Comment on "Weak Phase γ Using Isospin Analysis and Time-Dependent Asymmetry in $B_d \to K_S\pi^+\pi^-$"

In a recent interesting Letter [1] Deshpande, Sinha, and Sinha propose to determine the weak phase γ in $B \to K\pi\pi$ decays. They use the CP asymmetry in $B^0(t) \to K_S\pi^+\pi^-$, and an isospin triangle relation among the amplitudes for $B^+ \to K^0(\pi^+\pi^0)_e$, $B^0 \to K^0(\pi^+\pi^-)_e$, and $B^0 \to K^0(\pi^0\pi^0)_e$, in which the two pions are in an even angular momentum state. A crucial assumption is that electroweak penguin and tree amplitudes contributing to $B^+ \to K^0(\pi^+\pi^0)_e$ involve a common strong phase. Such a property was shown to hold in the SU(3) symmetry limit for the $I = 3/2$ amplitude in $B \to K\pi$ [2,3], and in the isospin symmetry limit for the $I = 2 B \to \pi\pi$ amplitude [3,4].

Here we will clarify the condition under which tree and electroweak amplitudes can be related to each other, showing that this condition is not fulfilled in the case studied in [1].

The effective Hamiltonian describing charmless $\Delta S = 1$ (or $\Delta S = 0$) decays [5] consists of current-current operators Q_1 and Q_2, QCD penguin operators Q_i, $i = 3$–6, and electroweak penguin (EWP) operators Q_i, $i = 7$–10. The operators Q_1 and Q_2, multiplying Wilson coefficients c_1 and c_2, respectively, and Cabibbo-Kobayashi-Maskawa (CKM) coefficients $V^*_{ub}V_{us}$ (or $V^*_{ub}V_{ud}$), will be named tree operators. EWP operators involve CKM factors $V^*_{tb}V_{ts}$ (or $V^*_{tb}V_{td}$). The EWP operators Q_9 and Q_{10} with the dominant Wilson coefficients, c_9 and c_{10}, have the same $(V$–$A)(V$–$A)$ structure as the tree operators, and would have approximately the same matrix elements if they had also identical flavor SU(3) and isospin structure.

One may decompose the tree and electroweak $\Delta S = 1$ four quark operators into a sum of $\overline{\mathbf{15}}$, $\mathbf{6}$, and $\overline{\mathbf{3}}$ [3],

$$\mathcal{H}_T = -\frac{G_F}{\sqrt{2}}V^*_{ub}V_{us}\left[\frac{c_1 - c_2}{2}(\overline{3}^{(a)}_0 + \mathbf{6}_1) + \frac{c_1 + c_2}{2} \times \left(\overline{\mathbf{15}}_1 + \frac{1}{\sqrt{2}}\overline{\mathbf{15}}_0 - \frac{1}{\sqrt{2}}\overline{\mathbf{3}}^{(s)}_0\right)\right], \quad (1)$$

$$\mathcal{H}_{\rm EWP} = -\frac{G_F}{\sqrt{2}}\frac{3V^*_{tb}V_{ts}}{2}\left[\frac{c_9 - c_{10}}{2}\left(\frac{1}{3}\overline{\mathbf{3}}^{(a)}_0 + \mathbf{6}_1\right) + \frac{c_9 + c_{10}}{2} \times \left(-\overline{\mathbf{15}}_1 - \frac{1}{\sqrt{2}}\overline{\mathbf{15}}_0 - \frac{1}{3\sqrt{2}}\overline{\mathbf{3}}^{(s)}_0\right)\right], \quad (2)$$

where subscripts denote the isospin of corresponding operators. The representation $\overline{\mathbf{3}}$ appears both symmetric and antisymmetric under the interchange of two quarks. Both the $\mathbf{6}$ and $\overline{\mathbf{15}}$ operators include a $\Delta I = 1$ component.

Equations (1) and (2) imply two proportionality relations [6]:

$$\mathcal{H}_{\rm EWP}(\overline{\mathbf{15}}) = -\frac{3}{2}\frac{c_9 + c_{10}}{c_1 + c_2}\frac{V^*_{tb}V_{ts}}{V^*_{ub}V_{us}}\mathcal{H}_T(\overline{\mathbf{15}}), \quad (3)$$

$$\mathcal{H}_{\rm EWP}(\mathbf{6}) = \frac{3}{2}\frac{c_9 - c_{10}}{c_1 - c_2}\frac{V^*_{tb}V_{ts}}{V^*_{ub}V_{us}}\mathcal{H}_T(\mathbf{6}). \quad (4)$$

The two proportionality constants are approximately equal in magnitudes but differ in sign [5], $(c_9 + c_{10})/(c_1 + c_2) \approx (c_9 - c_{10})(c_1 - c_2)$. Therefore, EWP and tree amplitudes in B decay processes which obtain contributions from either the $\overline{\mathbf{15}}$ or the $\mathbf{6}$ operator, but not from both, are proportional to each other and involve a common strong phase. This property does not hold when the two operators contribute because of the opposite signs in Eqs. (3) and (4).

In the case of $B \to (K\pi)_{I=3/2}$ [2,3], the K and π are in an S-wave state, which is symmetric under an interchange of the two SU(3) octets. This state is a pure $\mathbf{27}$. The only SU(3) operator which contributes to this transition is the $\overline{\mathbf{15}}$. Consequently, the EWP and tree amplitudes are proportional to each other in the SU(3) approximation. The same holds true in the isospin symmetry limit for the EWP and tree amplitudes of $B \to (\pi\pi)_{I=2}$, since only the $\overline{\mathbf{15}}$ contains a $\Delta I = 3/2$ component [3,4]. On the other hand, in $B^+ \to K^0(\pi^+\pi^0)_e$ studied in [1] the final state has $I = 3/2$, $S = 1$ and can be in a $\mathbf{27}$ and in a $\mathbf{10}$, to which the $\Delta I = 1$ components of both the $\overline{\mathbf{15}}$ and the $\mathbf{6}$ operators contribute. Hence, the condition for proportional EWP and tree amplitudes and for a common strong phase does not hold. Although this proportionality does not follow from symmetry considerations alone, it would be interesting to study possible dynamical assumptions that can lead to such a situation.

I thank Jonathan Rosner for useful discussions, and I am grateful to the Enrico Fermi Institute at the University of Chicago for its kind hospitality. This work was supported in part by the U.S. Department of Energy through Grant No. DE FG02 90ER40560.

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Received 23 May 2003; published 26 September 2003
DOI: 10.1103/PhysRevLett.91.139101
PACS numbers: 12.15.Hh

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