Vorton Existence and Stability

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We present the first concrete evidence for the classical stability of vortons, circular cosmic string loops stabilized by the angular momentum of the charge and current trapped on the string. We begin by summarizing what is known about vorton solutions and, in particular, their analytic stability with respect to a range of radial and nonradial perturbations. We then discuss numerical results of vorton simulations in a full 3D field theory, that is, Witten's original bosonic superconducting string model with a modified potential term. For specific parameter values, these simulations demonstrate the long-term stability of sufficiently large vorton solutions created with an elliptical initial ansatz.

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Introduction.—Topological defects are known to play an important role in many physical contexts and they may also impact cosmology, the relativistic setting for this present study (for a review, see Ref. [1]). Among the possible cosmic defects, vortex strings have a prominent position because they naturally evolve into a scale-invariant configuration, therefore avoiding analogs of the monopole problem of cosmological domination. These strings might be responsible for a variety of astrophysical phenomena, such as cosmic rays, gravitational wave radiation, or gravitational lensing. The richness of their phenomenology comes in part from the possibility of additional internal structure, making them superconducting [2].

These superconducting strings have been widely studied, and this article follows a companion paper [3], which investigated in detail the model studied here. Let us mention that the potential existence of vorton solutions has been postulated recently in other important physical contexts such as QCD [4] and high-T_c superconductivity [5], and some new cosmological vorton scenarios have been suggested [6]. We note that if we are able, here, to establish vorton stability in vacuum (i.e., resisting collapse due to the powerful tension of a relativistic string), then these results bode well for their as yet untested stability in less extreme physical situations. We emphasize that here we are focusing on classical vorton stability, while for a discussion of quantum stability the reader is referred to [7] and references therein.

The structure of this letter is as follows: After summarizing our previous results for vorton equilibrium states, we will examine additional relevant perturbations which we have observed as a result of our work. Finally, we will give an account of our numerical results on vortons and their long-term stability given elliptically perturbed initial conditions.

The framework.—Our study is based on the model first proposed in Ref. [2] :

$$\mathcal{L} = (\partial_\mu \phi)(\partial^\mu \phi)^+ + (\partial_\mu \sigma)(\partial^\mu \sigma)^+ - \frac{\lambda_\phi}{4}(|\phi|^2 - \eta_\phi^2)^2 - \frac{\lambda_\sigma}{4}(|\sigma|^2 - \eta_\sigma^2)^2 - \beta|\phi|^2|\sigma|^2, \quad (1)$$

where the two complex scalar fields ϕ and σ are minimally coupled, each being invariant under $U(1)$ transformations.

If the constants of the theory (the couplings and the vacuum expectation values) are chosen carefully, one can break the ϕ symmetry, leading to $|\phi| = \eta_\phi$, while keeping $\sigma = 0$ in the vacuum, because of the nonvanishing interaction term. Along a ϕ cosmic string, the interaction vanishes, and σ can form a condensate. (For a detailed account of this model, and for the bounds on parameters, see Ref. [3].)

This $|\sigma|$ condensate can also carry charge and currents along the string (taken to lie on the z axis), as can be readily seen from the ansatz:

$$\sigma = |\sigma|(r)e^{i(\omega t + kz)}, \quad (2)$$

which induces a (Noether) charge Q and a current J on the string world sheet:

$$Q = \omega \int dz \int dS\, |\sigma|^2, \qquad J = k \int dz \int dS\, |\sigma|^2, \quad (3)$$

as well as a topologically conserved quantity, the winding number:

$$N = \int dz \, \frac{k}{2\pi}. \quad (4)$$

These features alter significantly the standard string cosmology scenario, with perhaps the most striking consequence of superconductivity being the classical stability of string loops. These loops, which were dubbed *vortons* in Refs. [8,9], cannot decay because of the angular momentum of their charge carriers.

We present in Fig. 1 a typical, straight superconducting string profile. In general, one can expect a condensate somewhat broader than the underlying vortex: $\delta_\sigma > \delta_\phi$, or equivalently, $\beta\eta_\phi^2 - \frac{1}{2}\lambda_\sigma\eta_\sigma^2 = m_\sigma^2 < m_\phi^2 = \frac{1}{2}\lambda_\phi\eta_\phi^2$. It is probably worth emphasizing here the model dependence of vorton studies: the parameter space is very broad, and we need a specific particle physics model fixing the couplings to make more precise predictions about their properties.

In our previous paper, we have been able to analyticaly characterize the equilibrium states proving, in principle, that vortons should occur for every initial nonzero value of Q and N. If, as usual, we call μ the string tension, and we define

$$\Sigma = \int dS\,|\sigma|^2, \qquad \Sigma_4 = \int dS\,|\sigma|^4, \tag{5}$$

we can recall the following equilibrium conditions. In the *chiral* case $\omega^2 = k^2$, vortons will shrink or expand until:

$$\omega^2 = k^2 = \left(\mu - \frac{1}{4}\lambda_\sigma\Sigma_{4o}\right)\Big/(2\Sigma_o), \tag{6}$$

where the suffix o denotes the chiral value of a quantity (or equivalently, when $\omega = k = 0$). In the *electric* ($\omega^2 > k^2$) or *magnetic* ($\omega^2 < k^2$) regimes, given that $\Sigma_{QN} = Q/N$, we showed that the vorton state would minimize the following function of $\omega^2 - k^2 = u^2$:

$$\mathcal{E} = N\left[\frac{[\mu - \frac{\lambda_\sigma}{4}\Sigma_4(u^2)]}{\Sigma(u^2)}\sqrt{\frac{\Sigma_{QN}^2 - \Sigma^2(u^2)}{u^2}} + 2\Sigma_{QN}^2\sqrt{\frac{u^2}{\Sigma_{QN}^2 - \Sigma^2(u^2)}}\right], \tag{7}$$

which clearly admits such a minimum. Given an initial state with small seed charges and currents, we demonstrated that the smaller final equilibrium state will generically be located away from the chiral state (which is *not* an attractor—see Ref. [3] for details).

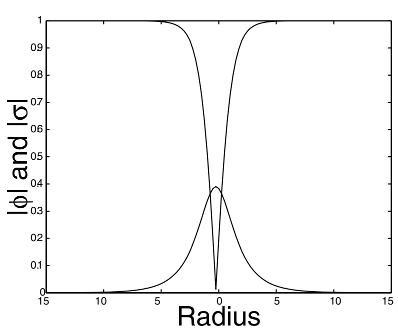

FIG. 1. Profiles of ϕ and σ, plotted in units of η_ϕ and η_σ; the parameters here are $\lambda_\phi = 1.5$, $\eta_\phi = 1.0$, $\lambda_\sigma = 10.0$, $\eta_\sigma = 0.5$, $\beta = 1.5$

Stablity analysis.—We employed numerical methods to investigate the behavior of these equilibrium vorton states, which enabled us to identify and analytically characterize various instabilities corresponding to different regimes of the loop. We have already proved in [3] that a straight superconducting string cannot evolve too deeply into the magnetic regime ($\omega^2 < k^2$), because of nonlinear effects that force the condensate to become pinched locally and to unravel, thus losing winding number.

Our analytical analysis, and a suitable ansatz for Σ and Σ_4, gave a precise criterion for the threshold value $k_{\rm inst}$ of this instability. Let us define $\alpha = \frac{\lambda_\sigma}{4}\Sigma_{4o}/\Sigma_o$, and k_c, the maximum k value associated with a nonvanishing condensate. Then,

$$\frac{1}{k_{\rm inst}^2} = \frac{1}{\alpha} + \frac{1}{k_c^2}. \tag{8}$$

We have also studied the chiral and electric cases, and we were unable to find an instability of this kind, suggesting that these regimes are stable against a "pinching" perturbation. There is, however, another potential instability particularly relevant in the electric regime.

In the model presented in Eq. (1), the radial loop equilibrium discussed above in (7) is valid only if one assumes that the charge and the current remain localized on the string's world sheet. When we consider small loops, especially those tractable numerically, this assumption is not necessarily valid. Indeed, one can imagine the interaction energy as some sort of a potential well for the condensate. If the loop gets too small, the energy stored in the rotating the charge carriers, proportional to r^{-2}, may become larger than this potential barrier, and therefore the two fields will break apart. Using the usual profiles to evaluate the interaction energy, we obtain the following stability criterion (see Ref. [10] for a more detailed analysis):

$$\frac{R}{\delta_\sigma} \gtrsim \frac{\lambda_\phi}{2\beta}\frac{\mu_\phi}{\sigma_o^2}, \tag{9}$$

where R is the radius of the vortex loop. Generically, the right-hand side of (9) is quite large, and so stable loops require $R \gg \delta_\sigma$. This is a problem for real vortons in the electric regime. The width of the condensate diverges as they go deeper into the electric regime, and so these loops can prove quite hard to stabilize. Numerically, memory limitations are such that we cannot achieve more than $R \sim 20\delta_\sigma$, and thus we are faced with this potential splitting instability even in the chiral regime, since this is just the order of magnitude given by (9), as we have demonstrated in our simulations.

To overcome this difficulty, we have modified the interaction term in Eq. (1), $V_{\rm int} = \beta|\phi|^2|\sigma|^2$, by considering a toy model with $V_{\rm int} = \beta'|\phi|^6|\sigma|^2$. Because of the

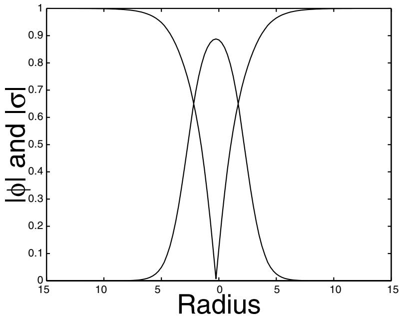

FIG. 2. Profiles of ϕ and σ in the modified Witten model, plotted in units of η_ϕ and η_σ; the parameters here are $\lambda_\phi = 0.5$, $\eta_\phi = 1.0$, $\lambda_\sigma = 18.0$, $\eta_\sigma = 0.35$, with the new interaction parameter $\beta' = 3.3$

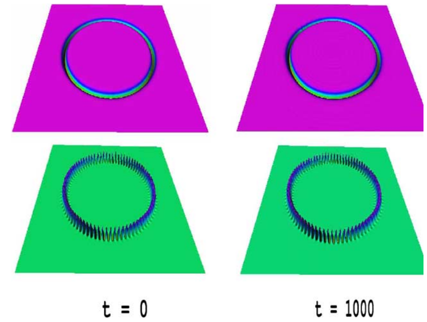

FIG. 3 (color online). Vorton simulation, showing $|\phi|$ (above) and the real part of σ, at $t = 0$ and $t = 1000$ (note the uniform winding of σ, meaning a uniform current).

higher power of ϕ involved, the effective potential seen by the condensate is much closer to a square well, which allows σ to build up a higher and more robust condensate, as can be seen from Fig. 2. This represents only a quantitative, rather than qualitative, modification of the vorton model with the new criterion for the splitting instability becoming less stringent. Then, the constraint given by (9) is relaxed to $R/\delta_\sigma \lesssim 5$, and we can create numerical configurations in which the condensate and the underlying vortex are held together tightly.

Vorton simulations.—The theory defined in the previous section is ideal for numerical simulations. Our code evolves the fields according to the full 3D equations of motion arising from Eq. (1), using a lattice-inspired Hamiltonian formalism [11] modified from [12]. energy and charge conservation in all simulations was maintained to below 1% accuracy.

First, we study the perfectly circular chiral case. The initial configuration was obtained using an SOR relaxation routine to calculate the radial profiles. We then used the ansatz:

$$\sigma(r, \theta, \phi) = |\sigma|(r, \theta) e^{i(k\phi + \omega t)}, \tag{10}$$

which describes a homogeneous chiral vorton. Here, we have neglected small corrections due to the curvature.

We then let the loop evolve with our code, using (Dirichlet) reflective boundary conditions which do not act to stabilize the configuration. We could observe the loop slowly oscillating around its equilibrium position, in agreement with the radial analysis we have given (see also [13–16]). As can be seen in Fig. 3, the whole structure appears to be remarkably stable.

We now turn our attention to vorton stability with respect to perturbations in the eccentricity ϵ, by considering a loop with $\epsilon < 1$. To ensure that the current is initially homogeneous, we have to consider the modified ansatz:

$$\sigma(r, \theta, s) = |\sigma|(r, \theta) e^{ks + \omega t}, \tag{11}$$

where s is the arclength along the string. This arclength is given by an elliptic integral which we evaluate accurately using the Gauss-Tschebychev algorithm (as can be found in, e.g., [17]).

Our observations are summarized in Fig. 4, where we can see clearly that the loop is actually oscillating between its initial configuration, and another, somewhat

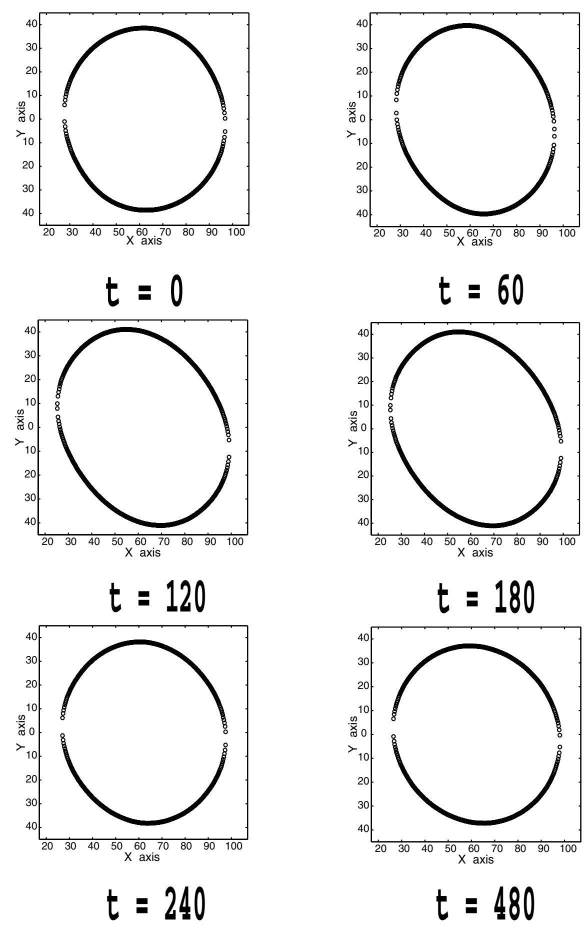

FIG. 4. Plots of the zeros of ϕ in the transverse plane of an elliptical chiral vorton: the first four plots show the first period, and the last two are taken after two and three periods (initially, $\epsilon = 0.4$).

larger loop with higher eccentricity, tilted in the direction opposite to the current flow. We note also that this behavior satisfies $T \simeq L$, where T is the period of the movement, and L is the length of the loop.

This can be understood if one considers the effect on the current of squashing a circular vorton, to make it look like an ellipse. To the lowest order, the net effect on the angular momentum of the current is to induce a correction of the form:

$$M = M_0\left(1 + \frac{\epsilon^2}{4}\right). \tag{12}$$

Thus, the angular momentum of the current is increased with the eccentricity, and this has to be compensated by a rotation of the loop in the opposite direction (we discuss the transfer of momentum at length elsewhere [10]). Despite the oscillating eccentricity, ultimately these loops tend to evolve towards more circular configurations, as can be seen from the last two plots in Fig. 4.

Eccentric loop configurations retained their identity for more than 10 000 time steps (many light-crossing times), and so these simulations appear to establish that stable vortons should form during the evolution of the Universe. Two small caveats to this conclusion remain. First, in the very longest simulations of over 30 000 time steps, the buildup of background radiation (due to the reflective boundary conditions) causes some friction on the time-varying current which is eventually driven towards the less stable magnetic regime. We are developing absorbing boundary conditions for massive radiation to test the significance of this boundary artifact. Second, there is the sensitivity of these objects to their initial conditions, since a slight mismatch in the phase of the condensate can have dramatic consequences on the subsequent evolution of the loop. This problem is still under active investigation and will be discussed at greater length elsewhere.

Conclusions and discussion.—We have presented in this paper the first simulations of vortons, which appear to strongly indicate their stability, and hence their cosmological relevance (as well as in other physical contexts). Stable vortons may have profound consequences for cosmology. These objects can be very massive, and they could account for dark matter, or even dominate the universe [18,19]. They are naturally associated with very high energies, and this makes them prime candidates for high-energy astrophysical puzzles, such as cosmic rays [20] or gamma ray bursts [21]. Further study is required to test these hypotheses, but there can be little doubt that a deeper understanding of the microphysics of superconducting strings will further the confrontation between vortons and observations.

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