Supersolid versus Phase Separation in Atomic Bose-Fermi Mixtures

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We show that a two-dimensional atomic mixture of bosons and fermions cooled into their quantum degenerate states and subject to an optical lattice develops a supersolid phase characterized by the simultaneous presence of a nontrivial crystalline order and phase order. This transition is in competition with phase separation. We determine the phase diagram of the system and propose an experiment allowing for the observation of the supersolid phase.

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Cooling atoms to the nK regime allows for the realization and study of new thermodynamic phase transitions and their associated phases, with an interesting synergy emerging between the fields of quantum atom optics and condensed matter physics. Recent trends are the study of the superfluid to Mott-insulator phase transition appearing in cold bosonic systems subject to an optical lattice [1,2] and the striving for the realization of a BCS-type condensate in a fermionic system [3,4]. In this Letter, we investigate the possibility to realize a nontrivial supersolid phase in a mixed boson-fermion system sympathetically cooled into their corresponding quantum degenerate states [5,6]. We identify a promising system where this novel phase can be observed and determine the relevant phase diagram.

Supersolids simultaneously exhibit two types of order which usually appear in competition with each other—these are the diagonal long-range order (DLRO) associated with the periodic density modulation in a crystal and the off-diagonal long-range order (ODLRO) associated with the phase order in the condensate [7]. Supersolids have been proposed to exist in the strongly interacting ^{4}He system [8,9] and in various model systems describing interacting bosons on a lattice and analyzed numerically [10–13]. Here, we investigate the possibility to use a specifically tuned boson-fermion mixture to realize a supersolid phase in a controlled experiment.

The basic idea underlying our scheme is to share tasks between the fermions and the bosons: the fermions are tuned through a density wave instability establishing crystalline order (DLRO), while the condensate bosons provide the ODLRO. The interaction with the fermions imprints an additional density modulation also in the bosonic density field, hence resulting in a supersolid phase. In order to trigger a density wave instability in the fermions, we confine the mixed boson-fermion system to two dimensions and subject it to an optical lattice providing perfect Fermi-surface nesting at half filling [14]. Note that the resulting crystalline order relevant for the DLRO component in the supersolid is not due to the density modulation enforced by the optical lattice but is the superstructure triggered by the density wave instability.

The supersolid transition triggered by the fermions competes with an instability towards phase separation in the boson system [15,16]. Given the dimensionality and the lattice geometry of our system, the presence of Van Hove singularities strongly enhances the tendency towards phase separation and produces new and interesting features in this transition: an arbitrary weak interaction between the bosons and the fermions is sufficient to drive the phase separation at low temperatures. However, proper tuning of the parameters allows us to supersede this phase separation through the supersolid transition shown in Fig. 1. In the following, we first investigate the two instabilities towards phase separation and density wave formation and then analyze the supersolid phase within a mean-field scheme. We focus on weak interactions allowing us to make use of linear response theory and apply Bogoliubov mean-field schemes to both fermionic and bosonic systems.

The Hamiltonian for interacting bosons and fermions subject to an optical lattice takes the form $H = H_{\rm B} + H_{\rm F} + H_{\rm int}$ with ($\alpha = $ F,B)

$$\begin{aligned} H_\alpha &= \int d\mathbf{x}\,\psi_\alpha^+\Big(-\frac{\hbar^2}{2m_\alpha}\Delta + V_\alpha(\mathbf{x})\Big)\psi_\alpha, \\ H_{\rm int} &= \int d\mathbf{x}\Big(g_{\rm FB}\psi_{\rm B}^+\psi_{\rm B}\psi_{\rm F}^+\psi_{\rm F} + \frac{1}{2}g_{\rm B}\psi_{\rm B}^+\psi_{\rm B}^+\psi_{\rm B}\psi_{\rm B}\Big). \end{aligned} \tag{1}$$

Here, we assume a repulsive interaction $g_{\rm B} = 4\pi a_s\hbar^2/m$ between the bosons, with the scattering length $a_s > 0$. The coupling $g_{\rm FB} = 2\pi a_{\rm FB}\hbar^2/\mu$ accounts for the interaction between the fermions and the bosons, with μ the relative mass and $a_{\rm FB}$ the boson-fermion scattering length. Furthermore, we restrict the analysis to spinless fermions; such a spinless fermionic atom gas is naturally achieved via spin polarization. The s-wave scattering length in the fermion system vanishes, while p-wave scattering is small in general and is neglected in the following analysis. The optical lattice with wavelength λ provides an $a = \lambda/2$-periodic potential for the bosons and fermions with $V_{\rm F,B}(\mathbf{x}) = V_{\rm F,B}[\sin^2(\pi x/a) + \sin^2(\pi y/a)]$. We expand the bosonic and fermionic field operators $\psi_{\rm B,F}$ in the Bloch wave functions $v_{k,n}$ and $w_{k,n}$ of

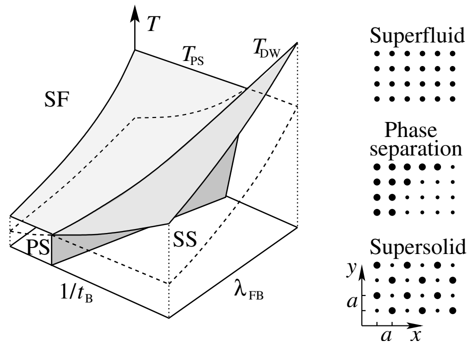

FIG. 1. Left: sketch of the λ_{FB}-$1/t_B$-T phase diagram. For $T_{DW} > T_{PS}$ the low temperature phase at $T < T_{DW}$ is a supersolid (SS), while phase separation (PS) emerges at $T < T_{PS}$ for $T_{PS} > T_{DW}$. Right: bosonic density $n_B(x, y)$ for the superfluid (SF), phase separated, and supersolid phases.

the single particle problem in a periodic potential (we restrict the analysis to the lowest Bloch band),

$$\psi_B(\mathbf{x}) = \sum_{\mathbf{k}\in K} b_{\mathbf{k}} w_{\mathbf{k}}(\mathbf{x}), \qquad \psi_F(\mathbf{x}) = \sum_{\mathbf{k}\in K} c_{\mathbf{k}} v_{\mathbf{k}}(\mathbf{x}). \quad (2)$$

Here, K denotes the first Brillouin zone, while $b_{\mathbf{k}}$ and $c_{\mathbf{k}}$ are the bosonic and fermionic annihilation operators. For a strong optical lattice $V_{F,B} > E^r_{F,B} = 2\hbar^2\pi^2/\lambda^2 m_{F,B}$, the restriction to the lowest Bloch band is justified, and the Hamiltonian simplifies to [1]

$$H = \sum_{\mathbf{k}\in K} \epsilon_B(\mathbf{k}) b^+_{\mathbf{k}} b_{\mathbf{k}} + \frac{U_B}{2N} \sum_{\{\mathbf{k},\mathbf{k}',\mathbf{q},\mathbf{q}'\}} b^+_{\mathbf{k}} b^+_{\mathbf{q}} b_{\mathbf{k}'} b_{\mathbf{q}'}$$
$$+ \sum_{\mathbf{q}\in K} \epsilon_F(\mathbf{q}) c^+_{\mathbf{q}} c_{\mathbf{q}} + \frac{U_{FB}}{N} \sum_{\{\mathbf{k},\mathbf{k}',\mathbf{q},\mathbf{q}'\}} b^+_{\mathbf{k}} b_{\mathbf{k}'} c^+_{\mathbf{q}} c_{\mathbf{q}'}, \quad (3)$$

with quantization volume $V = Na^2$ and N the number of unit cells (below, $n_{F,B}$ denote the number of particles per unit cell). The summation $\{\mathbf{k}, \mathbf{k}', \mathbf{q}, \mathbf{q}'\}$ is restricted to $\mathbf{k}, \mathbf{k}', \mathbf{q}, \mathbf{q}' \in K$ with the momentum conservation $\mathbf{k} - \mathbf{k}' + \mathbf{q} - \mathbf{q}' = \mathbf{K}_m$; the reciprocal lattice vector $\mathbf{K}_m$ accounts for umklapp processes. Here, $U_{FB} = g_{FB} \int d\mathbf{x} |\tilde{w}|^2 |\tilde{v}|^2$ and $U_B = g_B \int d\mathbf{x} |\tilde{w}|^4$, with $\tilde{w}(\mathbf{x})$ and $\tilde{v}(\mathbf{x})$ the Wannier functions associated with the Bloch band $w_{\mathbf{k}}$ and $v_{\mathbf{k}}$ [1], while $\epsilon_{F,B}(\mathbf{k})$ denote the energy dispersion of the fermions and bosons, respectively. For a strong optical lattice only nearest neighbor hopping survives and the dispersion relation takes the form

$$\epsilon_F(\mathbf{q}) = -2J_F[\cos(q_x a) + \cos(q_y a)] \quad (4)$$

with J_F the hopping energy. The Fermi surface at half filling $n_F = 1/2$ is shown in Fig. 2 and exhibits perfect nesting for $\mathbf{k}_{DW} = (\pi/a, \pi/a)$ and Van Hove singularities at $\mathbf{k} = (0, \pm\pi/a), (\pm\pi/a, 0)$. The bosonic energies are $\epsilon_B(\mathbf{q}) = 2J_B[2 - \cos(q_x a) - \cos(q_y a)]$ with $\epsilon_B(0) = 0$.

Integrating out the fermions provides an effective interaction for the bosons which depends on the temperature T of the fermionic atom gas. Within linear response theory, the boson density operator $n_B(\mathbf{q})$ drives the fermionic density $\langle n_F(\mathbf{q})\rangle = U_{FB}\chi(T, \mathbf{q}) n_B(\mathbf{q})$ with $\chi(T, \mathbf{q})$ the fermionic response function. A perturbed fermionic density in turn acts as a drive for the bosons, leading to an effective boson-boson interaction

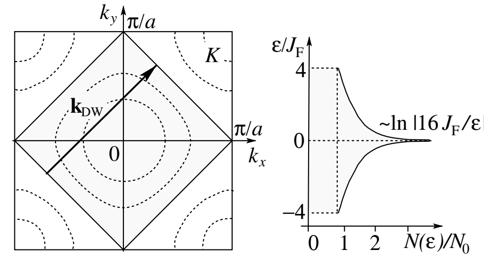

FIG. 2. Left: first Brillouin zone K and Fermi surface of 2D fermions in an optical lattice. The solid lines denote the Fermi surface at half filling. Right: density of states with logarithmic Van Hove singularities $N(\epsilon) \sim N_0 \ln|16J_F/\epsilon|$.

$$H_{int} = \frac{1}{2N} \sum_{\{\mathbf{k},\mathbf{k}',\mathbf{q},\mathbf{q}'\}} [U_B + U^2_{FB}\chi(T, \mathbf{q} - \mathbf{q}')] b^+_{\mathbf{k}} b_{\mathbf{k}'} b^+_{\mathbf{q}} b_{\mathbf{q}'}.$$

The fermionic response is given by the Lindhard function

$$\chi(T, \mathbf{q}) = \int_K \frac{d\mathbf{k}}{v_0} \frac{f[\epsilon_F(\mathbf{k})] - f[\epsilon_F(\mathbf{k} + \mathbf{q})]}{\epsilon_F(\mathbf{k}) - \epsilon_F(\mathbf{k} + \mathbf{q}) + i\eta} \quad (5)$$

with $v_0 = (2\pi/a)^2$ the volume of the first Brillouin zone. The temperature T enters via the Fermi distribution function $f(\epsilon) = 1/[1 + \exp(\epsilon/T)]$ ($\mu_F = 0$ at half filling); in our weak coupling analysis we are interested in temperatures well below the superfluid transition temperature T_{KT} of the bosons. Using the fermionic dispersion relation (4), the Lindhard function exhibits two logarithmic singularities, which give rise to instabilities in the system: the singularity at $\mathbf{q} = 0$ induces an instability towards phase separation, while the singularity at $\mathbf{k}_{DW}$ drives a density wave and provides a supersolid phase. In the following, we discuss these two instabilities in detail.

For a fermionic system with a regular density of states, the Lindhard function at $\mathbf{q} = 0$ reduces to $\chi(T \to 0, 0) = -N(0)$ at low temperatures, with $N(0)$ the fermionic density of states. However, fermions on a square lattice within a tight-binding scheme exhibit a logarithmic Van Hove singularity in the density of states, $N(\epsilon) = N_0 K[\sqrt{1 - \epsilon^2/16J_F^2}] \sim N_0 \ln|16J_F/\epsilon|$ with $N_0 = 1/(2\pi^2 J_F)$ and $K[k]$ the complete elliptic integral of the first kind [17]; see Fig. 2. The response at half filling $n_F = 1/2$ then behaves as ($c_1 = 2\exp(C)/\pi \approx 1.13$)

$$\chi(T \to 0, 0) = \int d\epsilon N(\epsilon) \partial_\epsilon f(\epsilon) \sim -N_0 \ln\frac{16c_1 J_F}{T}. \quad (6)$$

The coupling between the bosons and the fermions induces an attraction between the bosons, and the effective long distance scattering parameter takes the form $U_{eff} = U_B + U^2_{FB}\chi(T, 0)$. The thermodynamic stability of a superfluid condensate at low temperatures requires a positive effective interaction $U_{eff} > 0$ and the condition $U_{eff}(T_{PS}) = 0$ defines the critical temperature T_{PS} for phase separation. Using Eqs. (6), we find for the critical temperature

$$T_{\rm PS} = 16 c_1 J_{\rm F} \exp[-1/\lambda_{\rm FB}] \quad (7)$$

with $\lambda_{\rm FB} = U_{\rm FB}^2 N_0 / U_{\rm B} \ll 1$ the ratio between the induced attraction and the intrinsic repulsion between the bosons. Below the critical temperature $T_{\rm PS}$, the effective interaction $U_{\rm eff}$ turns negative, providing a negative compressibility and rendering the Bose system unstable. The new ground state with fixed averaged densities $n_{\rm B}$ and $n_{\rm F}$ exhibits phase separation with areas of increased and decreased local densities coexisting; cf. Fig. 1.

Note that this transition towards phase separation exhibits two major differences as compared to the phase separation discussed in Refs. [15,18]. First, our phase separation appears as an instability, i.e., for arbitrary small coupling $U_{\rm FB}$ between the bosons and fermions. Second, the increase/decrease in the bosonic density drives the fermionic density away from half filling, providing a regular $\chi(T, 0)$ which stabilizes the system.

This phase separation is in competition with a second instability in the system triggered by the singularity in the Lindhard function at $\mathbf{k}_{\rm DW}$. Using (5) and the symmetry $\epsilon_{\rm F}(\mathbf{q} + \mathbf{k}_{\rm DW}) = -\epsilon_{\rm F}(\mathbf{q})$, the Lindhard function becomes

$$\chi(T, \mathbf{k}_{\rm DW}) = \int d\epsilon N(\epsilon) \frac{\tanh(\epsilon/2T)}{-2\epsilon} \sim -\frac{N_0}{2}\left[\ln\frac{16 c_1 J_{\rm F}}{T}\right]^2.$$

The combination of Van Hove singularities and perfect nesting produces the $[\ln T]^2$ singular behavior known to produce a T_c enhancement in superconductivity [19]. Within Bogoliubov theory, the bosonic quasiparticle spectrum becomes

$$E_{\rm B}(\mathbf{q}) = \sqrt{\epsilon_{\rm B}^2(\mathbf{q}) + 2 n_{\rm B} \epsilon_{\rm B}(\mathbf{q})[U_{\rm B} + U_{\rm FB}^2 \chi(T, \mathbf{q})]}. \quad (8)$$

The induced attraction between the bosons reduces the energy of quasiparticles providing a "roton" minimum at $\mathbf{k}_{\rm DW}$ (see also [20]), which vanishes $[E_{\rm B}(\mathbf{k}_{\rm DW}) = 0]$ at the critical temperature

$$T_{\rm DW} = 16 c_1 J_{\rm F} \exp[-\sqrt{(2 + t_{\rm B})/\lambda_{\rm FB}}] \quad (9)$$

with $t_{\rm B} = 8 J_{\rm B}/n_{\rm B} U_{\rm B}$ the ratio between the kinetic and the interaction energy of the bosons (weak coupling requires $t_{\rm B} \gg \lambda_{\rm FB}$). Below this critical temperature, the boson mode $b_{\mathbf{k}_{\rm DW}}$ becomes macroscopically occupied and its interference with the condensate produces a bosonic density wave; see below. In this new phase, a (quasi-)condensate characterized by an off-diagonal (quasi-)long-range order with a finite superfluid stiffness coexists with a density wave providing diagonal long-range order, thus establishing a supersolid phase. Note that decreasing the hopping $t_{\rm B}$ drives a superfluid to Mott-insulator transition for commensurate densities [1] below $t_{\rm B} < t_{\rm SF-MI} \approx 1/3$, providing an additional competing phase at strong coupling $t_{\rm B} < 1$.

Next, we study the supersolid phase within a mean-field description. We introduce the mean fields $\langle b_0 \rangle = \sqrt{n_0 N} \exp(i\varphi_0)$ and $\langle b_{\mathbf{k}_{\rm DW}} \rangle = (\Delta/2U_{\rm FB})\sqrt{N/n_0} \exp(i\varphi)$ and neglect thermal excitations of bosonic quasiparticles at $T \ll T_{\rm KT}$, implying the constraint $n_{\rm B} = n_0 + \Delta^2/(4 n_0 U_{\rm FB}^2)$. The bosonic density takes the form (to be evaluated at lattice sites)

$$n_{\rm B}(x, y) = n_{\rm B} + \frac{\Delta \cos\theta}{U_{\rm FB}}\left[\cos\frac{\pi x}{a}\cos\frac{\pi y}{a}\right] \quad (10)$$

with $\theta = \varphi_0 - \varphi$; this bosonic density wave appears as a result of the interference between the two condensates $\langle b_0 \rangle$ and $\langle b_{\mathbf{k}_{\rm DW}} \rangle$. Neglecting terms independent of Δ, the Hamiltonian (3) per unit cell reduces to

$$\frac{H}{N} = 2 J_{\rm B} \frac{\Delta^2}{n_{\rm B} U_{\rm FB}^2} + \frac{U_{\rm B} \Delta^2 \cos^2\theta}{2 U_{\rm FB}^2} + \frac{H_{\rm F}}{N} + O(\Delta^4). \quad (11)$$

The first and second terms describe the increase in the kinetic and interaction energies of the bosons, while $H_{\rm F}$ accounts for the nesting of fermions with $\mathbf{q} \in K$ and $\mathbf{q}' = \mathbf{q} - \mathbf{k}_{\rm DW} + \mathbf{K}_m$ (the reciprocal lattice vector $\mathbf{K}_m$ ensures the constraint $\mathbf{q}' \in K$),

$$H_{\rm F} = \frac{1}{2}\sum_{\mathbf{q} \in K} (c_{\mathbf{q}}^+, c_{\mathbf{q}}^+) \begin{pmatrix} \epsilon_{\rm F}(\mathbf{q}) & \Delta\cos\theta \\ \Delta\cos\theta & \epsilon_{\rm F}(\mathbf{q}') \end{pmatrix} \begin{pmatrix} c_{\mathbf{q}} \\ c_{\mathbf{q}'} \end{pmatrix}. \quad (12)$$

Diagonalizing, we obtain the fermionic quasiparticle excitation spectrum $\tilde{\epsilon}_{\rm F}(\mathbf{k}, \Delta) = \pm[\epsilon_{\rm F}^2(\mathbf{k}) + \cos^2\theta \Delta^2]^{1/2}$. Minimizing the thermodynamic potential $\Omega(T, \Delta, \theta)$ provides the constraint $\theta = s\pi$, with s an integer, and the self-consistency relation ($\partial_\Delta \Omega = 0$)

$$\frac{1}{\lambda_{\rm FB}}(2 + t_{\rm B}) = \frac{1}{N_0}\int_K \frac{d\mathbf{k}}{v_0} \frac{\tanh[\tilde{\epsilon}_{\rm F}(\mathbf{k}, \Delta)/2T]}{\tilde{\epsilon}_{\rm F}(\mathbf{k}, \Delta)}. \quad (13)$$

Setting $\Delta(T_{\rm DW}) = 0$, we reproduce the critical temperature (9). Using the density of states $N_\Delta(\epsilon) = N(\sqrt{\epsilon^2 - \Delta^2})\ |\epsilon|/\sqrt{\epsilon^2 - \Delta^2}$, the gap at $T = 0$ becomes $\Delta(0) = 32 J_{\rm F} \exp[-\sqrt{(2 + t_{\rm B})/\lambda_{\rm FB}}]$, and we obtain the standard BCS relation $2\Delta(0)/T_{\rm DW} = 2\pi/e^C \approx 3.58$. The mean field $\langle b_0 \rangle = \sqrt{n_0 N} \exp[i\varphi_0]$ breaks the continuous U(1) symmetry of the system and describes the off-diagonal quasi-long-range order. The excitation spectrum exhibits a linear dispersion around $q = 0$ and the quasi-long-range order is sufficient to provide a finite superfluid stiffness [9]. On the other hand, the mean field $\langle b_{\mathbf{k}_{\rm DW}} \rangle$ characterizes a commensurate density wave. Its phase is locked to the condensate phase via the constraint $\theta = s\pi$ and the excitation spectrum around $\mathbf{q} = \mathbf{k}_{\rm DW}$ is gapped. Note that the optical lattice reduces the continuous translation symmetry to a discrete symmetry providing a trivial crystalline order. The supersolid transition then breaks this discrete translation symmetry and establishes a nontrivial diagonal long-range order; breaking a discrete symmetry, long-range order can exist even at finite temperatures.

The competition between the two instabilities at $\mathbf{q} = 0$ and $\mathbf{k}_{DW}$ provides the phase diagram shown in Fig. 1. For $T_{DW} > T_{PS}$ the system undergoes a transition into a supersolid phase at T_{DW}; the gap in the fermionic excitation spectrum then removes the instability towards phase separation luring at lower temperatures. In turn, for $T_{PS} > T_{DW}$ the instability towards phase separation wins over the density wave formation and drives the fermionic density away from half filling; the nesting at $\mathbf{k}_{DW}$ is quenched and the instability towards density wave formation disappears. The projection of the critical line $T_{PS} = T_{DW}$ onto the λ_{FB}-$1/t_B$ plane satisfies the relation $1/t_B = \lambda_{FB}/(1 - 2\lambda_{FB})$; with decreasing coupling λ_{FB} we enter the supersolid phase, while increasing the hopping t_B drives the system towards phase separation.

Finally, we estimate the relevant experimental parameters for an atomic mixture of fermionic ^{40}K and bosonic ^{87}Rb with scattering lengths $a_B = 5.77$ nm and $a_{FB} \approx 15$ nm [6]. The 2D setup is realized through application of an anisotropic 3D optical lattice ($\lambda = 830$ nm and $V_F/V_B \approx 3/7$), with $V_F^z \gg V_F$ and $V_B^z \gg V_B$ quenching the interplane hopping (we express $V_{F,B}$ and $V_{F,B}^z$ via the recoil energies $E_{F,B}^r = 2\pi^2\hbar^2/\lambda^2 m_{F,B}$). The hopping amplitudes $J_{F,B}$ derive from the 1D Mathieu equation,

$$J_{F,B} = (4/\sqrt{\pi}) E_{F,B}^r V_{F,B}^{3/4} \exp(-2\sqrt{V_{F,B}}), \qquad (14)$$

while the interactions U_{FB} and U_B are given as [1]

$$\frac{U_{FB}}{E_F^r} = 8\sqrt{\pi} \frac{1 + m_F/m_B}{(1 + \sqrt{V_F/V_B})^{3/2}} \frac{a_{FB}}{\lambda\gamma} (V_F^z)^{1/4} V_F^{1/2} \qquad (15)$$

and $U_B/E_B^r = 4\sqrt{2\pi}(a_B/\lambda\gamma)(V_B^z)^{1/4}V_B^{1/2}$. Using a finite angle between the laser beams producing the standing light waves, we allow the relative size of the in- and out-of-plane lattice constants a and a_z to change. The parameter $\gamma = 2a_z/\lambda$ then denotes the increase in the unit cell volume and allows the interaction strengths U_B and U_{FB} to tune independently of $J_{F,B}$ (alternatively, Feshbach resonances allow a_{FB} to tune). Fixing $V_F^z = 20$, $n_F = 1/2$, and $n_B = 3/2$ in the trap center, we obtain for $\gamma = 3$ and $V_F = 4.5$ the coupling parameters $\lambda_{FB} \approx 0.39$, $t_B \approx 0.55$ ($T_{DW} \approx 48$ nK); we enter a regime at the border of validity of our weak coupling analysis. Note that the opening of a fermionic excitation gap in the supersolid phase pins the fermion density at $n_F = 1/2$ over many lattice sites for $\Delta \gg \hbar\omega_{trap}$ with ω_{trap} the trapping frequency. The system then exhibits a supersolid plateau in the center of the trap surrounded by a superfluid ring. Using the above estimates, the V_F-γ phase diagram is shown in Fig. 3; we find that changing the strength of the optical lattice V_F allows us to drive the supersolid into phase separation. The supersolid is easily identified via the coherence peak of a bosonic condensate in an optical lattice and the additional appearance of coherence peaks at $\mathbf{k}_{DW}$; using the above parameters, the weight of these additional peaks involve 15% of the total particle number.

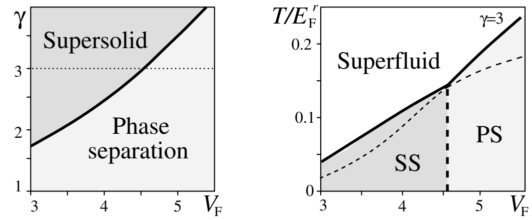

FIG. 3. Left: V_F-γ phase diagram at low temperatures. Increasing the strength of the optical lattice in an experiment allows us to drive the supersolid into phase separation. Right: transition temperatures for $\gamma = 3$ ($E_F^r = 347$ nK).

In conclusion, we have identified a new supersolid phase in a 2D Fermi-Bose gas mixture subject to an optical lattice; the bosons then play the role of the phonons in a condensed matter system. The perfect Fermi-surface nesting leads to the appearance of a fermionic density wave and the condensation of the bosons at $\mathbf{k}_{DW}$. The interference of this $\mathbf{k}_{DW}$ condensate with the usual $\mathbf{q} = 0$ condensate establishes the bosonic density wave characteristic of the supersolid phase.

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