}(E)$ creates a spin-up hole plane wave with energy E (counted from the superconducting chemical potential μ_S) in lead $1A$, going out from the normal reservoirs towards the superconductor, and similarly for the other operators. The γ operators obey fermionic commutation relations

To obtain the state of the quasiparticles going out from the superconductor, $|\Psi_{\rm out}\rangle$, we note that the operators creating and destroying outgoing quasiparticles are related [17,19] to the operators of the incoming quasiparticles via a scattering matrix. The amplitude for a hole injected in arm $1A$, to be backreflected as an electron in arm $2A$, is denoted $r_{2A,1A}^{eh}$ and similarly for the other amplitudes. In the tunnel limit under consideration, the amplitude to backscatter as the same type of quasiparticle is $r_{j\eta,j'\eta'}^{hh} \sim 1$. The Andreev reflection amplitude, $r_{j\eta,j'\eta'}^{eh}$, is given by $-i\Gamma/4$ (independent of energy and spin). Processes where a hole incident on barrier 1(2) is backscattered as an electron at barrier 2(1), i.e., when a pair in the superconductor breaks up, are exponentially suppressed with the distance between the two emission points [6] and can be neglected in the present setup.

The tunneling limit $\Gamma \ll 1$ makes it relevant to change the perspective from a quasiparticle picture to an all-electron picture. An Andreev reflection, occurring with a small probability Γ^2, can be considered as a perturbation of the ground state $|\bar{0}\rangle$ in the normal reservoirs (a filled Fermi sea of electrons at energies $E < -eV$). It creates an excitation consisting of a pair of electrons (see Fig. 2). Performing a Bogoliubov transformation [i.e., $\gamma_{j\eta}^{\uparrow}(E) = c_{j\eta}^{\uparrow\dagger}(-E)$ and $\gamma_{j\eta}^{\downarrow}(E) = -c_{j\eta}^{\downarrow\dagger}(-E)$] to electron creation operators $c_{j\eta}^{\sigma\dagger}(E)$, the state of the quasiparticles going out from the superconductor becomes [20] to first order in Γ

$$|\Psi_{\rm out}\rangle = |\bar{0}\rangle + |\tilde{\Psi}\rangle + |\Psi\rangle, \quad (8)$$

where the states

$$|\tilde{\Psi}\rangle = \frac{i\Gamma}{4}\int_0^{eV} dE \sum_{j=1,2}\sum_{\eta=A,B} [c_{j\eta}^{\uparrow\dagger}(E)c_{j\eta}^{\downarrow\dagger}(-E) - c_{j\eta}^{\downarrow\dagger}(E)c_{j\eta}^{\uparrow\dagger}(-E)]|\bar{0}\rangle,$$
$$|\Psi\rangle = \frac{i\Gamma}{4}\int_{-eV}^{eV} dE[c_{1A}^{\uparrow\dagger}(E)c_{1B}^{\downarrow\dagger}(-E) - c_{1A}^{\downarrow\dagger}(E)c_{1B}^{\uparrow\dagger}(-E) + c_{2A}^{\uparrow\dagger}(E)c_{2B}^{\downarrow\dagger}(-E) - c_{2A}^{\downarrow\dagger}(E)c_{2B}^{\uparrow\dagger}(-E)]|\bar{0}\rangle \quad (9)$$

describe orbitally entangled electron "wave packet" pairs, i.e., a superposition of pairs of electrons at different energies [21]. In first quantization, the state $|\Psi\rangle$ is just the state in Eq. (2). We emphasize that the change from a quasiparticle to an all-electron picture, providing a clear picture of the entanglement, does not alter the physics.

The detection of the entanglement is done via the zero-frequency current cross correlators $S_{\alpha\beta}$ in Eq. (6). To obtain $S_{\alpha\beta}$, we insert the current operator [19], $\hat{I}_{\alpha\eta}(t) = (e/h)\int dEdE' \exp[i(E-E')t/\hbar]\sum_\sigma c_{\alpha\eta}^{\sigma\dagger}(E')c_{\alpha\eta}^{\sigma}(E)$, into Eq. (6) and average with respect to the state $|\Psi\rangle$ in Eqs. (8) and (9). The splitters are taken into account by relating the $c_{j\eta}^{\sigma}$ operators in arms $1A$ and $2A$ ($1B$ and $2B$) to the $c_{\alpha\eta}^{\sigma}$ operators in the reservoirs $+A$ and $-A$ ($+B$ and $-B$) via the scattering matrix in Eq. (1). The average current, equal in all arms $\alpha\eta$, is $\langle I_{\alpha\eta}\rangle \equiv I = (e^2/2h)\Gamma^2 V$, independent of the splitter transparency.

The ground state $|\bar{0}\rangle$ in Eq. (8) does not contribute to the correlator. Moreover, we find that $|\tilde{\Psi}\rangle$, describing two electrons emitted into the same normal lead, only contributes [14] to the cross correlator at order Γ^4 and can be neglected. From Eq. (6) we find $S_{\alpha\beta}(\phi_A, \phi_B) = P_0 P_{\alpha\beta}(\phi_A, \phi_B)$, where $P_{\alpha\beta}$ is given in Eq. (5), just as announced. It is the structure of $|\Psi\rangle$ in 12-space, i.e., $|\Psi_{12}\rangle$, that determines the angle dependence of $P_{\alpha\beta}$; all properties of $|\Psi_{AB}\rangle$ just gives rise to the constant P_0. Moreover, the calculations show that the correlator $\langle \delta\hat{I}_{1\alpha}(t)\delta\hat{I}_{2\beta}(0)\rangle$ vanishes as $(\tau_c/t)^2$ for $t \gg \tau_c$; thus $S_{\alpha\beta}(\phi_A, \phi_B)$ can be inserted in Eq. (4) and be used to violate the BI, Eq. (3).

Until now, ideal conditions have been considered. One possible source of disturbance is dephasing. In our system, dephasing can quite generally be expressed in terms of a density matrix $\rho = [|11\rangle\langle 11| + |22\rangle\langle 22| + \gamma(|11\rangle\langle 22| + |22\rangle\langle 11|)]/2$, where the off-diagonal elements, giving rise to the entanglement, are suppressed by a phenomenological dephasing parameter $0 \leq \gamma \leq 1$. The correlators $E(\phi_A, \phi_B)$ in Eq. (4) then take the form

$$E = \cos(2\phi_A)\cos(2\phi_B) + \gamma\sin(2\phi_A)\sin(2\phi_B). \quad (10)$$

By adjusting the four angles ϕ_A, ϕ_A', ϕ_B, and ϕ_B' we find that the maximal Bell parameter in Eq. (3) is

$$S = 2\sqrt{1+\gamma^2}, \quad (11)$$

which violates the BI for any $\gamma > 0$. The optimal violation angles, all in the first quadrant, are $\tan(2\phi_A) = -\gamma\cot(\phi_S)$, $\tan(2\phi_A') = \gamma\tan(\phi_S)$, and $\tan(\phi_B - \phi_B') = \mathrm{sgn}[\cos(2\phi_A)]\{[\tan^2(\phi_S) + \gamma^2]/[\gamma^2\tan^2(\phi_S) + 1]\}^{1/2}$, where $\phi_S = \phi_B + \phi_B'$ can be chosen at will. The corresponding transmission probabilities $T_\eta = \cos^2(\phi_\eta)$ are shown for $\gamma = 1$ and $\gamma = 0.3$ in Fig. 2.

The BI can thus in principle be violated for any amount of dephasing. However, it might be difficult to produce splitters which can reach all transmission probabilities between 0 and 1. This is not a problem in the absence of dephasing, $\gamma = 1$, a violation can be obtained for a large, order of unity, fraction of the "transmission probability space." However, in the limit of strong dephasing, $\gamma \ll 1$, the set of probabilities for optimal violation contains transmissions close to both 0 and 1; see Fig. 2. Expecting unity transmission to be most complicated to reach experimentally, we note that by instead choosing $T_A = T_B = 0$, $T'_B = 1/2$, and $T'_A \ll \gamma$, the inequality in Eq. (3) becomes $2|1 + \gamma T'_A| \leq 2$. This gives a violation, although not maximal, for all $\gamma \ll 1$.

Apart from dephasing there are several other effects, such as additional scattering phases, impurity scattering, or asymmetric tunnel barriers, which might alter the possibility to violate the BI. All these effects can be taken into account by replacing $\gamma \rightarrow \gamma' \cos(\phi_0)$ in Eq. (11), with the important conclusion that none of these effects will destroy the possibility to violate the BI.

The phase factor ϕ_0 is the sum of possible scattering phases from the splitters [the amplitudes in Eq. (1) are taken real], phases $\sim k_F \Delta L$ due to a difference in length, ΔL, between the normal arms (see Fig. 1), scattering phases from weak impurities, and a possible phase difference between the superconductor at the two tunnel contacts 1 and 2. As a consequence, e.g., the superconducting phase (in a loop geometry) can be modulated to compensate for the other phases.

The factor γ' plays the same role as dephasing. One possible contribution to γ' is energy dependent phases which oscillate rapidly on a scale of eV, suppressing the entangled part of the current correlator. For different lengths of the normal arms, there is always a phase $\sim E\Delta L/\hbar v_F$. This phase can, however, be neglected for $\Delta L \ll \hbar v_F/eV$, which for $eV \ll \Delta$ is fulfilled for ΔL smaller than $\hbar v_F/\Delta$. Another possibility is that, due to asymmetries of the tunnel barriers $\Gamma_1 \neq \Gamma_2$, the amplitude for the process where the pair is emitted to $|11\rangle$ is different from the process where it is emitted to $|22\rangle$. This gives rise to a state, in 12-space, $(\Gamma_1|11\rangle + \Gamma_2|22\rangle)/\sqrt{\Gamma_1^2 + \Gamma_2^2})$. In this case [22] $\gamma' = 2\Gamma_1\Gamma_2/(\Gamma_1^2 + \Gamma_2^2)$. Thus, it is in principle possible to violate BI for arbitrary asymmetry. In contrast, we find that the constraint on a single mode splitter cannot easily be relaxed.

In conclusion, we have investigated a spin-independent scheme to generate and detect two-particle orbital entanglement in a mesoscopic normal-superconductor system. The cross correlator between the currents in the two leads depends in a nonlocal way on transparencies of beam splitters in the two leads. These nonlocal correlations arise due to the entanglement of the injected pair. For appropriate choices of transparencies, the correlators give a violation of a BI for arbitrary strong dephasing.

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