Quasi-Spin-Wave Quantum Memories with a Dynamical Symmetry

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For the two-mode exciton system formed by the quasi-spin-wave collective excitation of many Λ atoms fixed at the lattice sites of a crystal, we discover a dynamical symmetry depicted by the semidirect product algebra $SU(2)\overline{\otimes}h_2$ in the large N limit with low excitations. With the help of the spectral generating algebra method, we obtain a larger class of exact zero-eigenvalue states adiabatically interpolating between the initial state of photon-type and the final state of quasi-spin-wave exciton-type. The conditions for the adiabatic passage of dark states are shown to be valid, even with the presence of the level degeneracy. These theoretical results can lead to the proposal of a new protocol for implementing quantum memory robust against quantum decoherence.

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Recent progress in quantum information has stimulated the development of new concept technologies, such as quantum computation, quantum cryptography, and quantum teleportation [1]. The practical implementation of these quantum protocols relies on the construction of both quantum memories (QMEs) and quantum carriers (QCAs) free of quantum decoherence. While photons can be generally taken as quantum carriers, quantum memories should correspond to localized systems capable of storing and releasing quantum states reversibly. Moreover, to control the coherent transfer of information, there should be a time dependent mechanism for turning on and turning off the interaction between QME and QCA at appropriate instants of time.

A single atom in a cavity QED system [2] seems to satisfy the above mentioned requirements for QME using the Raman adiabatic passage mechanism [3]. To achieve the *strong coupling* required for a practical QME a very elegant method has been proposed recently [4–6], where ensembles of Λ-type atoms are used to store and transfer the quantum information of photons by the collective atomic excitations through electromagnetically induced transparency (EIT) [7]. Some experiments [8,9] have already demonstrated the central principle of this technique — the group velocity reduction. The recent success in experiment also shows the power of such an atomic ensemble QME [10] and motivates additional theoretical work [11]. But there still exists the decoherence problem. An ensemble consists of many moving atoms and atoms in different spatial positions may experience different couplings to the controlling external fields. This results in decoherence in quantum information processing [12]. To avoid the spatial-motion induced decoherence, one naturally considers the case that each Λ atom is fixed at a lattice site of a crystal. The most recent experiment of the ultraslow group velocity of light in a crystal of $Y_2S_iO_5$ [13] proposes the possibility of implementing robust quantum memories by utilizing the solid state exciton system.

In this Letter, we study a system consisting of the quasi-spin-wave collective excitations of many Λ-type atoms. In this system, most spatially fixed atoms stay in the ground state, the two quasispin collective excitations to two excited states behave as two types of bosons, and thus a two-mode exciton system forms. We prove that in the large N limit with low excitations, this excitonic system possesses a hidden dynamical symmetry described by the semidirect product $SU(2)\overline{\otimes}h_2$ of quasispin $SU(2)$ and the exciton Heisenberg-Weyl algebra h_2. With the help of the spectrum generating algebra theory [14] based on $SU(2)\overline{\otimes}h_2$, we can construct the eigenstates of the two-mode exciton-photon system including the collective dark states as a special class. Since the external classical field is controllable, the quantum information can be coherently transferred from the cavity photon to the exciton system and vice versa. Therefore, the two-mode quasi-spin-wave exciton system can serve as a robust quantum memory.

The model system we consider consists of a crystal with N lattice sites as shown in Fig. 1. There are N three-level atoms of the Λ-type with the excited state $|a\rangle$, the relative ground state $|b\rangle$, and the metastable lower state $|c\rangle$. They interact with two single-mode optical fields. The transition from $|a\rangle$ to $|b\rangle$ of each atom is approximately resonantly coupled to a quantized radiation mode (with coupling constant g and annihilation operator a), and the transition from $|a\rangle$ to $|c\rangle$ is driven by an exactly resonant classical field of Rabi frequency Ω. In recent years, for the similar exciton system in a crystal slab with spatially fixed two level atoms, both quantum decoherence and the fluorescence process have been extensively studied [15].

For convenience we introduce the notation $\mathbf{j} = (a_x j_x, a_y j_y, a_z j_z)$ to denote the position of the $\mathbf{j}$th site where a_u is the length of the crystal cell along the

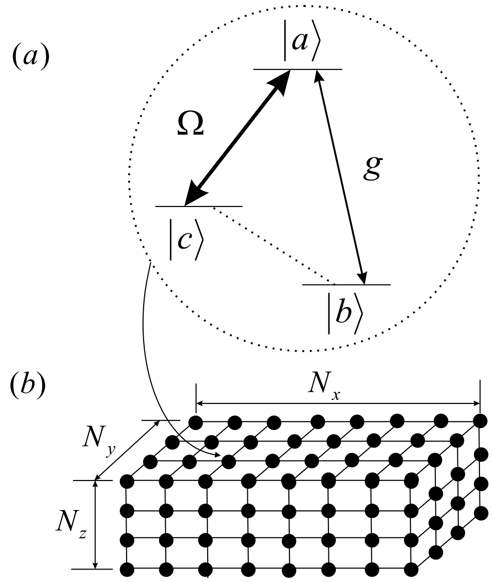

FIG. 1. Configuration of the proposed quantum memory with Λ-type atoms. (a) Located at lattice sites of crystal and (b) resonantly coupled to a control classical field and a quantized probe field.

u direction and $j_u = 1, 2, \cdots, N_u$ for $u = x, y, z$. Then the quantum dynamics of the total system is described by the following Hamiltonian in the interaction picture:

$$H = ga \sum_{\mathbf{j}=1}^{N} \exp(i\mathbf{K}_{ba} \cdot \mathbf{j})\sigma_{ab}^{\mathbf{j}} + \Omega \sum_{\mathbf{j}=1}^{N} \exp(i\mathbf{K}_{ca} \cdot \mathbf{j})\sigma_{ac}^{\mathbf{j}} + \text{H.c.}, \tag{1}$$

where $N = N_x N_y N_z$, and $\mathbf{K}_{ba}$ and $\mathbf{K}_{ca}$ are, respectively, the wave vectors of the quantum and classical light fields. The quasispin operators $\sigma_{\alpha\beta}^{\mathbf{j}} = |\alpha\rangle_{\mathbf{jj}}\langle\beta|$ $(\alpha, \beta = a, b, c)$ for $\alpha \neq \beta$ describe the transition between the levels of $|a\rangle$, $|b\rangle$, and $|c\rangle$.

The function of quantum memory is understood in terms of its quantum state mapping technique. This motivates us to define the basic quantum states according to the concrete form of the interaction. We denote by $|v\rangle = |b_1, b_2, \cdots, b_N\rangle$ the collective ground state with all N atoms staying in the same single particle ground state $|b\rangle$. It is obvious that, from the ground state $|v\rangle$, the first order and second order perturbations of the interaction can create the one exciton quasispin wave states $|1_a\rangle$ and $|1_c\rangle$:

$$|1_s\rangle = \frac{1}{\sqrt{N}} \sum_{\mathbf{j}=1}^{N} e^{i\mathbf{K}_{bs}\cdot\mathbf{j}} |b, b, \cdots, \overbrace{s}^{\mathbf{j}\text{th}}, \cdots, b\rangle, \tag{2}$$

for $s = a, c$, respectively. The wave vector $\mathbf{K}_{bc} = \mathbf{K}_{ba} - \mathbf{K}_{ca}$ is introduced to depict the second order transition process from $|b\rangle$ to $|c\rangle$ as shown in Fig. 2(a). Its collective effect can be described by operator

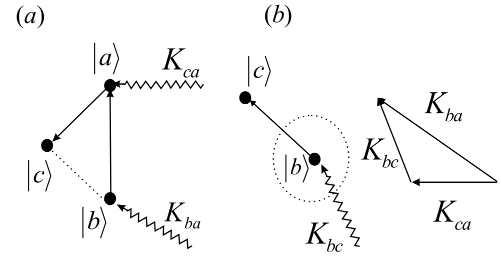

FIG. 2. Illustration of the second order process $|b\rangle \rightarrow |a\rangle \rightarrow |c\rangle$ induced by the classical and quantized lights.

$$C = \frac{1}{\sqrt{N}} \sum_{\mathbf{j}=1}^{N} e^{-i\mathbf{K}_{bc}\cdot\mathbf{j}} \sigma_{bc}^{\mathbf{j}}, \tag{3}$$

which gives $|1_c\rangle \equiv C^\dagger|v\rangle$. Correspondingly, the collective excitation from $|b\rangle$ to $|a\rangle$ is described by

$$A = \frac{1}{\sqrt{N}} \sum_{\mathbf{j}=1}^{N} e^{-i\mathbf{K}_{ba}\cdot\mathbf{j}} \sigma_{ba}^{\mathbf{j}}, \tag{4}$$

which gives $|1_a\rangle \equiv A^\dagger|v\rangle$. In the large N limit with the low excitation condition that there are only a few atoms occupying $|a\rangle$ or $|c\rangle$ [16], the two-mode quasi-spin-wave excitations behave as two bosons since in this case they satisfy the bosonic commutation relations $[A, A^\dagger] = 1$, $[C, C^\dagger] = 1$. In the same limit, it is worth pointing out that $[A, C] = 0$ and $[A, C^\dagger] = -T_-/N \rightarrow 0$, thus these quasi-spin-wave low excitations are independent of each other.

In terms of these two-mode exciton operators, the interaction Hamiltonian reads

$$H = g\sqrt{N}aA^\dagger + \Omega T_+ + \text{H.c.}, \tag{5}$$

where the collective operators

$$T_- = \sum_{\mathbf{j}=1}^{N} e^{-i\mathbf{K}_{ca}\cdot\mathbf{j}} \sigma_{ca}^{\mathbf{j}}, \quad T_+ = (T_-)^\dagger, \tag{6}$$

generate the SU(2) algebra together with the third generator $T_3 = \sum_{\mathbf{j}=1}^{N}(\sigma_{aa}^{\mathbf{j}} - \sigma_{cc}^{\mathbf{j}})/2$. It is very interesting to observe that the exciton operators and the SU(2) generators span a larger Lie algebra. By a straightforward calculation we obtain

$$[T_-, C] = -A, [T_+, A] = -C. \tag{7}$$

Denote by h_2 the Lie algebra generated by A, $A^\dagger$, C, and $C^\dagger$. It then follows that $[\text{SU}(2), h_2] \subset h_2$. This means that in the large N limit with the low excitation condition the operators A, $A^\dagger$, C, $C^\dagger$, T_3, $T_\pm$, and the identity 1 span a semidirect product Lie algebra $\text{SU}(2)\overline{\otimes}h_2$. In the following discussion we focus on this case unless otherwise explicitly specified. Since the Hamiltonian H can be expressed as a function of the generators of $\text{SU}(2)\overline{\otimes}h_2$, one says that the two-mode exciton system possesses a dynamical symmetry governed by the dynamical "group" (or dynamical algebra) $\text{SU}(2)\overline{\otimes}h_2$. The discovery of this dynamical symmetry leads us, by the spectrum generating algebra method [14], to find H-invariant subspaces, in which one can diagonalize the Hamiltonian easily.

To that cause, we define

$$D = a\cos\theta - C\sin\theta \tag{8}$$

with $\theta(t)$ satisfying $\tan\theta(t) = g\sqrt{N}/\Omega(t)$. It mixes the electromagnetic field and collective atomic excitations of quasispin wave. Evidently, $[D, D^\dagger] = 1$ and $[D, H] = 0$. Thus the Heisenberg-Weyl group h generated by D and $D^\dagger$ is a symmetry group of the two-mode exciton-photon system. We introduce the state $|\mathbf{0}\rangle = |v\rangle \otimes |0\rangle_L$, where $|0\rangle_L$ is the vacuum of the electromagnetic field; we find $D|\mathbf{0}\rangle = 0$, and it is an eigenstate of H with zero eigenvalue. Consequently, a degenerate class of eigenstates of H with zero eigenvalue can be constructed naturally as follows:

$$|d_n\rangle = [n!]^{-1/2} D^{\dagger n}|\mathbf{0}\rangle. \tag{9}$$

Physically, the above dressed state is canceled by the interaction Hamiltonian and thus is called a dark state or a dark-state polariton (DSP). A DSP traps the electromagnetic radiation from the excited state due to quantum interference canceling. For the case with an ensemble of free moving atoms, the similar DSP was obtained in Refs. [4–6] to clarify the physics of the state-preserving slow light propagation in EIT associated with the existence of quasiparticles.

Now starting from these dark states $|d_n\rangle$, we can use the spectrum generating algebra method to build other eigenstates for the total system. To this end we introduce the bright-state polariton operator

$$B = a\sin\theta + C\cos\theta. \tag{10}$$

It is easy to check that $[B, B^\dagger] = 1$ and $[D, B^\dagger] = [D, B] = 0$. Evidently $[A, B] = [A, B^\dagger] = 0$; this amounts to the fact that A commutes with C and $C^\dagger$ in the large N limit with low excitations. What is crucial for our purpose is the commutation relations $[H, Q_\pm^\dagger] = \pm\epsilon Q_\pm^\dagger$ for $Q_\pm = (1/\sqrt{2})(A \pm B)$, where $\epsilon = \sqrt{g^2N + \Omega^2}$. Thanks to these commutation relations we can construct the eigenstates

$$|e(m, k; n)\rangle = [m!k!]^{-1/2} Q_+^{\dagger m} Q_-^{\dagger k}|d_n\rangle, \tag{11}$$

as the dressed states of the two-mode exciton system. The corresponding eigenvalues are

$$E(m, k) = (m - k)\epsilon, m, k = 0, 1, 2 \cdots. \tag{12}$$

We notice that for each given pair of indices (m, k), $\{|e(m, k; n)\rangle | n = 0, 1, 2, \cdots\}$ defines a degenerate set of eigenstates. Physically the spectral structure of the dressed two-mode exciton system resembles that of a two-mode harmonic oscillator, but its energy level number is finite and each energy level possesses a very large degeneracy.

The above equations show that there exists a larger class $\mathbf{S} : \{|e(m, m; n)\rangle \equiv |d(m, n)\rangle | m, n = 0, 1, 2, \cdots\}$ of states of zero eigenvalue $E(m, m) = 0$. They are constructed by acting $Q_+^\dagger Q_-^\dagger$ m times on $|d_n\rangle$:

$$|d(m, n)\rangle = \sum_{k=0}^{m} \frac{A^{\dagger 2(m-k)} B^{\dagger 2k}}{2^m (m - k)! k!} |d_n\rangle. \tag{13}$$

This larger degeneracy is physically rooted in the larger symmetry group h_2 generated by $Q_+^\dagger Q_-^\dagger$ and Q_+Q_- together with D and $D^\dagger$. The original quantum memory defined by $\{|d_n\rangle = |d(0, n)\rangle\}$ in Refs. [4–6] actually is a special subset of the larger class.

Now we consider whether these states of zero eigenvalue can work well as a quantum memory by the adiabatic manipulation [4–6]. The quantum adiabatic theorem [17,18] for degenerate cases tells us that, under the adiabatic condition

$$\left|\frac{\langle e(m, k; n)|\partial_t|d(m, n)\rangle}{E(m, k)}\right| \sim \frac{g\sqrt{N}|\dot{\Omega}(t)|}{\epsilon^3} \ll 1, \tag{14}$$

the adiabatic evolution of any degenerate system will keep itself within the block $\mathbf{S}$ of dark states with the same instantaneous eigenvalue zero. However, it does not forbid transitions within states in this block $\mathbf{S}$, such as those between $\{|d_n\rangle = |d(0, n)\rangle\}$ and $\{|d(m, n)\rangle (m \neq 0)\}$. So it is important to consider whether there exists any dynamical mechanism to forbid such transitions. Actually this issue has been uniformly ignored in all previous studies even for the degenerate set $\{|d_n\rangle\}$.

We can generally consider this problem by defining the zero-eigenvalue subspaces $\mathbf{S}^{[m]}:\{|d(m,n)\rangle | n = 0, 1, 2, \cdots\}$, $\mathbf{S}^{[0]} = \mathbf{S}$. The complementary part of the direct sum $\mathbf{DS} = \mathbf{S}^{[0]} \oplus \mathbf{S}^{[1]} \oplus \cdots$ of all dark-state subspaces is $\mathbf{CS} = \{|e(m, k, n)\rangle | k \neq m, n = 0, 1, 2, \cdots\}$ in which each $|e(m, k, n)\rangle$ has a nonzero eigenvalue. Any state $|\phi(t)\rangle = \sum_{m,n} c_n^{[m]}(t)|d(m, n)\rangle$ in $\mathbf{S}^{[m]}$ evolves according to

$$i\frac{d}{dt} c_n^{[m]}(t) = \sum_{m',n'} D_{mn}^{m'n'} c_{n'}^{[m']}(t) + F[\mathbf{CS}], \tag{15}$$

where $F[\mathbf{CS}]$, which can be ignored under the adiabatic conditions [17,18], represents a certain functional of the complementary states and $iD_{m'n'}^{mn} = \langle d(m', n')|\partial_t|d(m, n)\rangle$. Considering $\partial_\theta B = D$ and $\partial_\theta D = -B$, we have $iD_{m'n'}^{mn} = \dot{\theta}\langle d(m', n')|\partial_\theta|d(m, n)\rangle$. The equation about $\partial_\theta|d(m, n)\rangle$ contains four terms: $|e(m, m \mp 1; n \pm 1)\rangle$ and $|e(m \mp 1, m; n \pm 1)\rangle$. This implies the exact result $\langle d(m', n')|\partial_\theta|d(m, n)\rangle = 0$, showing there is indeed no mixing among the dark states during the adiabatic evolution. Viewed from the physical aspect, this can also be understood as the adiabatic change of external parameters does not lead the system to enter into the complementary space $\mathbf{CS}$. Notice that only for the nonadiabatic evolution will the nonzero matrix elements $\langle e(m', k, n')|\partial_\theta|d(m, n)\rangle$ be a cause for state mixing. The same physics has been considered in the context of the Abelianization of the non-Abelian gauge structure induced by an adiabatic process [17,18]. This argument gives the necessary theoretical support for the practical realization of the original scheme of quantum memory by Fleischhauer, Lukin, and their collaborators [4–6].

Based on the above consideration, we thus claim that, for each fixed $m \neq 0$, each subspace $\mathbf{S}^{[m]}$ can work formally as a quantum memory different from that in Refs. [4–6]. We introduce the notation

$$|\mathbf{A}, \mathbf{P}, m\rangle = \frac{1}{2^m m!}(A^{\dagger 2} - P^{\dagger 2})^m |\mathbf{0}\rangle \tag{16}$$

for $\mathbf{P} = \mathbf{a}, \mathbf{C}$. Both the initial state $|d(m,n)\rangle|_{\theta=0} = |\mathbf{A}, \mathbf{C}, m\rangle \otimes |n\rangle_L$ and the final state $|d(m,n)\rangle|_{\theta=\pi/2} = |n\rangle_C \otimes (-1)^n |\mathbf{A}, \mathbf{a}, m\rangle$ have factorization structure. Thus we can use the general initial state $|s(0)\rangle = \sum_n c_n |n\rangle_L$ of single-mode light to record quantum information and prepare the exciton in a paired state $|\mathbf{A}, \mathbf{C}, m\rangle$. When one rotates the mixing angle θ from zero to $\pi/2$ by changing the coupling strength $\Omega(t)$ adiabatically, the total system will reach the final state $|S(t)\rangle = (\sum_n c_n |n\rangle_C) \otimes |\mathbf{A}, \mathbf{a}, m\rangle$ with the c-mode quasi-spin-wave decoupling with the other parts. From the viewpoint of quantum measurement the decoding process is then to average over the states of the photon and the A exciton and to obtain the pure state density matrix $\rho_C = \sum_{n,m} c_n c_m^* |n\rangle_{cc}\langle m|$, which is the same as that for the initial photons. Therefore, the above discussion suggests a new protocol of storing quantum information when the decay of the excited state is small enough during adiabatic manipulation.

Before concluding, we address the fact that the individual atoms in the generalized states $|d(m,n)\rangle$ have excited state components, and therefore $|d(m,n)\rangle$ is not totally dark in practice. If the excited state decays faster, the generalized states $|d(m,n)\rangle$ would also decay during slow adiabatic manipulation. This metastable nature leads to an undesirable effect for memory application. We also point out that the present treatment is valid only for the low density excitation regime where the bosonic modes of the quasi-spin-wave excitations can be used effectively. Therefore the above down Fock state formally written as $A^{\dagger m} C^{\dagger n} |\mathbf{0}\rangle$ does not make sense when m or n is large. By the mathematical duality, the situation with extremely high excitation can be dealt with in a similar manner. In fact, the serious difficulty lies only in the region where the excitation is neither very low nor very high. In that case, it turns out that the boson commutation relation of the excitaton operators must be modified, for example, to the q-deformed one $[q = 1 - O(\frac{1}{N})]$ [16]. Physically this modification will cause quantum decoherence of the collective degrees of freedom in the exciton system. Finally, we emphasize that, though in our model system assumed to be located at regular lattice sites as in a crystal, our results (at least in mathematical formulation) remain valid for an ensemble of atoms with random spatial positions, provided that we can ignore the kinetic energy terms (of the center of mass motion) of the atom. It seems that the ensemble of free atoms can function as quantum memory of the same kind. However, the strict treatment of the atomic ensemble based quantum memory should include the kinetic energy terms of the atom center of mass. The momentum transfer of atomic center of mass can induce additional quantum decoherence [12]. In our present protocol, this decoherence effect is partly overcome by fixing atoms at lattice sites and thus neglecting the kinetic energy terms.

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