Towards Quantum Superpositions of a Mirror

William Marshall,[1,2] Christoph Simon,[1] Roger Penrose,[3,4] and Dik Bouwmeester[1,2]

[1]*Department of Physics, University of Oxford, Oxford OX1 3PU, United Kingdom*
[2]*Department of Physics, University of California, Santa Barbara, California 93106, USA*
[3]*Center for Gravitational Physics and Geometry, The Pennsylvania State University, University Park, Pennsylvania 16802, USA*
[4]*Department of Mathematics, University of Oxford, Oxford OX1 3LB, United Kingdom*

(Received 30 September 2002; published 23 September 2003; publisher error corrected 25 September 2003)

We propose an experiment for creating quantum superposition states involving of the order of 10^{14} atoms via the interaction of a single photon with a tiny mirror. This mirror, mounted on a high-quality mechanical oscillator, is part of a high-finesse optical cavity which forms one arm of a Michelson interferometer. By observing the interference of the photon only, one can study the creation and decoherence of superpositions involving the mirror. A detailed analysis of the requirements shows that the experiment is within reach using a combination of state-of-the-art technologies.

DOI: 10.1103/PhysRevLett.91.130401 PACS numbers: 03.65.Ta, 03.65.Yz, 42.50.Ct

Introduction.—In 1935 Schrödinger pointed out that according to quantum mechanics even macroscopic systems can be in superposition states [1]. The associated quantum interference effects are expected to be hard to detect due to environment induced decoherence [2]. Nevertheless, there have been proposals on how to create and observe macroscopic superpositions in various systems [3–7], as well as experiments demonstrating superposition states of superconducting devices [8] and large molecules [9]. One long-term motivation for this kind of experiment is the search for unconventional decoherence processes [5,10].

In several of the above proposals a small quantum system (e.g., a photon [4–6] or a superconducting island [7]) is reversibly coupled to a large system (e.g., a moveable mirror [4–6] or a cantilever [7]) in order to create a macroscopic superposition. The existence of the quantum superposition of the large system is verified by observing the disappearance and reappearance of interference for the small system, as the large system is driven into a superposition and then returns to its initial state. The challenge is to find a feasible implementation of this idea.

Our proposal develops on the ideas in Refs. [4,5]. We also use results from Ref. [6], which relies on coupling between atoms and photons in a microcavity to create and detect superposition states of a moveable mirror. In particular, the formalism used in Ref. [6], based on Refs. [11,12], is applicable to our case. The main purpose here is to show that our purely optical proposal has the potential to be performed with current technology.

Principle.—The proposed setup, shown in Fig. 1, consists of a Michelson interferometer which has a high-finesse cavity in each arm. The cavity in arm (A) contains a tiny mirror attached to a micromechanical oscillator, similar to the cantilevers in atomic force microscopes. The cavity is used to enhance the radiation pressure of the photon on the mirror. The initial superposition of the photon being in either arm causes the system to evolve into a superposition of states corresponding to two distinct locations of the mirror. The observed interference of the photon allows one to study the creation of coherent superposition states of the mirror.

The system can be described by a Hamiltonian [6,11]

$$H = \hbar\omega_c a^\dagger a + \hbar\omega_m b^\dagger b - \hbar G a^\dagger a(b + b^\dagger), \qquad (1)$$

where ω_c and a are the frequency and creation operator for the photon in the cavity, ω_m and b are the frequency and phonon creation operator for the center of mass motion of the mirror, and $G = (\omega_c/L)\sqrt{(\hbar/2M\omega_m)}$ is the coupling constant, where L is the cavity length and M is the mass of the mirror.

Let us suppose that initially the photon is in a superposition of being in either arm A or B, and the mirror is in

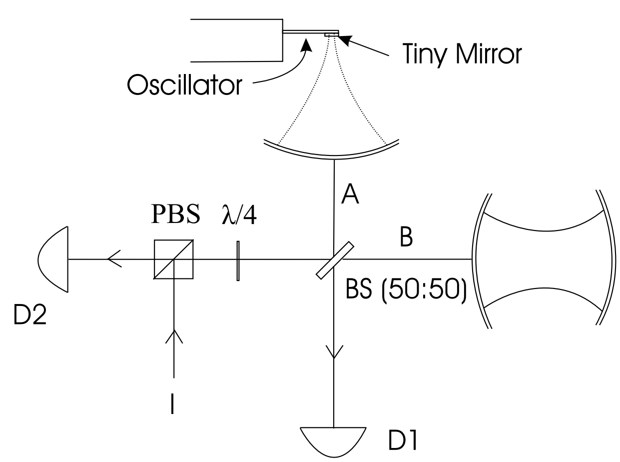

FIG. 1. The proposed setup: a Michelson interferometer for a single photon, where in each arm there is a high-finesse cavity. The cavity in arm A has a very small end mirror mounted on a micromechanical oscillator. The single photon comes in through I. If the photon is in arm A, the motion of the small mirror is affected by its radiation pressure. The photon later leaks out of either cavity and is detected at D1 or D2.

its ground state $|0\rangle_m$. Then the initial state is $|\psi(0)\rangle = (1/\sqrt{2})(|0\rangle_A|1\rangle_B + |1\rangle_A|0\rangle_B)|0\rangle_m$. After a time t the state of the system will be given by [6,12]

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}} e^{-i\omega_c t}[|0\rangle_A|1\rangle_B|0\rangle_m + e^{i\kappa^2(\omega_m t - \sin\omega_m t)}|1\rangle_A|0\rangle_B \times |\kappa(1 - e^{-i\omega_m t})\rangle_m], \quad (2)$$

where $\kappa = G/\omega_m$, and $|\kappa(1 - e^{-i\omega_m t})\rangle_m$ denotes a coherent state with amplitude $\kappa(1 - e^{-i\omega_m t})$. In the second term on the right-hand side the mirror moves under the influence of the radiation pressure of the photon in cavity A. The mirror oscillates around a new equilibrium position determined by the driving force. The parameter κ quantifies the displacement of the mirror in units of the size of the ground state wave packet.

The maximum interference visibility for the photon is given by twice the modulus of the off-diagonal element of the photon's reduced density matrix. By tracing over the mirror one finds from Eq. (2) that the off-diagonal element has the form $\frac{1}{2} e^{-\kappa^2(1-\cos\omega_m t)} e^{i\kappa^2(\omega_m t - \sin\omega_m t)}$. The first factor is the modulus, reaching a minimum after half a period at $t = \pi/\omega_m$, when the mirror is at its maximum displacement. The second factor gives the phase, which is identical to that obtained classically due to the varying length of the cavity.

In the absence of decoherence, after a full period, the system is in the state $(1/\sqrt{2})(|0\rangle_A|1\rangle_B + e^{i\kappa^2 2\pi}|1\rangle_A|0\rangle_B) \times |0\rangle_m$, such that the mirror is again disentangled from the photon. Full interference can be observed if the photon is detected at this time, provided that the phase factor $e^{i\kappa^2 2\pi}$ is taken into account. This revival, shown in Fig. 2, demonstrates the coherence of the superposition state that exists at intermediate times. For $\kappa^2 \gtrsim 1$ the superposition involves two distinct mirror positions. If the environment of the mirror "remembers" that the mirror has moved, then, even after a full period, the photon will still be entangled with the mirror's environment, and thus the revival will not be complete. Therefore the setup can be used to measure the decoherence of the mirror.

Here we have assumed that the mirror starts out in its ground state. We will argue below that optical cooling close to the ground state should be possible. However, in Ref. [6] it was shown that this is not necessary for observing the revival, although for a thermal mirror state with an average phonon number $\bar{n} = 1/(e^{\hbar\omega_m/kT} - 1)$ the revival peak is narrowed by a factor of $\sqrt{\bar{n}}$, leading to stricter requirements on the stability; see Fig. 2 and the discussion below. We now discuss the experimental requirements for achieving a superposition of distinct mirror positions and for observing the revival at $t = 2\pi/\omega_m$.

Conditions for displacement by ground state size.—We require $\kappa^2 \gtrsim 1$, which implies the momentum imparted by the photon has to be larger than the initial quantum uncertainty of the mirror's momentum. Let N denote the number of round-trips of the photon in the cavity during one period of the mirror's motion, such that $2NL/c = 2\pi/\omega_m$. The condition $\kappa^2 \gtrsim 1$ can be written

$$\frac{2\hbar N^3 L}{\pi c M \lambda^2} \gtrsim 1, \quad (3)$$

where λ is the wavelength of the light. The factors entering Eq. (3) are not all independent. The achievable N, determined by the quality of the mirrors, and the minimum mirror size (and hence M) both depend on λ. The mirror's lateral dimension should be an order of magnitude larger than λ to limit diffraction losses. The thickness required in order to achieve sufficiently high reflectivity depends on λ as well.

Equation (3) allows one to compare the viability of different wavelength ranges. While the highest values for N are achievable for microwaves using superconducting mirrors (up to 10^{10}), this is counteracted by their longer wavelengths. On the other hand, there are no good mirrors for highly energetic photons. The optical regime is optimal, given current mirror technology. We propose an experiment with λ around 630 nm.

The cavity mode needs to have a sharp focus on the tiny mirror, which requires the other cavity end mirror to be large due to beam divergence. The maximum cavity length is therefore limited by the difficulty of making large high-quality mirrors. We propose a cavity length of 5 cm, and a small mirror size of $10 \times 10 \times 10$ μm, leading to a mass of order 5×10^{-12} kg.

Such a mirror on a mechanical oscillator can be fabricated by coating a silicon cantilever with alternating layers of SiO_2 and a metal oxide. The best current optical mirrors are made in this way. A larger silicon oscillator has been coated with SiO_2/Ta_2O_5 and used as part of a high-finesse cavity in Ref. [13].

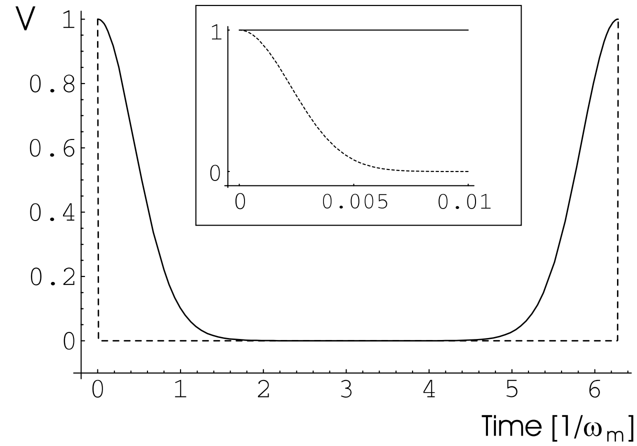

FIG. 2. Time evolution of the interference visibility V of the photon over one period of the mirror's motion for the case where the mirror has been optically cooled close to its ground state ($\bar{n} = 2$, solid line) and for $T = 2$ mK, which corresponds to $\bar{n} = 100\,000$ (dashed line—see also inset). The visibility decays after $t = 0$, but in the absence of decoherence there is a revival of the visibility after a full period. The width of the revival peak scales like $1/\sqrt{\bar{n}}$.

For the above dimensions the condition Eq. (3) is satisfied for $N = 5.6 \times 10^6$. Correspondingly, photon loss per reflection must be smaller than 3×10^{-7}, about a factor of 4 below reported values for such mirrors [14] and for a transmission of 10^{-7}, consistent with a 10 μm mirror thickness. For these values, about 1% of the photons are still left in the cavity after a full period of the mirror. For the above values of N and L one obtains a frequency $\omega_m = 2\pi \times 500$ Hz. This corresponds to a spread of the mirror's ground state wave function of order 10^{-13} m.

The fact that a relatively large L is needed to satisfy Eq. (3) implies that the creation of superpositions following the microcavity based proposal of Ref. [6] imposes requirements beyond current technology. A large L is helpful because, for a given N, it allows a lower frequency ω_m, and thus a more weakly bound mirror that is easier to displace by the photon.

Decoherence.—The requirement of observing the revival puts a bound on the acceptable environmental decoherence. To estimate the expected decoherence we model the mirror's environment by an (Ohmic) bath of harmonic oscillators. The effect of this can approximately be described by a decoherence rate $\gamma_D = \gamma_m k T_E M(\Delta x)^2/\hbar^2$ governing the decay of off-diagonal elements between different mirror positions [2]. Here γ_m is the damping rate for the mechanical oscillator, T_E is the temperature of the environment, which is constituted mainly by the internal degrees of freedom of the mirror and cantilever, and Δx is the separation of two coherent states that are originally in a superposition. This approximation is strictly valid only for times much longer than $2\pi/\omega_m$ and for Δx large compared to the width of the individual wave packets. Here we assume that the order of magnitude of the decoherence is well captured by γ_D. If the experiment achieves $\kappa^2 \gtrsim 1$, i.e., a separation by the size of a coherent state wave packet, $\Delta x \sim \sqrt{(\hbar/M\omega_m)}$, the condition $\gamma_D \lesssim \omega_m$ can be cast in the form

$$Q \gtrsim \frac{kT_E}{\hbar\omega_m}, \tag{4}$$

where $Q = \omega_m/\gamma_m$ is the quality factor of the mechanical oscillator. For $Q \gtrsim 10^5$, which has been achieved [15] for silicon cantilevers of approximately the right dimensions and frequency, this implies that the temperature of the environment has to be of the order of 2 mK, which is achievable with state-of-the-art dilution refrigerators.

Optical cooling.—Cooling the mirror's center of mass motion significantly eases the stability requirements for the proposed experiment. A method for optical cooling of a mirror via feedback was first proposed in Ref. [16]. By observing the phase of the output field of a cavity, its length can be measured with high precision. This can be used to implement a feedback mechanism that cools the center of mass motion of the mirror far below the temperature of its environment. A variation of the original scheme was experimentally implemented in Ref. [17], where a vibrational mode of a macroscopic mirror was cooled using a feedback force proportional to the natural damping force, but larger by a gain factor g. The size of g determines the achievable final temperature for a given T_E. For a tiny mirror, large gain values are realistic using the radiation pressure of a second laser beam to implement the feedback force. To analyze cooling to the quantum regime, one has to take into account the fact that measurement and feedback introduce noise, Ref. [18].

For our proposed experiment the constant component of the feedback laser has to balance the force from the measurement field, since otherwise the mirror would start to oscillate when the light is turned off. Adapting Ref. [19], the final energy of the cooled mirror is given by

$$E_c = \frac{\hbar\omega_m}{2}\frac{1}{2(1+g)}\left[\frac{4k_B T_E}{\hbar\omega_m} + 2\zeta + \frac{g^2}{\eta\zeta}\right], \tag{5}$$

where T_E is the temperature of the mirror's environment, $\zeta = (64\pi c P/M\gamma_m\omega_m\lambda\gamma_c^2 L^2)$, with P the light intensity incident on the measurement cavity and γ_c the cavity decay rate, and η the detection efficiency. The first term in Eq. (5) comes from the original thermal fluctuations, which are suppressed by the feedback. The second term is the back action noise from the measurement and feedback light. It differs from the formula of Ref. [19] by a factor of 2 to include the noise from the feedback laser. The third term is the noise due to imperfect measurement. Increasing the light intensity in the cavity improves the measurement precision, but also increases the back action noise.

The energy of the mirror can be made very close to its ground state energy choosing realistic parameter values; $E_c = \hbar\omega_m$ can be achieved with $g = 6 \times 10^5$, $T_E = 2$ mK, $P = 10^{-8}$ W, $\gamma_c = 3 \times 10^7$ s^{-1}, $\lambda = 800$ nm, $\eta = 0.8$, $\gamma_m = 0.03$ s^{-1}, and M, ω_m, L as before. The necessary feedback force for such a high value of g can be achieved with a feedback laser intensity modulation of $\Delta P_{fb} = 10^{-6}$ W. To balance the measurement field, the constant component of the feedback laser should be $\bar{P}_{fb} = 4 \times 10^{-6}$ W. The relatively large value of γ_c can be achieved in the cavity used in the superposition experiment by working at a wavelength away from where the mirrors are optimal.

Once the mirror has been cooled close to its ground state, which is reached in a time of order $1/(\gamma_m g)$ [20], the measurement and feedback laser fields should be turned off simultaneously. Then the experiment proceeds as described above. Reheating of the mirror happens at a time scale of $1/\gamma_m$ [20] and thus is not a problem for a high-Q oscillator. After every run of the experiment, the mirror has to be reset to its initial state by the optical cooling procedure.

Stability.—The distance between the large cavity end mirror and the equilibrium position of the small mirror has to be stable to of order $\lambda/20N = 0.6 \times 10^{-14}$ m over the whole measurement time, which is determined as follows. A single run of the experiment starts by sending a weak pulse into the interferometer, such that on average 0.1 photons go into either cavity. This probabilistically prepares a single-photon state as required to a good approximation. The two-photon contribution has to be kept low because it causes noise in the interferometer. Considering the required low value of ω_m and the fact that approximately 1% of the photons remain after a full period for the assumed loss, this implies a detection rate of approximately 10 photons per minute in the revival interval. Thus we demand stability to of order 10^{-14} m over a few minutes. Stability of order 10^{-13} m/min for an STM at 8 K was achieved with a rather simple suspension [21]. Gravitational wave observatories using interferometers also require very high stability in order to have a length sensitivity of 10^{-19} m over time scales of a ms or greater, for arm lengths of order 1 km [22]. If the mirror is in a thermal state, the revival peak is narrowed by a factor $\sqrt{\bar{n}}$ [6], leading to lower count rates in the revival interval and thus making the stability requirements stricter by the same factor, cf. Fig. 2.

The experiment also requires ultrahigh vacuum conditions in order to ensure that events where an atom hits the cantilever are sufficiently rare not to cause significant errors, which is at the level of about 5/s. Background gas particle densities of order $100/\mathrm{cm}^3$ have been achieved [23] and are sufficient for our purposes.

Outlook and conclusions.—In principle the proposed setup has the potential to test wave function reduction models, in particular, the one of Ref. [5]. We estimate that the ratio Q/T needs to be improved by about 6 orders of magnitude from the values discussed in this Letter ($Q = 10^5$ and $T = 2$ mK) to make the predicted wave function decoherence rate comparable to the environmental decoherence rate. However, temperatures as low as 60 μK have been achieved with adiabatic demagnetization [24], while Q is known to increase with decreasing temperature [15] and through annealing [25].

We have performed a detailed study of the experimental requirements for the creation and observation of quantum superposition states of a mirror consisting of 10^{14} atoms, approximately 9 orders of magnitude more massive than any superposition observed to date. Our analysis shows that, while very demanding, this goal appears to be within reach of current technology.

This work was supported by the E.U. (IST-1999-10033). W. M. is supported by EPSRC (Contract No. 00309297). C. S. is supported by a Marie Curie Fund of the E.U. (No. HPMF-CT-2001-01205) and thanks the University of California Santa Barbara for its hospitality. R. P. thanks the NSF for support under Contract No. 00-90091 and the Leverhulme Foundation. We would like to thank S. Bose, M. Davies, M. de Dood, T. Knuuttila, R. Lalezari, A. Lamas-Linares, and J. Pethica for useful discussions.

[1] E. Schrödinger, Naturwissenschaften **23**, 807 (1935).
[2] E. Joos *et al.*, *Decoherence and the Appearance of a Classical World in Quantum Theory* (Springer, Berlin, 1996); W. H. Zurek, Phys. Today **44**, 36 (1991).
[3] J. Ruostekoski, M. J. Collett, R. Graham, and D. F. Walls, Phys. Rev. A **57**, 511 (1998); J. I. Cirac, M. Lewenstein, K. Molmer, and P. Zoller, Phys. Rev. A **57**, 1208 (1998).
[4] D. Bouwmeester, J. Schmiedmayer, H. Weinfurter, and A. Zeilinger, in *Gravitation and Relativity: At the Turn of the Millennium*, edited by N. Dadhich and J. Narlikar (IUCAA, Pune, 1998). This paper was based on discussions between J. Schmiedmayer, R. Penrose, D. Bouwmeester, J. Dapprich, H. Weinfurter, and A. Zeilinger (1997).
[5] R. Penrose, in *Mathematical Physics 2000*, edited by A. Fokas *et al.* (Imperial College, London, 2000).
[6] S. Bose, K. Jacobs, and P. L. Knight, Phys. Rev. A **59**, 3204 (1999).
[7] A. D. Armour, M. P. Blencowe, and K. C. Schwab, Phys. Rev. Lett. **88**, 148301 (2002).
[8] C. H. van der Wal *et al.*, Science **290**, 773 (2000); J. R. Friedman, V. Patel, W. Chen, S. K. Tolpygo, and J. E. Lukens, Nature (London) **406**, 43 (2000).
[9] M. Arndt *et al.*, Nature (London) **401**, 680 (1999); W. Schöllkopf and J. P. Toennies, Science **266**, 1345 (1994).
[10] G. C. Ghirardi, A. Rimini, and T. Weber, Phys. Rev. D **34**, 470 (1986); G. C. Ghirardi, P. Pearle, and A. Rimini, Phys. Rev. A **42**, 78 (1990); I. C. Percival, Proc. R. Soc. London A **447**, 189 (1994); D. I. Fivel, Phys. Rev. A **56**, 146 (1997); L. Diósi, Phys. Rev. A **40**, 1165 (1989).
[11] C. K. Law, Phys. Rev. A **51**, 2537 (1994); C. K. Law, Phys. Rev. A **49**, 433 (1993).
[12] S. Mancini, V. I. Man'ko, and P. Tombesi, Phys. Rev. A **55**, 3042 (1997).
[13] I. Tittonen *et al.*, Phys. Rev. A **59**, 1038 (1999).
[14] G. Rempe, R. J. Thompson, H. J. Kimble, and R. Lalezari, Opt. Lett. **17**, 363 (1992); C. J. Hood, H. J. Kimble, and J. Ye, Phys. Rev. A **64**, 033804 (2001).
[15] H. J. Mamin and D. Rugar, Appl. Phys. Lett. **79**, 3358 (2001).
[16] S. Mancini, D. Vitali, and P. Tombesi, Phys. Rev. Lett. **80**, 688 (1998).
[17] P. F. Cohadon, A. Heidmann, and M. Pinard, Phys. Rev. Lett. **83**, 3174 (1999).
[18] J.-M. Courty, A. Heidmann, and M. Pinard, Eur. Phys. J. D **17**, 399 (2001).
[19] D. Vitali, S. Mancini, L. Ribichini, and P. Tombesi, quant-ph/0211102.
[20] M. Pinard, P. F. Cohadon, T. Briant, and A. Heidmann, Phys. Rev. A **63**, 013808 (2000).
[21] B. C. Stipe, M. A. Rezaei, and W. Ho, Rev. Sci. Instrum. **70**, 137 (1999).
[22] S. Rowan and J. Hough, Living Rev. Relativity **3**, 2000 (2000).
[23] G. Gabrielse *et al.*, Phys. Rev. Lett. **65**, 1317 (1990).
[24] W. Yao *et al.*, J. Low Temp. Phys. **120**, 121 (2000).
[25] J. Yang, T. Ono, and M. Esashi, Appl. Phys. Lett. **77**, 3860 (2000).