Negative Lateral Shift of a Light Beam Transmitted through a Dielectric Slab and Interaction of Boundary Effects

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It is found that when a light beam travels through a slab of optically denser dielectric medium in air, the lateral shift of the transmitted beam can be negative. This is a novel phenomenon that is reversed in comparison with the geometrical optic prediction according to Snell's law of refraction. A Gaussian-shaped beam is analyzed in the paraxial approximation, and a comparison with numerical simulations is made. Finally, an explanation for the negativity of the lateral shift is suggested, in terms of the interaction of boundary effects of the slab's two interfaces with air.

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Light is reflected and transmitted at dielectric interfaces. It is well known that the totally reflected light beam is laterally shifted from the position predicted by geometrical optics. This phenomenon is referred to as the Goos-Hänchen (GH) shift [1,2]. The investigation of the GH shift has been extended to the partial reflection regime [3–9] and to other areas of physics, such as acoustics, quantum mechanics, plasma physics, nonlinear optics [2], and surface physics [10]. There have also been papers dealing with negative GH shifts in reflection in some complicated circumstances such as negative-permittivity media [11,12], absorptive media [13–16], and negatively refractive media [17]. Apart from the lateral shift, the reflected beam may also undergo a focal shift, angular shift, and beam-waist modification [3,9] with respect to the prediction of geometrical optics. The behavior of the transmitted beams did not draw as much attention as those of reflected beams. Hsue and Tamir [4] once discussed the lateral shift of a transmitted beam in a transmitting-layer configuration. But they concluded that it is always shifted in a forward direction. The main purpose of this Letter is to report a novel phenomenon of a transmitted beam through a dielectric slab in air, the negative lateral shift from the position predicted by geometrical optics, according to Snell's law of refraction. It is shown at the same time that the lateral shift of the reflected beam can also be negative in this simple configuration.

For simplicity, we consider a nonmagnetic dielectric slab in air. Denote by a, ε, and n the thickness, dielectric constant, and refractive index of the slab, extending from 0 to a, as is shown in Fig. 1. A two-dimensional ($\partial/\partial z = 0$) light beam of TE polarization (TM polarization can also be discussed in the same way) and of angular frequency ω comes from the left with an incidence angle θ_0 specified by its axis. Let $E_{\rm in}(\vec{x}) = A\exp(i\vec{k}\cdot\vec{x})$ be the electric field of a Fourier component of the incident beam, where $\vec{k} = (k_x, k_y) = (k\cos\theta, k\sin\theta)$, $k = (\varepsilon_0\mu_0\omega^2)^{1/2}$, where θ denotes the incidence angle of the contributed plane wave, and time dependence $\exp(-i\omega t)$ is implied and suppressed. The corresponding Fourier component of the transmitted electric field is determined by Maxwell's equations and boundary conditions to be $E_t(\vec{x}) = TA\exp\{i[k_x(x-a) + k_y y]\}$, where the phase $\varphi = \varphi(k_y)$ of the transmission coefficient $T = T(k_y)$ and the reciprocal $1/|T|$ of its absolute value are, respectively, the phase and norm of complex number $\cos k'_x a + \frac{i}{2}(\frac{k'_x}{k_x} + \frac{k_x}{k'_x})\sin k'_x a$, so that

$$\varphi(k_y) = \mathrm{int}\left(\frac{k'_x a}{\pi} + \frac{1}{2}\right)\pi + \tan^{-1}\left[\frac{1}{2}\left(\frac{k'_x}{k_x} + \frac{k_x}{k'_x}\right)\tan k'_x a\right], \tag{1}$$

$k'_x = k'\cos\theta'$, $k' = (\varepsilon\mu_0\omega^2)^{1/2}$, θ' is determined by Snell's law, $n\sin\theta' = \sin\theta$, and int(.) means the integer part of involved number. Since $\varepsilon > \varepsilon_0$, $\theta' < \theta$, we know that $k'_x > k_x$.

When measured in the same way as the lateral shift of reflected beam as is indicated in Fig. 1, the lateral shift of transmitted beam is defined as $-d\varphi/dk_y$ [18,19] and is given here by

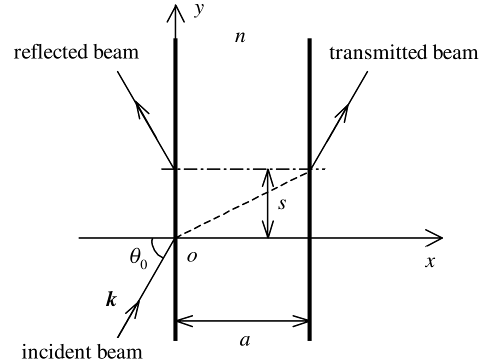

FIG. 1. Schematic diagram of a light beam propagating through a slab of denser dielectric medium in the air.

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$$s = \frac{2k_{y0}a}{k_{x0}} \frac{k_{x0}^2(k_{x0}^2 + k_{x0}'^2)/k_0^4 - \sin(2k_{x0}'a)/2k_{x0}'a}{4k_{x0}^2 k_{x0}'^2/k_0^4 + \sin^2 k_{x0}'a}, \quad (2)$$

where $k_{x0} = k\cos\theta_0$, $k_{y0} = k\sin\theta_0$, $k_{x0}' = k'\cos\theta_0'$, θ_0' is determined by $n\sin\theta_0' = \sin\theta_0$, and $k_0^2 = k_{x0}'^2 - k_{x0}^2 = k'^2 - k^2$. It is seen that when inequality

$$k_{x0}^2(k_{x0}^2 + k_{x0}'^2)/k_0^4 < \sin(2k_{x0}'a)/2k_{x0}'a \quad (3)$$

holds, the lateral shift is negative. It is reversed in comparison with the prediction of Snell's law of refraction that the lateral shift of transmitted beam would be $a\tan\theta_0'$ and be always positive. Since $\sin(2k_{x0}'a)/2k_{x0}'a \leq 1$, Eq. (3) leads to the following necessary condition, $k_{x0}^2(k_{x0}^2 + k_{x0}'^2)/k_0^4 < 1$, which means that the incidence angle θ_0 satisfies

$$\cos\theta_0 < \left(\frac{n^2 - 1}{2}\right)^{1/2} \equiv \cos\theta_t. \quad (4)$$

This shows that if the incidence angle satisfies Eq. (4), that is to say if θ_0 is larger than the threshold angle θ_t, one can always find a thickness a of the slab at which the lateral shift of transmitted beam is negative. Furthermore, Eq. (4) is satisfied by any incidence angle if $n > \sqrt{3}$, which means that the phenomenon of negative lateral shift is more easily observed experimentally in media of larger refractive indices. The inequality (3) also shows that negative lateral shifts are more easily implemented at larger angles of incidence because the larger the angle of incidence is, the more easily the inequality is satisfied.

A typical dependence of the lateral shift on a is shown in Fig. 2, where the dielectric medium of the slab is chosen to be perspex of refractive index $n = 1.605$ ($\theta_t = 27.4°$) at wavelength $\lambda = 32.8$ mm [8], a is rescaled by $k_{x0}'a$. In order to obtain large negative shifts, a large incidence angle is chosen, $\theta_0 = 80.2°$. Calculation under these conditions shows that the lateral shift is equal to -63.8 mm for $a = 14.2$ mm and is even equal to -82.4 mm for $a = 1.0$ mm

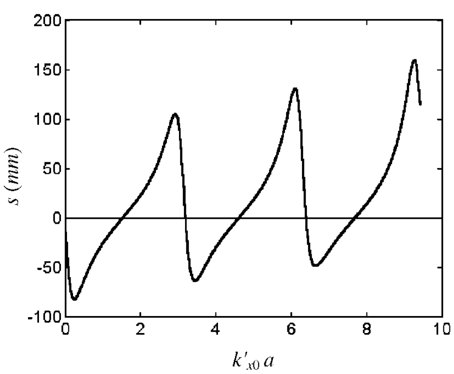

FIG. 2. Dependence of the lateral shift s on the thickness a of the slab, where the denser medium is chosen to be perspex of refractive index $n = 1.605$ at wavelength $\lambda = 32.8$ mm, the incidence angle is $\theta_0 = 80.2°$, a is rescaled by $k_{x0}'a$.

Now let us look briefly at the reflected beam. Denoted by $RA\exp[i(-k_x x + k_y y)]$ the Fourier component of reflected electric field, the reflection coefficient $R = R(k_y)$ is determined by Maxwell's equations and boundary conditions to be

$$R(k_y) = \frac{\exp(i\pi/2)}{4g^2}\left(\frac{k_x'}{k_x} - \frac{k_x}{k_x'}\right) \times \left[\sin 2k_x'a + i\left(\frac{k_x'}{k_x} + \frac{k_x}{k_x'}\right)\sin^2 k_x'a\right]. \quad (5)$$

The factor that determines the phase of reflection coefficient is

$$\sin 2k_x'a + i\left(\frac{k_x'}{k_x} + \frac{k_x}{k_x'}\right)\sin^2 k_x'a. \quad (6)$$

If we denote it by $g'\exp(i\varphi')$, then the phase of reflection coefficient will be $\varphi' + \pi/2$. Obviously, we have

$$\tan\varphi' = \tan\varphi = \frac{1}{2}\left(\frac{k_x}{k_x'} + \frac{k_x'}{k_x}\right)\tan k_x'a.$$

What is meant by this equation is that the local properties of φ' with respect to k_y are the same as those of φ. So the lateral shift of reflected beam is locally given by Eq. (2) [6,18,19].

Since the imaginary part of complex number (6) is non-negative, φ' is defined over a finite interval $[0, \pi]$ and is a periodical function of $k_x'a$ with period π, as is shown by the real curve in Fig. 3, where the physical parameters are the same as in Fig. 2. For comparison, in Fig. 3 is also shown by the dotted curve the dependence of φ on $k_x'a$ under the same condition. The relation of φ' with $k_x'a$ as a whole is different from that of φ. φ' is not continuous at $k_x'a = m\pi$ ($m = 1, 2, 3...$), while φ is.

The noncontinuity of φ' at $k_x'a = m\pi$ is understandable. Mathematically, complex number (6) is equal to zero when $k_x'a = m\pi$, so that its phase is undefined. Physically speaking, the reflection coefficient is equal to zero when $k_x'a = m\pi$, as Eq. (5) shows, so that its phase is

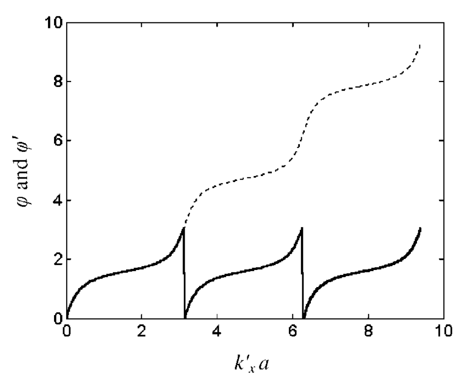

FIG. 3. Dependence of the phases φ' and φ on $k_x'a$, where $\lambda = 32.8$ mm, $n = 1.605$, $\theta = 80.2°$. φ' is shown by the real curve, and φ is shown by the dotted curve.

meaningless. In this case, the reflected beam has very low intensity and is severely distorted so that it cannot be described in terms of a shifted beam [4]. All these amount to a conclusion that when resonant transmission does not occur, the lateral shifts of reflected and transmitted beams are the same in this symmetric configuration when measured the same way.

Of course, when measured with reference to the prediction of Snell's law, the lateral shift of transmitted beam will be equal to $s - a\tan\theta_0'$. So when the lateral shift of reflected beam is negative with reference to geometrical reflection, the lateral shift of transmitted beam is even more negative with reference to the prediction of Snell's law, especially at large angle of incidence.

For a Gaussian-shaped incident beam, $E_{\rm in}(\vec{x})|_{x=0} = \exp(-y^2/2w_y^2 + ik_{y0}y)$, which has the Fourier integral of the following form,

$$E_{\rm in}(\vec{x})|_{x=0} = \frac{1}{\sqrt{2\pi}}\int A(k_y)\exp(ik_y y)dk_y,$$

where $w_y = w_0\sec\theta_0$, w_0 is the width of the beam at the waist, and $A(k_y) = w_y\exp[-(w_y^2/2)(k_y - k_{y0})^2]$ is the angular spectral distribution, the electric field of the transmitted beam can be written as

$$E_t(\vec{x}) = \frac{1}{\sqrt{2\pi}}\int T(k_y)A(k_y)\exp\{i[k_x(x-a) + k_y y]\}dk_y. \tag{7}$$

If the incident light beam is well collimated, $A(k_y)$ is a sharply distributed Gaussian function around k_{y0}. In this case, the transmission coefficient $T(k_y)$ can be approximated, by writing it in an exponential form, expanding the exponent in Taylor series at k_{y0} and retaining the first two terms, to be

$$\begin{aligned} T(k_y) &\approx \exp\left[\ln T(k_{y0}) + \frac{1}{T(k_{y0})}\frac{dT}{dk_{y0}}(k_y - k_{y0})\right] \\ &= T(k_{y0})\exp\left[\left(\frac{1}{|T(k_{y0})|}\frac{d|T|}{dk_{y0}} + i\frac{d\varphi}{dk_{y0}}\right)(k_y - k_{y0})\right], \end{aligned} \tag{8}$$

where d/dk_{y0} denotes the derivative with respect to k_y evaluated at $k_y = k_{y0}$, i.e., at $\theta = \theta_0$. Substituting Eq. (8) into Eq. (7) and using paraxial approximation, $k_x \approx k_{x0} - (k_y - k_{y0})\tan\theta_0$, we finally obtain for the electric field of transmitted beam,

$$\begin{aligned} E_t(\vec{x}) &= T(k_{y0})\exp\left(i\eta\frac{d\varphi}{dk_{y0}}\right)\exp\left(\frac{\eta^2 w_y^2}{2}\right) \\ &\times \exp\left\{-\frac{1}{2w_y^2}\left[y + \frac{d\varphi}{dk_{y0}} - (x-a)\tan\theta_0\right]^2\right\} \\ &\times \exp\{i[(k_{x0} - \eta\tan\theta_0)(x-a) + (k_{y0} + \eta)y]\}, \end{aligned} \tag{9}$$

where $\eta = [1/w_y^2|T(k_{y0})|](d|T|/dk_{y0})$. It can be seen from this expression that:

(a) The transmitted beam is of the same Gaussian shape as that of the incident beam as is indicated by the fourth factor. The locus of its peak is given by

$$y - (x-a)\tan\theta_0 + d\varphi/dk_{y0} = 0. \tag{10}$$

Its lateral shift along $x = a$ line from point $(a, 0)$ is then $-d\varphi/dk_{y0}$. This is what is given by Eq. (2) and is independent of the waist width of the beam in this approximation. Equation (10) also tells us that the locus is parallel to the wave vector (k_{x0}, k_{y0}) that is expected from Snell's law for transmitted beam.

(b) The impact of $d|T|/dk_{y0}$, the dependence of $|T|$ upon k_y, on the transmitted beam is different from that of $d\varphi/dk_{y0}$. The final factor shows that when η cannot be omitted, the propagation wave vector of transmitted beam is $(k_{x0} - \eta\tan\theta_0, k_{y0} + \eta)$, rather than (k_{x0}, k_{y0}). So the propagation direction specified by the modified wave vector is different from the prediction of Snell's law and is thus not parallel to the locus of the peak of the transmitted beam. Denoting by $\theta_0 + \delta$ the angle of propagation direction, it is found that $\delta \approx \sin\delta = \eta/k_{x0}$, which means that the angle deflection δ of propagation direction depends closely on the waist width of the beam. When the width of the beam is very large, the deflection disappears. In addition, the third factor shows that the magnitude of the transmitted beam is modified also by $d|T|/dk_{y0}$. This modification depends on the width of the beam, too.

The approximating theory for the lateral shift presented here is in good agreement with numerical simulations to within an error of 10% when the beam-waist w_0 is as small as 5 times the wavelength for $\lambda = 32.8$ mm, $n = 1.605$, $\theta_0 = 80.2°$. The numerical result for the lateral shift is obtained by performing the integration in Eq. (7) from $k_y = 0$ to $k_y = k$. The discrepancy between theoretical approximation and numerical simulation is mainly due to the fact that the symmetric center, $k_{y0} = k\sin 80.2° = 0.985k$, of the angular spectral distribution $A(k_y)$ is far from the center, $k/2$, of the integral limits. This is because the theoretical calculations are in better agreement with numerical simulations when the center of the angular spectral distribution is closer to the center of the integral limits. When the two centers coincide, that is, when the angle of incidence is equal to 30°, the difference between theoretical and numerical results for the lateral shift is less than 0.3%.

If higher-order terms of Taylor series are retained in the exponent of the exponential form of $T(k_y)$, other effects will be expected [3].

The previously discovered negative lateral shifts of reflected beam as mentioned at the beginning have all their own bases, such as negative permittivity, absorption, and negative index of refraction; how do we understand the present negative lateral shift? To this end, we rewrite Eq. (2) as

$$s = \frac{2k_{x0}k_{y0}(k_{x0}^2 + k_{x0}'^2)/k_0^4}{4k_{x0}^2k_{x0}'^2/k_0^4 + \sin^2 k_{x0}'a} a - \frac{k_{y0}}{k_{x0}k_{x0}'} \frac{\sin 2k_{x0}'a}{4k_{x0}^2k_{x0}'^2/k_0^4 + \sin^2 k_{x0}'a}, \quad (11)$$

which consists of two parts. One is a thickness-proportional term multiplied by a periodical factor with respect to $k_{x0}'a$, the other itself is periodical. It is the second term that makes the lateral shift to be negative. By averaging the two periodical functions over $k_{x0}'a$ in one period π, we simply get

$$\bar{s} = a \tan\theta_0', \quad (12)$$

which is exactly what we expect from Snell's law. This may be explained as follows.

The GH shift and other effects for reflected beam with respect to the prediction of geometrical optics at a single dielectric interface result from the interaction of the beam with the interface. When the angle of incidence is smaller than the critical angle for total reflection, the effect of a single interface for the reflected beam leads to an angular shift instead of a lateral one [20,21]. Here the periodical functions in Eq. (11) can be viewed as the result of the interaction between the effects of the two interfaces in the present situation which produces lateral as well as angular shifts. The averaging over $k_{x0}'a$ just effaces the interaction so as to produce what the geometrical optics predicts, in much the same way as the average of a quantum observable over a quantum state gives a good representation of the classical variable [22].

To summarize, after obtaining the expression for the lateral shift by the stationary-phase approximation, we analyzed a Gaussian-shaped incident beam in paraxial approximation and compared the theoretically approximating results with numerical simulations. It is shown that the lateral shift of the transmitted beam is equal to that of the reflected beam and can be either forward or backward when they are all measured from the normal to the interfaces (the x axis) at which the incidence point is located. If compared with the prediction of Snell's law, the lateral shift of the transmitted beam is more backward when the reflected beam is shifted backward with respect to geometric reflection. It is also shown that the locus of the transmitted beam defined by the peak of the intensity is not parallel to its propagation direction specified by the modified wave vector. Though the lateral shift is basically independent of the waist width of the beam, the deflection of propagation direction depends closely upon it. An explanation for the negativity of lateral shift is advanced at last in terms of the interaction of the boundary effects of the slab's two interfaces. Of course, the energy is conserved and the energy flow can be discussed by approaches of Tamir and Bertoni [11] or Lai *et al.* [12]. The predicted effects here may have potential applications in optical modulations. For instance, a small change in the refractive index of the slab can result in a significant variation of the lateral shift of the transmitted beam.

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