

## Coulomb Drag by Small Momentum Transfer between Quantum Wires

M. Pustilnik,<sup>1,2</sup> E. G. Mishchenko,<sup>3,4</sup> L. I. Glazman,<sup>1</sup> and A. V. Andreev<sup>3,4</sup>

<sup>1</sup>*Theoretical Physics Institute, University of Minnesota, Minneapolis, Minnesota 55455, USA*

<sup>2</sup>*School of Physics, Georgia Institute of Technology, Atlanta, Georgia 30332, USA*

<sup>3</sup>*Bell Laboratories, Lucent Technologies, Murray Hill, New Jersey 07974, USA*

<sup>4</sup>*Department of Physics, University of Colorado, Boulder, Colorado 80390, USA*

(Received 13 August 2002; published 19 September 2003)

We demonstrate that in a wide range of temperatures Coulomb drag between two weakly coupled quantum wires is dominated by processes with a small interwire momentum transfer. Such processes, not accounted for in the conventional Luttinger liquid theory, cause drag only because the electron dispersion relation is not linear. The corresponding contribution to the drag resistance scales with temperature as  $T^2$  if the wires are identical, and as  $T^5$  if the wires are different.

DOI: 10.1103/PhysRevLett.91.126805

PACS numbers: 73.63.Nm, 71.10.Pm, 72.15.Nj, 73.23.Ad

Electrons moving in a conductor generate a fluctuating electric field around it. This field gives rise to an unusual transport phenomenon, Coulomb drag between two closely situated conductors [1]. The structure of the fluctuating field is determined by electron correlations within the conductors. Correlations are stronger in conductors of lower dimensionality. The Tomonaga-Luttinger model captures some aspects of the correlations in the case of one-dimensional conductors (quantum wires). Within this model, the Coulomb drag was studied in [2,3]. In a typical setup [4,5] a dc current flows through the active wire (1), while the bias applied to the passive wire (2) sets  $I_2 = 0$ ; see Fig. 1. The “drag resistivity” (drag resistance per unit length of the interacting region) is then defined as

$$r = -\lim_{I_1 \rightarrow 0} \frac{e^2}{2\pi\hbar L} \frac{1}{dI_1} \frac{dV_2}{dI_1}. \quad (1)$$

The only source of drag in the Luttinger liquid is interwire backscattering, associated with a large momentum transfer between the wires. The model predicts a distinctive temperature dependence of the corresponding contribution  $r_{2k_F}$  to the drag resistivity [Eq. (1)]. In the case of identical wires  $r_{2k_F} \propto l_{2k_F}^{-1} e^{\Delta/T}$  at the lowest temperatures [2,3]. Here  $l_{2k_F}$  is the scattering length characterizing the interwire backscattering. At temperatures  $T$  above the gap  $\Delta$ , this exponential dependence is replaced by a power law,  $r_{2k_F} \sim l_{2k_F}^{-1} (T/\epsilon_F)^{1-\gamma}$ , where  $\epsilon_F$  is the Fermi energy. The exponential temperature dependence of  $r_{2k_F}$  indicates the formation of a zigzag charge order due to the  $2k_F$  component of the interwire interaction [2,3]. To the contrary, the exponent  $\gamma > 0$  in the power-law portion of the function  $r_{2k_F}(T)$  is determined by the interactions within the wires;  $\gamma = 0$  in the absence of interactions [6]. This renormalization of  $r_{2k_F}$  is similar in origin to the suppression of the conductance of a Luttinger liquid with an impurity [7].

However, forward scattering between the wires also induces drag. To see this, one has to go beyond the

Tomonaga-Luttinger model and account for the nonlinearity of the electronic dispersion relation. If the electron velocity depends on momentum, then even small (compared to  $2k_F$ ) momentum transfer results in drag.

The small ( $r_0$ ) and large ( $r_{2k_F}$ ) momentum transfer contributions to the drag are inversely proportional to the corresponding scattering lengths  $l_0$  and  $l_{2k_F}$ , respectively. Their ratio  $l_0/l_{2k_F} \propto e^{-4k_F d}$  depends strongly on the distance  $d$  between the wires. If the drag effect is employed to study the correlations within a wire, rather than the zigzag order induced by interwire interaction, then  $d$  must be large:  $k_F d > 1$ . In this case the gap  $\Delta \sim \epsilon_F (k_F l_{2k_F})^{-1/\gamma}$  becomes narrow, and the role of forward scattering increases.

In this Letter we demonstrate that the drag resistivity between weakly coupled wires is dominated by the forward scattering in a wide temperature range. Even for identical wires, which is the most favorable for the backscattering case,  $r_0 \propto T^2$  wins over  $r_{2k_F}$  at all  $T$  above  $T^* \sim \epsilon_F (l_0/l_{2k_F})^{1/(1+\gamma)}$ . For different wires,  $r_{2k_F}$  is exponentially small at  $T \lesssim u\delta n$ , whereas  $r_0$  has a power-law low-temperature asymptotics; here  $\delta n$  is the mismatch of the electron densities between the wires and  $u$  is the characteristic plasma velocity; hereafter we set  $\hbar = 1$ .

The Hamiltonian of the system,  $H = H_1 + H_2 + H_{12}$ , is given by the sum of the Hamiltonians of the two isolated wires  $i = 1, 2$ ,

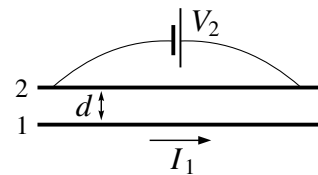


FIG. 1. Coulomb drag between quantum wires. A dc current  $I_1$  flows through the active wire (1). A voltage bias  $V_2$  is applied to the passive wire (2) in such a way that  $I_2 = 0$ .

$$H_i = \int dx \psi_i^\dagger(x) \frac{\hat{p}^2}{2m} \psi_i(x) + H_{\text{int}}, \quad \hat{p} = -id/dx, \quad (2)$$

$$H_{\text{int}} = \int dx dy \rho_i(x) U_{ii}(x-y) \rho_i(y),$$

and of the Hamiltonian of the interwire interaction,

$$H_{12} = \int dx_1 dx_2 \rho_1(x_1) U_{12}(x_1 - x_2) \rho_2(x_2); \quad (3)$$

here  $\rho_i(x) = \psi_i^\dagger(x) \psi_i(x)$ .

We assume that the interactions are screened by the nearby metallic gates. The screening length  $d_s$  is set by the distance to the gates and is typically [4,5] of the order of the separation  $d$  between the wires,  $d_s \sim d$ . The short-distance cutoff  $d_{ij}$  of the Coulomb potential is provided by the wire width  $d_0$  or the interwire distance  $d$  for  $i = j$  or  $i \neq j$ , respectively. The Fourier transforms  $U_{ij}(k) = \int dx e^{ikx} U_{ij}(x)$  are rapidly decreasing functions of  $k$  with  $U_{ij}(k) \propto e^{-|k|d_s}$  for  $|k| \gg 1/d_s$ , and  $U_{ij}(k) \approx \text{const}$  for  $|k| \ll 1/d_{ij}$ . Note that  $d_{ij} \sim d_s \sim d$  for interwire interaction  $U_{12}(k)$ .

Because of the interaction  $H_{12}$ , electrons in wire 2 experience a force [2] whose density is given by

$$\mathcal{F}_2 = \int dx [dU_{12}(x)/dx] \langle \rho_1(x) \rho_2(0) \rangle. \quad (4)$$

Since there is no current in wire 2, this force must be balanced by an external electric field,  $en_2 \mathcal{E}_2 + \mathcal{F}_2 = 0$ , where  $\mathcal{E}_2 = V_2/L$  and  $n_i = \langle \rho_i \rangle$  is the concentration of electrons in the wire  $i$ . At  $T \gg \Delta$  (see the discussion above) the correlation function in the right-hand side of Eq. (4) can be evaluated in the first order in  $U_{12}$ ,

$$\frac{V_2}{L} = \frac{1}{en_2} \int \frac{dk d\omega}{(2\pi)^2} k U_{12}^2(k) \tilde{S}_1(k, \omega) \tilde{S}_2(-k, -\omega), \quad (5)$$

where  $\tilde{S}_i(k, \omega)$  are the dynamic structure factors,

$$\tilde{S}_i(k, \omega) = \int dx dt e^{i\omega t - ikx} \langle \rho_i(x, t) \rho_i(0, 0) \rangle,$$

calculated in the presence of a finite current  $I_1$  in wire 1. The structure factor  $\tilde{S}_2(k, \omega)$  in wire 2 coincides with its equilibrium value,  $S_2(k, \omega)$ . The electronic subsystem in wire 1 is in equilibrium in the reference frame moving with the drift velocity  $v_d = I_1/en_1$  in the direction of the current. Therefore the structure factor  $\tilde{S}_1$  is obtained from the equilibrium value  $S_1$  using the Galilean transformation:  $\tilde{S}_1(k, \omega) = S_1(k, \omega - qv_d)$ . Equations (1) and (5) then yield

$$r = \int dk d\omega \frac{k^2 U_{12}^2(k)}{8\pi^3 n_1 n_2} \frac{\partial S_1(k, \omega)}{\partial \omega} S_2(-k, -\omega). \quad (6)$$

Now we use the fluctuation-dissipation theorem,

$$S_i(k, \omega) = \frac{2A_i(k, \omega)}{1 - e^{-\omega/T}}$$

to further simplify Eq. (6),

$$r = \int_0^\infty dk \int_0^\infty d\omega \frac{k^2 U_{12}^2(k)}{4\pi^3 n_1 n_2 T} \frac{A_1(k, \omega) A_2(k, \omega)}{\sinh^2(\omega/2T)}. \quad (7)$$

Here  $A_i$  is the imaginary part of the retarded density-density correlation function. Equation (7) was derived by different means in [8]; similar expressions have been also obtained for noninteracting systems with disorder [9]. Here we demonstrated the validity of Eq. (7) for clean interacting systems.

We start with the evaluation of the drag resistivity for noninteracting electrons ( $U_{ii} = 0$ ). Concentrating on the small momentum transfer contribution to  $r$ , we consider the limit  $l_0/l_{2k_F} \rightarrow 0$ , thus setting  $r_{2k_F} = 0$ . In this case the main contribution to the integral over  $k$  in Eq. (7) comes from small momenta  $k \ll k_F$  and small energies  $\omega \ll \epsilon_F$ . At these values of  $k$  and  $\omega$ , functions  $A_i(k, \omega)$  are sharply peaked at  $\omega_i = v_i k$ , where  $v_i = \pi n_i/m$  are the Fermi velocities in the two wires. For a given  $k < 2k_F$  the widths of the peaks can be estimated as

$$\delta\omega(k, T) = \max\{k^2/m, kT/k_F\}. \quad (8)$$

Equation (8) and the exact  $f$ -sum rule,

$$\int_0^\infty d\omega \omega A_i(k, \omega) = \frac{\pi n_i k^2}{m} \frac{1}{2}, \quad (9)$$

allow us to estimate the peak heights:  $A_i \sim k/2\delta\omega$ . If the difference between the Fermi velocities is small,

$$\delta v = |v_1 - v_2| \ll v_F = \pi n/m, \quad n = (n_1 + n_2)/2,$$

then Eq. (7) reduces to

$$r = \frac{1}{4\pi^3 n^2 T} \int_0^\infty dk \frac{k^2 U_{12}^2(k)}{\sinh^2(v_F k/2T)} \alpha(k, T), \quad (10)$$

$$\alpha(k, T) = \int_0^\infty d\omega A_1(k, \omega) A_2(k, \omega). \quad (11)$$

The function  $\alpha(k, T)$  depends on  $\delta v$ . If the wires are identical ( $\delta v = 0$ ), then Eq. (11) and the above estimates for  $A_i$  yield

$$\alpha(k, T) \approx \frac{k^2}{4\delta\omega(k, T)}. \quad (12)$$

There are two competing scales in the integrand of Eq. (10). The first scale,  $k_0 \sim 1/d \ll k_F$ , characterizes the  $k$  dependence of the interwire interaction  $U_{12}(k)$ . The typical wave vector of thermally excited electron-hole pairs,  $T/v_F$ , defines the second scale. The two scales coincide at  $T = T_0 = v_F k_0$ . At  $T \ll T_0$  one can replace  $U_{12}(k)$  by  $U_{12}(0)$  in Eq. (10). Furthermore, we use  $\alpha$  in the form of Eq. (12) at  $T = 0$ , which results in

$$r = \frac{c_1}{l_0} \left( \frac{T}{\epsilon_F} \right)^2, \quad \frac{1}{l_0} = \left[ \frac{U_{12}(0)}{2\pi v_F} \right]^2 n \quad (13)$$

with  $c_1 = \pi^4/12$ . Use of the exact form of  $A_i(k, \omega)$  in Eq. (11) changes only the numerical coefficient,  $c_1 = \pi^2/4$ .

The increase of temperature  $T$  above  $T_0$  results in a saturation of the drag resistivity. Indeed, at  $T_0 \ll T \ll \epsilon_F$  one can expand  $\sinh(v_F k/2T)$  in Eq. (10) and use  $\delta\omega = kT/k_F$  for the peak width in (12). This yields

$$r \sim \frac{1}{l_0} \int_0^\infty \frac{kdk}{n^2} \frac{U_{12}^2(k)}{U_{12}^2(0)} \sim \frac{1}{l_0} \left(\frac{T_0}{\epsilon_F}\right)^2. \quad (14)$$

A further increase of  $T$  leads to the decay of the drag,

$$r \propto l_0^{-1} (T_0/\epsilon_F)^2 (T/\epsilon_F)^{-3/2}, \quad T \gg \epsilon_F, \quad (15)$$

similar to the two-dimensional case [10].

We now consider wires with slightly different Fermi velocities  $\delta v > 0$ . In this case the peaks of  $A_i(k, \omega)$  are separated in  $\omega$  by  $k\delta v$ . We define a new temperature scale  $T_1 = k_F \delta v$  by equating the separation to the peak width (8). We assume this scale is small,  $T_1 \ll T_0$ . The difference between velocities does not affect the drag at  $T \gg T_1$ . However, at  $T \ll T_1$  the drag resistivity is suppressed exponentially. To obtain the leading asymptotics of  $r(T)$  it is sufficient to use the  $T=0$  limit [11] of  $A_i(k, \omega)$  in Eq. (11),  $\alpha(k, 0) = (m/4k)(k - m\delta v) \times$

$\theta(k - m\delta v)$ . Equation (10) then results in

$$r = \frac{\pi^2/4}{l_0} \left(\frac{T_1}{\epsilon_F}\right)^2 \frac{T}{T_1} e^{-T_1/T}, \quad T \ll T_1. \quad (16)$$

The activation temperature dependence Eq. (16) holds for all  $T \ll T_1$ , because for noninteracting electrons at  $T=0$  the product  $A_1(k, \omega)A_2(k, \omega)$  is exactly zero [11] at  $k < m\delta v$ . If electrons interact, some overlap of  $A_1$  and  $A_2$  exists even at small  $k \ll m\delta v$ . This yields a further contribution to  $r$ , which has a power-law temperature dependence. We evaluate this contribution for weak intrawire interaction.

It is convenient to write  $A(k, \omega)$  (we suppress the index  $i$  in the following) in the form  $A(k, \omega) = [S(k, \omega) - S(-k, -\omega)]/2$  and use the Lehmann representation for the dynamic structure factor:

$$S(k, \omega) = \frac{2\pi}{L} \sum_n |\langle n | \rho_k | g_s \rangle|^2 \delta(\omega - E_n + E_{g_s}). \quad (17)$$

Here  $L$  is the system size,  $|g_s\rangle$  is the ground state, and  $\rho_k = \sum_p \psi_{p+k}^\dagger \psi_p$ . We evaluate the matrix element in Eq. (17) in the first order in the intrawire interaction. The nonvanishing at the  $\omega - v_F k \gg \delta\omega$  contribution results from the processes in which the unperturbed final state  $|n\rangle$  in Eq. (17) has two electron-hole pairs:  $|n\rangle^{(0)} = \psi_{p+q}^\dagger \psi_p \psi_{p'-q}^\dagger \psi_{p'} |0\rangle$ . This contribution is

$$\begin{aligned} \delta S(k, \omega) &= \frac{1}{\pi^2} \int dp dp' dq dq' \delta(q - q' - k) \delta(\omega - \xi_{p+q} + \xi_p - \xi_{p'-q'} + \xi_{p'}) \\ &\times f_p (1 - f_{p+q}) f_{p'} (1 - f_{p'-q'}) K^2(p, p', q, q', \omega), \end{aligned} \quad (18)$$

where  $f_p$  are the Fermi functions,  $\xi_p = p^2/2m$ , and

$$\begin{aligned} K &= \frac{U(q') - U(p - p' + q')}{\omega - \xi_{p+q} + \xi_{p+q'}} \\ &- \frac{U(q') - U(p - p' + q)}{\omega - \xi_{p+q-q'} + \xi_p} + [p \leftrightarrow p', q \leftrightarrow -q']. \end{aligned}$$

Note that Eq. (18), unlike Eq. (17), accounts for a finite temperature. At  $\omega \ll \epsilon_F$  and  $k \ll k_F$ , Eq. (18) yields the interaction-induced correction to  $A(k, \omega)$ ,

$$\delta A(k, \omega) = \frac{\tilde{U}^2 k^4 \theta(\omega - v_F k)}{v_F m^2 \omega^2 - v_F^2 k^2}, \quad (19)$$

where  $\tilde{U} = [U(0) - U(2k_F)]/2\pi v_F \ll 1$ . This result is valid for  $\omega \ll \epsilon_F$ ,  $k \ll k_F$ , and  $|\omega - v_F k| \gg \max\{\tilde{U} v_F k, \delta\omega(k, T)\}$  and describes  $A(k, \omega)$  outside the interval Eq. (8). The limit of linear electron dispersion relation ( $m \rightarrow \infty$ ) corresponds [12] to  $\delta A(k, \omega) = 0$ .

We use Eq. (19) to evaluate the interaction-induced correction  $\delta r$  to the drag resistivity between nonidentical wires with  $T_1 = k_F \delta v \gg \epsilon_F \tilde{U}$ . At the lowest temperatures, Eqs. (10) and (11) yield

$$\delta r \sim \frac{\tilde{U}^2}{l_0} \left(\frac{T_1}{\epsilon_F}\right)^4 \left(\frac{T}{T_1}\right)^5. \quad (20)$$

With increasing temperature, the  $r(T)$  dependence changes from Eq. (20) to the activation law (16). At  $T \gg T_1$  the difference between wires does not affect  $r(T)$ .

We now argue that intrawire interactions do not change the quadratic temperature dependence of  $r(T)$  at  $T_1 \ll T \ll T_0$ ; see Eq. (13). At these temperatures, an estimate equivalent to Eq. (13) reads  $r \sim |U_{12}^2(0)| v_F^{-3} \delta\omega(k_T, T)$  and yields  $r \propto T^2$ ; here  $k_T \sim T/v_F \ll k_F$  is the wave vector of a typical electron-hole excitation. Interaction apparently does not affect the functional dependence of  $\delta\omega$  on  $k$  and  $T$ ; the estimate (8) still can be used, although the coefficients  $1/m$  and  $1/k_F$  in it are affected by the interaction.

The Tomonaga-Luttinger model is insufficient for the evaluation of  $\delta\omega$  in the presence of interaction: it implies linear electron spectrum, which yields [12]  $\delta\omega = 0$ . Accounting for the curvature of the electron spectrum complicates the treatment of the interaction greatly. The width  $\delta\omega$  can be explicitly evaluated in the

Calogero-Sutherland model which is characterized by a very specific interaction potential,

$$U_{ii}(x) = \frac{2\pi^2}{mL^2} \frac{\lambda(\lambda-1)}{\sin^2[\pi x/L]}. \quad (21)$$

The parameter  $\lambda$  here is related to the conventional interaction parameter  $g$  of the Luttinger liquid:  $g = 1/\lambda$ . This relation follows from the definition  $g = v_F/u$  in terms of the velocity of the collective mode (plasmon)  $u$ , and its value  $u = (\pi n/m)\lambda$  in the Calogero-Sutherland model [13,14]. For the rational values of  $\lambda$  and at  $T = 0$  the density-density correlation function is known exactly [13,14]. Because of the integrability of the model,  $A_i(k, \omega) \neq 0$  only in a finite interval of  $\omega$  around  $\omega = u_i k$  [15]. We found this interval for  $k \leq 2\pi n_i$ :

$$-(1/g) \frac{k^2}{2m} < \omega - u_i k < \frac{k^2}{2m}, \quad (22)$$

which yields for the width

$$\delta\omega(k, 0) = \frac{1+g}{2g} \frac{k^2}{m}. \quad (23)$$

In order to estimate  $r$  we note that Eq. (7) and the sum rule (9) remain valid in the presence of interactions within the wires. This allows us to follow the steps that led to Eq. (13). Replacing  $v_F$  by the plasma velocity  $u$  in Eq. (10) and using Eq. (23), we find

$$r = \frac{c_g}{l_0} \left( \frac{T}{\epsilon_F} \right)^2, \quad c_g \propto \frac{g^6}{1+g}, \quad (24)$$

which agrees with our expectation for the  $r(T)$  dependence. We are not aware of a reliable theory of  $A_i(k, \omega)$  beyond the exactly solvable case. However, the self-consistent Born approximation results [16] allow us to corroborate the estimate  $\delta\omega \propto k^2/m$  for the peak width, so, apparently, the  $r \propto T^2$  dependence is universal.

First observations of drag between ballistic quantum wires appeared recently [4,5]. In a limited temperature interval,  $0.2 \text{ K} < T < 0.9 \text{ K}$ , a threefold drop in the drag resistance was observed [4] with the increase of temperature. This drop was fit to a power law  $r \propto T^{-0.77}$  and interpreted as evidence of the Luttinger liquid behavior. However, the Fermi wave vector in the wires of Ref. [4] was estimated to be  $k_F = 6 \times 10^4 \text{ cm}^{-1}$ , which yields  $\epsilon_F = \hbar^2 k_F^2 / 2m^* \approx 0.2 \text{ K}$  (we used here  $m^* = 0.068m_0$  known for GaAs). It thus appears that the measurements of Ref. [4] correspond to a nondegenerate or weakly degenerate regime incompatible with the Luttinger liquid description. An alternative explanation of the observations [4,5] is provided by our theory. Indeed, using the values of  $k_F$  and  $d = 200 \text{ nm}$  of [4], we find  $k_F d = 1.2$ . Under this condition the small momentum transfer contribution dominates at  $T > T^*$ ; see Fig. 2. The observed [4,5] behavior of  $r(T)$  may correspond to the crossover

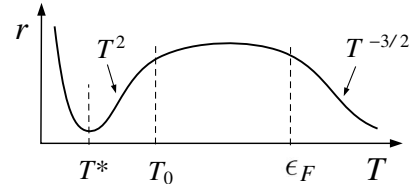


FIG. 2. Sketch of the temperature dependence of the drag resistivity between identical wires. The small momentum transfer contribution considered in this Letter dominates at  $T > T^*$ ; the ratio  $T^*/\epsilon_F$  is exponentially small for  $k_F d > 1$ .

regime between the limits  $r(T) = \text{const}$  and  $r(T) \propto T^{-3/2}$  presented by Eqs. (14) and (15).

We thank KITP, Santa Barbara, for hospitality, and K. Flensberg, A. Kamenev, S. Tarucha, and A. Tsvelik for discussions. This work was supported by NSF Grants No. DMR97-31756, No. DMR-9984002, No. DMR02-37296, and No. EIA02-10736 and by the Packard Foundation.

- [1] A. G. Rojo, *J. Phys. Condens. Matter* **11**, R31 (1999).
- [2] Yu. V. Nazarov and D. V. Averin, *Phys. Rev. Lett.* **81**, 653 (1998).
- [3] R. Klesse and A. Stern, *Phys. Rev. B* **62**, 16 912 (2000).
- [4] P. Debray *et al.*, *J. Phys. Condens. Matter* **13**, 3389 (2001); P. Debray *et al.*, *Semicond. Sci. Technol.* **17**, R21 (2002).
- [5] M. Yamamoto *et al.*, *Physica (Amsterdam)* **12E**, 726 (2002).
- [6] B. Y.-K. Hu and K. Flensberg, in *Hot Carriers in Semiconductors*, edited by K. Hess *et al.* (Plenum, New York, 1996), p. 943.
- [7] C. L. Kane and M. P. A. Fisher, *Phys. Rev. B* **46**, 15 233 (1992).
- [8] L. Zheng and A. H. MacDonald, *Phys. Rev. B* **48**, 8203 (1993).
- [9] A. Kamenev and Y. Oreg, *Phys. Rev. B* **52**, 7516 (1995); K. Flensberg *et al.*, *Phys. Rev. B* **52**, 14 761 (1995).
- [10] K. Flensberg and Ben Yu-Kuang Hu, *Phys. Rev. B* **52**, 14 796 (1995).
- [11] For noninteracting electrons at zero temperature  $A_i(k, \omega) = (m/2k)\theta(k^2/2m - |\omega - v_i k|)$ .
- [12] I. E. Dzyaloshinskii and A. I. Larkin, *Zh. Eksp. Teor. Fiz.* **65**, 411 (1973) [*Sov. Phys. JETP* **38**, 202 (1974)].
- [13] Z. N. C. Ha, *Phys. Rev. Lett.* **73**, 1574 (1994).
- [14] B. D. Simons, P. A. Lee, and B. L. Altshuler, *Phys. Rev. Lett.* **70**, 4122 (1993).
- [15] For a generic interaction  $A_i(k, \omega)$  has a tail  $\delta A_i(k, \omega)$  extending outside the interval (22); see Eq. (19). The corresponding contribution to the integral in Eq. (9) scales as  $k^4$  at small  $k$ . Accordingly, the presence of the tail does not affect the estimate of the height of the peak in  $A_i(k, \omega)$  and, therefore, the  $r \propto T^2$  dependence.
- [16] K. V. Samokhin, *J. Phys. Condens. Matter* **10**, L533 (1998).