Unity of Elementary Particles and Forces in Higher Dimensions

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The idea of unifying all the gauge and Yukawa forces as well as the gauge, Higgs, and fermionic matter particles naturally leads us to a simple gauge symmetry in higher dimensions with supersymmetry. We present a model in which, for the first time, such a unification is achieved in the framework of quantum field theory.

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The idea of grand unification in four dimensions (4D) was introduced to unify the three different forces of nature, strong, weak, and electromagnetic, as a single force in a simple gauge group such as $SU(5)$ [1]. The gauge symmetry is broken spontaneously at a high scale around 10^{16} GeV, called the grand unified theory scale, using Higgs mechanism. The supersymmetric versions of these theories are in good agreement with experiment. These theories also make several interesting predictions such as proton decay and neutrino masses. The shortcomings of such theories are that they do not unify all the forces. The Yukawa interactions remain unrelated to the gauge interactions. Also, all the particles are not unified: gauge, Higgs and matter fermions remain in different representations of the gauge symmetry group. In this Letter, we propose a simple model which unifies the forces (strong, electroweak, as well as Yukawa) in the gauge interaction of a unifying symmetry group, as well as unifies the elementary particles which include the gauge, Higgs, and two families of fermions in a single adjoint representation of this symmetry group.

An attractive motivation to extend the number of dimensions beyond the usual four is that the variety of particles in nature can be understood by means of a geometrical language. In the original idea by Kaluza-Klein [2], the 4D gauge fields are included in the higher dimensional metric tensor. Gauge symmetries can be broken and the matter fermions can be chiral by compactifying these extra dimensions in a suitable orbifold and using appropriate boundary conditions [3–5]. Certain generic problems in 4D grand unified theories, such as doublet-triplet splitting, are naturally solved in such a framework [5,6]. Another attractive feature in such higher dimensional theories is that the gauge fields with the extra dimensional components behave as scalar fields in 4D, and these can be used as Higgs fields which break gauge symmetry [3,7–10]. The masses of such scalar fields are prohibited by gauge invariance, and in supersymmetric theories, these remain massless at the low energy due to the nonrenormalization theorem. Thus those scalar fields can be good candidates for the low energy Higgs fields which break electroweak symmetry.

In the higher dimensional supersymmetric theories, the gauge multiplet, which is an extended supersymmetry multiplet, contains both the vector multiplet and the chiral supermultiplets in the language of 4D $N = 1$ supersymmetry. Assigning different transformation property between vector multiplet and chiral supermultiplets, we can make vector multiplet massless but chiral supermultiplets heavy in 4D, which means that the extended supersymmetry can be broken down to $N = 1$ supersymmetry. If we also break gauge symmetry through orbifold compactification simultaneously, the chiral supermultiplets which correspond to the broken generators can have zero modes, which remain massless at low energy. We can, then, identify such supermultiplets with the low energy fermion and Higgs fields. Several of these ideas have been separately realized, such as gauge-Higgs unification with gauge-Yukawa unification [11], or having three families of fermions in a higher dimensional multiplet [12,13], or gauge-Higgs-matter unification with one family of fermions [14].

We consider $N = 2$ supersymmetry in 6D with gauge group $SO(16)$. The gauge symmetry $SO(16)$ is broken down to Pati-Salam [15] symmetry with three extra $U(1)$'s in 4D through T^2/Z_6 orbifold compactification. The gauge bosons, Higgs fields, and the two families of quarks and leptons are unified in the 6D $N = 2$ gauge multiplet. Another family is naturally introduced as brane fields to cancel the gauge anomalies. Thus, this gives a three-family model. The numerical agreement of this gauge-Yukawa unification prediction for all the gauge and the third family Yukawa couplings is good as we shall see later.

The $N = 2$ supersymmetry in 6D corresponds to $N = 4$ supersymmetry in 4D, thus only the gauge multiplet can be introduced in the bulk. In terms of 4D $N = 1$ language, the gauge multiplet contains vector multiplet $V(A_\mu, \lambda)$ and three chiral multiplets Σ, Φ, and Φ^c in the adjoint representation of the gauge group. The fifth and sixth components of the gauge fields, A_5 and A_6, are contained in the lowest component of Σ, i.e., $\Sigma|_{\theta=\bar\theta=0} = (A_6 + iA_5)/\sqrt{2}$. The bulk action can be found in [16].

The T^2/Z_6 orbifold is constructed by identifying the complex coordinate z of the extra dimensions under

Z_6:$z \to \omega z$, where $\omega^6 = 1$. We can impose the transformation property of the gauge multiplet as [16,17]

$$V(x^\mu, \omega z, \bar{\omega}\bar{z}) = \mathcal{R}\cdot V(x^\mu, z, \bar{z}), \quad (1)$$

$$\Sigma(x^\mu, \omega z, \bar{\omega}\bar{z}) = \bar{\omega}\mathcal{R}\cdot \Sigma(x^\mu, z, \bar{z}), \quad (2)$$

$$\Phi(x^\mu, \omega z, \bar{\omega}\bar{z}) = \omega^l\mathcal{R}\cdot \Phi(x^\mu, z, \bar{z}), \quad (3)$$

$$\Phi^c(x^\mu, \omega z, \bar{\omega}\bar{z}) = \omega^m\mathcal{R}\cdot \Phi^c(x^\mu, z, \bar{z}), \quad (4)$$

where $\mathcal{R}$ acts on the gauge space satisfying $\mathcal{R}^6$ to be the identity mapping since $V(x^\mu, \omega^6 z, \bar{\omega}^6\bar{z})$ should be equal to $V(x^\mu, z, \bar{z})$. Nontrivial $\mathcal{R}$ breaks the gauge symmetry. These transformation properties break $N = 4$ supersymmetry down to $N = 1$ in 4D. Because the bulk action contains the trilinear term, $\Sigma[\Phi, \Phi^c]$, and the action must be invariant under the transformations (1)–(4), we have a relation $l + m \equiv 1 \pmod 6$. We will choose $l \equiv 4$, $m \equiv 3$.

The adjoint of $SO(16)$ is represented by 16×16 real antisymmetric matrices. The gauge twisted mapping $\mathcal{R}$ is represented as $\mathcal{R}\cdot V \equiv RVR^T$ where R is a matrix which satisfies that $R^6 = \pm I$ (I is the identity matrix). If we take the matrix $R = \mathrm{diag}(\overbrace{\omega, \ldots, \omega}^{8}, \overbrace{\bar{\omega}, \ldots, \bar{\omega}}^{8})$, $SO(16)$ is broken down to $SU(8) \times U(1)$. The adjoint $\mathbf{120}$ is decomposed as $\mathbf{120} = \mathbf{63}_0 + \mathbf{1}_0 + \mathbf{28}_{-1} + \overline{\mathbf{28}}_1$ under $SU(8) \times U(1)$. The $U(1)$ charges in some normalization are given by subscript. In the following, we denote this $U(1)$ symmetry by $U(1)_3$. The vector multiplet V is decomposed as $V_{\mathbf{120}} = V_{\mathbf{63}} + V_{\mathbf{1}} + V_{\mathbf{28}} + V_{\overline{\mathbf{28}}}$, and similarly for the chiral multiplets Σ, Φ, and Φ^c. The Z_6 charges are determined as 1 for $V_{\mathbf{63}} + V_{\mathbf{1}}$, ω^2 for $V_{\mathbf{28}}$, and $\bar{\omega}^2$ for $V_{\overline{\mathbf{28}}}$. The Z_6 charge assignments for Σ, Φ, and Φ^c are obtained by multiplying with $\bar{\omega}$, ω^l, and ω^m, respectively.

Now we take the matrix $R = \mathrm{diag}(\omega^{a/2}R_8, \bar{\omega}^{a/2}R_8^\dagger)$ where R_8 is an 8×8 unitary matrix which satisfies $R_8^6 = I$. The Z_6 transformation property for the vector multiplet V is easily obtained as

$$V_{\mathbf{63}}(x^\mu, \omega z, \bar{\omega}\bar{z}) = R_8 V_{\mathbf{63}}(x^\mu, z, \bar{z})R_8^\dagger, \quad (5)$$

$$V_{\mathbf{28}}(x^\mu, \omega z, \bar{\omega}\bar{z}) = \omega^a R_8 V_{\mathbf{28}}(x^\mu, z, \bar{z})R_8, \quad (6)$$

$$V_{\overline{\mathbf{28}}}(x^\mu, \omega z, \bar{\omega}\bar{z}) = \bar{\omega}^a R_8^\dagger V_{\overline{\mathbf{28}}}(x^\mu, z, \bar{z})R_8^\dagger, \quad (7)$$

$$V_{\mathbf{1}}(x^\mu, \omega z, \bar{\omega}\bar{z}) = V_{\mathbf{1}}(x^\mu, z, \bar{z}). \quad (8)$$

The transformation property for Σ, Φ, and Φ^c are obtained by multiplying with $\bar{\omega}$, ω^l, and ω^m, respectively. We choose the unitary matrix R_8 as

$$R_8 = \mathrm{diag}(\omega^b, \omega^b, \omega^b, \omega^b, \omega^c, \omega^c, \omega^d, \omega^d). \quad (9)$$

With this choice (a is odd and b, c, d are different numbers modulo 6), $SO(16)$ breaks down to $SU(4) \times SU(2)_L \times SU(2)_R \times U(1)^3$, and the 4D theory becomes $N = 1$ supersymmetric Pati-Salam model with three extra $U(1)$ symmetries.

The $SU(8) \times U(1)_3$ representations $\mathbf{63}_0$, $\mathbf{1}_0$, $\mathbf{28}_{-1}$, and $\overline{\mathbf{28}}_1$ are decomposed under the $SU(4) \times SU(2)_L \times SU(2)_R$ in the following matrix form:

$$\mathbf{63}_0 = \begin{bmatrix} (\mathbf{15},\mathbf{1},\mathbf{1}) & (\mathbf{4},\mathbf{2},\mathbf{1}) & (\mathbf{4},\mathbf{1},\mathbf{2}) \\ (\bar{\mathbf{4}},\mathbf{2},\mathbf{1}) & (\mathbf{1},\mathbf{3},\mathbf{1}) & (\mathbf{1},\mathbf{2},\mathbf{2}) \\ (\bar{\mathbf{4}},\mathbf{1},\mathbf{2}) & (\mathbf{1},\mathbf{2},\mathbf{2}) & (\mathbf{1},\mathbf{1},\mathbf{3}) \end{bmatrix} + (\mathbf{1},\mathbf{1},\mathbf{1}) + (\mathbf{1},\mathbf{1},\mathbf{1}), \quad (10)$$

$$\mathbf{1}_0 = (\mathbf{1},\mathbf{1},\mathbf{1}), \quad (11)$$

$$\mathbf{28}_{-1} = \begin{bmatrix} (\mathbf{6},\mathbf{1},\mathbf{1}) & (\mathbf{4},\mathbf{2},\mathbf{1}) & (\mathbf{4},\mathbf{1},\mathbf{2}) \\ & (\mathbf{1},\mathbf{1},\mathbf{1}) & (\mathbf{1},\mathbf{2},\mathbf{2}) \\ \text{(anti-sym)} & & (\mathbf{1},\mathbf{1},\mathbf{1}) \end{bmatrix}, \quad (12)$$

$$\overline{\mathbf{28}}_1 = \begin{bmatrix} (\mathbf{6},\mathbf{1},\mathbf{1}) & (\bar{\mathbf{4}},\mathbf{2},\mathbf{1}) & (\bar{\mathbf{4}},\mathbf{1},\mathbf{2}) \\ & (\mathbf{1},\mathbf{1},\mathbf{1}) & (\mathbf{1},\mathbf{2},\mathbf{2}) \\ \text{(anti-sym)} & & (\mathbf{1},\mathbf{1},\mathbf{1}) \end{bmatrix}, \quad (13)$$

The Z_6 transformation properties for these decomposed representations of the vector multiplet V are easily obtained from Eqs. (5)–(8), and similarly for the chiral multiplets Σ, Φ, and Φ^c.

We choose the matrix R_8 as

$$R_8 = \mathrm{diag}(1, 1, 1, 1, \omega^5, \omega^5, \omega^2, \omega^2) \quad (14)$$

to pick up one chiral family from $\mathbf{63}$. Then, the number a is chosen as $a = 3$ to make all the $V_{\mathbf{28}}$ and $V_{\overline{\mathbf{28}}}$ massive. We can extract zero modes in the chiral superfields Σ, Φ, and Φ^c through the transformation property (2)–(4) with $(l, m) \equiv (4, 3)$. The zero modes are listed in Table I. The L_i and $\bar{R}_i$ include left- and right-handed quarks and leptons and C_i includes vector-like colored Higgs. The model thus includes two chiral families in the bulk and three electroweak bidoublets H_i. The subscripts in Table I denote the charges under the $U(1)_1 \times U(1)_2 \times U(1)_3$ symmetry. The $U(1)_1$ and $U(1)_2$ symmetries originate from the $SU(8)$ generators: $\mathrm{diag}(1, 1, 1, 1, -1, -1, -1, -1)/2$ for $U(1)_1$ and $\mathrm{diag}(1, 1, 1, 1, 1, 1, -3, -3)/2$ for $U(1)_2$.

Since the three chiral multiplets Σ, Φ, and Φ^c are in the gauge multiplet, those chiral superfields have gauge interactions with each other in 6D. The trilinear gauge interaction term $\sqrt{2}\Sigma[\Phi, \Phi^c]$ in the action includes bulk

TABLE I. A list of the zero modes in chiral multiplets.

	$\mathbf{63}$	$\mathbf{28}$	$\overline{\mathbf{28}}$
Σ	L_3:$(\mathbf{4},\mathbf{2},\mathbf{1})_{1,0,0}$	S_1:$(\mathbf{1},\mathbf{1},\mathbf{1})_{-1,1,-1}$ S_2:$(\mathbf{1},\mathbf{1},\mathbf{1})_{-1,-3,-1}$	$\bar{R}_2$:$(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})_{0,1,1}$
Φ	$\bar{R}_3$:$(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})_{-1,-2,0}$	L_2:$(\mathbf{4},\mathbf{2},\mathbf{1})_{0,1,-1}$	H_3:$(\mathbf{1},\mathbf{2},\mathbf{2})_{1,1,1}$
Φ^c	H_1:$(\mathbf{1},\mathbf{2},\mathbf{2})_{0,2,0}$ H_2:$(\mathbf{1},\mathbf{2},\mathbf{2})_{0,-2,0}$	C_1:$(\mathbf{6},\mathbf{1},\mathbf{1})_{1,1,-1}$	C_2:$(\mathbf{6},\mathbf{1},\mathbf{1})_{-1,-1,1}$

superpotential for the zero modes,

$$S = \int d^6x \int d^2\theta y_6 (L_3 H_1 \bar{R}_3 + L_2 H_2 \bar{R}_2 + (H_1 S_2 - H_2 S_1) H_3 + \bar{R}_3 C_1 \bar{R}_2 + L_3 C_2 L_2) + \text{H.c.}, \qquad (15)$$

which includes Yukawa couplings.

Taking into account the normalization factors of the wave functions in the kinetic term, and using conventional normalization of the gauge coupling, we find that the 6D Yukawa coupling is equal to 6D gauge coupling, $y_6 = g_6$. The corresponding 4D couplings are derived as the coordinates of the extra dimensions are integrated out in the action. Thus, the 4D Yukawa and gauge couplings can be the same dimensionless number if the following conditions are satisfied [14]: (i) The brane-localized gauge and Yukawa interactions and their threshold corrections can be negligible. (ii) The zero modes of the fermions are not localized at different points on the orbifold. (iii) The 4D fields are not largely mixed with other brane-localized fields.

We now discuss the implications of the bulk interaction in Eq. (15). Suppose that the vacuum expectation values are given to S_1 and S_2 which are singlets under Pati-Salam symmetry. Then one linear combination of H_1 and H_2, as well as H_3 become heavy, and the following linear combination remains light:

$$H = (H_1 S_1 + H_2 S_2)/\sqrt{S_1^2 + S_2^2}. \qquad (16)$$

The Yukawa coupling terms are rewritten by using this light bidoublet Higgs as

$$L_3 H_1 \bar{R}_3 + L_2 H_2 \bar{R}_2 \to \frac{S_1 L_3 H \bar{R}_3 + S_2 L_2 H \bar{R}_2}{\sqrt{S_1^2 + S_2^2}}. \qquad (17)$$

The bulk superpotential term has a Z_2 flavor symmetry such as

$$L_3 \leftrightarrow L_2, \quad \bar{R}_3 \leftrightarrow \bar{R}_2, \quad H_1 \leftrightarrow H_2, \quad S_1 \leftrightarrow S_2, \quad H_3 \leftrightarrow -H_3, \quad C_1 \leftrightarrow -C_1, \quad C_2 \leftrightarrow -C_2. \qquad (18)$$

The vacuum expectation values of S_1 and S_2 break this Z_2 symmetry. Assuming $\langle S_1 \rangle \gg \langle S_2 \rangle$, we obtain the fermion mass hierarchy between the 3rd and 2nd family, supposing that L_3 and $\bar{R}_3$ are for the 3rd family and L_2 and $\bar{R}_2$ are for the 2nd family. Of course, this is just a toy structure, since we still have a wrong relation, $m_c/m_t = m_s/m_b = m_\mu/m_\tau$. We do not have flavor mixing in the bulk superpotential either. These problems can be solved by introducing brane-localized interaction. The vacuum expectation values of $SU(2)_R$ triplet and $SU(4)$ adjoint Higgs can break the wrong mass relation in a similar way as in the usual Pati-Salam model.

We briefly comment on the colored Higgs C_1 and C_2. These can get masses from a brane-localized term mC_1C_2. This mass term does not violate the baryon and lepton number conservation. The mass terms such as mC_1^2 or mC_2^2 violate the conservation, but such mass terms are forbidden by the extra $U(1)$ symmetries.

Since we project out the vector-like partners by Z_6, the remaining fermion components in Table I give rise to gauge anomalies for the two linear combinations of three extra $U(1)$ symmetries. Green-Schwarz mechanism [18] can be used to cancel out for only one linear combination. Thus we have to introduce other brane fields which are nonsinglets under Pati-Salam symmetry to cancel these anomalies. This can be interpreted as the origin of the 1st family. For example, if we introduce brane fields such as

$$L_1 : (\mathbf{4}, \mathbf{2}, \mathbf{1})_{-1,-1,1}, \quad \bar{R}_1 : (\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2})_{1,1,-1}, \quad H_4 : (\mathbf{1}, \mathbf{2}, \mathbf{2})_{-1,-1,1},$$

the anomalies such as $SU(4)^2 \times U(1)$, $SU(2)_L^2 \times U(1)$, and $SU(2)_R^2 \times U(1)$ are canceled out. Then, introducing appropriate singlet under Pati-Salam symmetry with nonzero extra $U(1)$ charges, we can obtain anomaly free particle contents. If we adopt the Green-Schwarz mechanism, we can make it anomaly free by introducing L_1 and $\bar{R}_1$ with appropriate $U(1)$ charges without introducing H_4. In any case, this model contains three families naturally.

We identify the two families originating from gauge multiplet with the 3rd and 2nd (or 1st) families. First (or the 2nd) family is identified to be the brane-localized fields at 3-brane fixed point. Then the Yukawa couplings for 3rd family, y_t, y_b, and y_τ, are unified to the gauge couplings at the compactification scale, if we neglect a small correction in Eq. (17), and the correction coming from brane-localized interactions by assuming large volume suppression. The hierarchy of Yukawa couplings for the second (or 1st) family is derived by assuming $\langle S_1 \rangle \gg \langle S_2 \rangle$ as we mentioned before, and the Yukawa couplings for 1st (or 2nd) family are naturally small since their values are suppressed by volume factor of the extra dimensions.

We have made a choice that L_3 and $\bar{R}_2$ are in the chiral multiplet Σ, and $\bar{R}_3$ and L_2 are in the Φ. Since the scalar components of Σ consist of gauge fields with extra dimensional coordinates, the gauge transformation of Σ is different from the transformation of Φ, and the brane-localized 4D Lagrangian is not left-right symmetric. The electroweak Higgs fields, H_1 and H_2, are in the chiral multiplets Φ^c and its gauge transformation is homogeneous, and thus we can introduce the brane-localized Yukawa coupling terms which give naturally small Cabibbo-Kobayashi-Maskawa mixing angles.

We can also introduce the brane-localized right-handed neutrino mass term for $\bar{R}_3$ with the brane fields $(\mathbf{4}, \mathbf{1}, \mathbf{2})$, and the neutrino mass becomes small through the seesaw mechanism. However, $\bar{R}_2$ is in the Σ whose gauge transformation is inhomogeneous, and hence we cannot introduce the brane-localized Majorana mass term for $\bar{R}_2$. Therefore, the Majorana mass term for $\bar{R}_2$

should arise from bulk interaction. Since $A \equiv \Phi^c \partial \Phi - \sqrt{2}\Phi^c[\Sigma, \Phi]$ is gauge covariant, $(\mathrm{Tr}A)^2 + \mathrm{Tr}A^2$ can be gauge invariant bulk interactions. Such higher order bulk interactions include Majorana mass terms of $\bar{R}_2$, if we assume that $(\mathbf{4}, \mathbf{1}, \mathbf{2})$ component in Φ and Φ^c get vacuum expectation values. The vacuum expectation values of $(\mathbf{4}, \mathbf{1}, \mathbf{2})$ component also break Pati-Salam symmetry down to the standard model. The vacua should satisfy the F- and D-flat conditions. If the vacua of Σ, Φ, and Φ^c are commutative and a holomorphic function of z or $\bar{z}$, then the F- and D- flat conditions are satisfied. Because of double periodicity, the vacua should be elliptic functions. We obtain the 4D Majorana mass terms by integrating out with respect to extra dimensions on the fundamental area of T^2/Z_6.

We now discuss the numerical predictions of our model. We assume that the compactification scale from 6D to 4D is the same scale where $SU(4) \times SU(2)_L \times SU(2)_R \times U(1)^3$ gauge symmetry are broken to the standard model, choosing appropriate Higgs superfields. So below the compactification scale, we have the usual MSSM particle contents with the gauge-Yukawa unification condition for the particles of the third family

$$g_1 = g_2 = g_3 = y_t = y_b = y_\tau, \tag{20}$$

where g_1, g_2, and g_3 correspond to the hypercharge (with proper normalization), weak, and strong interaction couplings and y_t, y_b, y_τ are the top, bottom, and tau Yukawa couplings, respectively.

Because of a crucial reduction of the number of the fundamental parameters from the gauge-Yukawa coupling unification, we are lead immediately to a number of distinctive predictions (in the absence of any large supersymmetric threshold corrections). Using the values of the electroweak parameters $\sin^2\theta_w = 0.2311 \pm 0.0001$ and $\alpha_{EM} = 127.92 \pm 0.02$ at M_Z scale [19], we can determine the unification scale and unified coupling constant. Then, evolving the remaining couplings from the unification scale to the low energy (the numerical calculation of gauge-Yukawa unification in a 4D model is demonstrated in Ref. [20]), we predict [14]

$$\begin{aligned} \alpha_3(M_Z) &= 0.123, \qquad m_t = 178\ \mathrm{GeV}, \\ m_b/m_\tau(M_Z) &= 1.77, \qquad \tan\beta = 51. \end{aligned} \tag{21}$$

These are in good agreement with the experimental data [19] except the small discrepancy for α_3 (world average value is $\alpha_3 = 0.117 \pm 0.002$ [19]). The small discrepancy for α_3 can be easily improved if we consider a unification scale threshold of $SU(4)$ sextet, C_1 and C_2. The prediction for $\tan\beta$ can be tested in the upcoming collider experiments.

In conclusion, we have presented a model in 6D with $N = 2$ supersymmetry which unify, for the first time, the strong, electroweak, and Yukawa forces as well as the elementary particles in the framework of local quantum field theory [21].

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