Interaction Effects in Conductivity of Si Inversion Layers at Intermediate Temperatures

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We compare the temperature dependence of resistivity $\rho(T)$ of Si-metal-oxide-semiconductor field-effect transistors with the recent theory by Zala *et al.* In this comparison, the effective mass m^* and g^* factor for mobile electrons have been determined from independent measurements. An anomalous increase of ρ with temperature, which has been considered as a signature of the "metallic" state, can be described quantitatively by the interaction effects in the ballistic regime. The in-plane magnetoresistance $\rho(B_\|)$ is only qualitatively consistent with the theory; the lack of quantitative agreement indicates that the magnetoresistance is more sensitive to sample-specific effects than $\rho(T)$.

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The theory of quantum corrections due to single-particle (weak localization) and interaction effects has been very successful in describing the low-temperature electron transport in low-dimensional conductors (see, e.g., [1]). Serious quantitative discrepancies between the experimental data and the theory of interaction corrections were noticed two decades ago (see, e.g., [2–4]); this disagreement became dramatic for high-mobility Si devices [5,6]. The attempts to attribute the "anomalous metallic behavior" of Si-metal-oxide-semiconductor field-effect transistors (MOSFETs) in the low-density (dilute) regime to the temperature dependent screening [7–9] could not reconcile the disagreement between the theory and experimental data (for reviews, see [10,11]).

Recently, important progress has been made in both experiment and theory, which allows solving this long-standing problem. First, the interaction corrections to the conductivity have been calculated beyond the diffusive regime in terms of the Fermi-liquid (FL) interaction parameters [12]. This is crucial because the most pronounced increase of ρ with temperature is observed in the ballistic regime (see below). Second, FL parameters for a 2D electron system have been found from measurements of Shubnikov–de Haas (SdH) oscillations in Si-MOSFETs [13,14]. All this allows one to perform quantitative comparison of experimental data with theory.

In this Letter, we show that the most prominent feature of the metallic state in Si-MOSFETs, the strong temperature dependence of the conductivity $\sigma(T)$ at large values of $\sigma \gg e^2/h$, can be accounted for by the theory [12] over a wide range of carrier densities and temperatures. Thus, the metallic conductivity of high-mobility Si-MOSFETs, which was considered anomalous for a decade, can now be explained by the interaction effects in the ballistic regime. The interaction effects in Si-MOSFETs are strongly enhanced due to the valley degeneracy [15] and renormalization of the FL parameters at low n. The experiment, however, still deviates from the theory at very low temperatures and in strong magnetic fields $B_\| \gg k_BT/g\mu_B$, as well as at low electron densities, where sample-specific effects come into play [16].

The ac (13 Hz) transport measurements have been performed on five (100) Si-MOS samples from different wafers: Si15b (peak mobility $\mu^{\rm peak} = 4$ m^2/V s), Si2Ni (3.4 m^2/V s), Si22 (3.3 m^2/V s), Si6-14 (2.4 m^2/V s), and Si43 (1.96 m^2/V s); more detailed description of the samples can be found elsewhere [16,17].

The theory introduced recently by Zala *et al.* [12] considers backscattering of electrons at the scattering centers and at the Friedel oscillations of the density of surrounding electrons. The interference between these scattering processes gives rise to quantum corrections to the Drude conductivity $\sigma_D = e^2 n\tau/m^*$ (τ is the momentum relaxation time, m^* is the effective mass of carriers), which can be expressed (in units of $e^2/\pi\hbar$) as follows [12]:

$$\sigma(T, B_\|) - \sigma_D = \delta\sigma_C + 15\delta\sigma_T + 2[\sigma(E_Z, T) - \sigma(0, T)] + 2[\sigma(\Delta_v, T) - \sigma(0, T)] + [\sigma(E_Z + \Delta_v, T) - \sigma(0, T)] + [\sigma(E_Z - \Delta_v, T) - \sigma(0, T)] + \delta\sigma_{\rm loc}(T). \quad (1)$$

Here $\delta\sigma_C = x[1 - \frac{3}{8}f(x)] - \frac{1}{2\pi}\ln(\frac{E_F}{T})$ and $\delta\sigma_T = A(F_0^a)x[1 - \frac{3}{8}t(x, F_0^a)] - [1 - B(F_0^a)]\frac{1}{2\pi}\ln(\frac{E_F}{T})$ are the interaction contributions in the singlet and triplet channels, respectively; $\delta\sigma_{\rm loc}(T) = \frac{1}{2\pi}\ln[\tau/\tau_\varphi(T)]$ is the weak localization contribution. The terms $\sigma(Z, T) - \sigma(0, T)$ reduce the triplet contribution when the Zeeman energy ($Z = E_Z = 2\mu_B B_\|$), the valley splitting ($Z = \Delta_v$), or a combination of these factors ($Z = E_Z \pm \Delta_v$) exceed the temperature. The prefactor 15 to $\delta\sigma_T$ reflects the enhanced number of triplet components due to the valley-degenerate electron spectrum in (100) Si-MOSFETs [15].

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Because of this enhancement, the "negative" correction to the conductivity due to the triplet channel, $d\delta\sigma_T/dT < 0$, overwhelms the "positive" correction due to the singlet channel and weak localization, $d(\delta\sigma_C + \delta\sigma_{\rm loc})/dT > 0$. Equation (1) predicts the linear dependence $\sigma(T)$ in the ballistic regime $T\tau \gg 1$ and the logarithmic dependence in the diffusive regime $T\tau \ll 1$; the crossover between the two regimes occurs at $T \approx (1 + F_0^a)/(2\pi\tau)$ [12].

The terms in Eq. (1) are functions of $x = T\tau/\hbar$, Z, and F_0^a; their explicit expressions are given in Ref. [12]. The FL interaction constant $F_0^a \equiv F_0^\sigma$, which controls the renormalization of the g^* factor [$g^* = g_b/(1 + F_0^a)$, where $g_b = 2$ for Si], has been independently determined in Ref. [14]. The momentum relaxation time τ is found from the Drude resistivity $\rho_D \equiv \sigma_D^{-1}$ using the renormalized effective mass m^* determined in Ref. [14]. Earlier, there were attempts to apply the theory [12] for fitting the experimental data on $\rho(T)$ and $\rho(B)$ in p-type GaAs [18] and Si-MOSFETs [19] using a number of fitting parameters. In contrast, our approach provides a rigorous test of applicability of the theory [12] to Si-MOSFETs, as we determined the two FL constants in independent measurements.

Figure 1 illustrates the central result of this paper: the experimental dependences $\rho(T)$ can be quantitatively described by the theory of electron-electron corrections in the ballistic regime [12]. For the samples studied, the ballistic regime extends down to $T \lesssim 0.2$ K [12]. The solid lines show the $\rho(T)$ dependences calculated according to Eq. (1). Throughout the paper, we assume $\Delta_v = 0$; small values $\Delta_v \lesssim 1$ K do not affect the theoretical curves at intermediate temperatures.

In the comparison, the Drude resistivity is needed for both, calculating the magnitude of ρ and determining τ in Eq. (1). The theory [12] suggests a recipe for finding the classical ρ_D value by extrapolating the high-temperature quasilinear $\rho(T)$ dependence in the ballistic regime to $T = 0$. One needs to do an accurate nonlinear extrapolation, according to Eq. (1), to account for the contribution of the $\ln(T/E_F)$ terms and nonlinear crossover functions $t(T\tau)$ and $f(T\tau)$. Both nonlinear terms extend far into the ballistic regime. The obtained ρ_D values differ from the result of the simplified linear extrapolation by ~1 to 10% as n changes from 40×10^{11} cm^{-2} to 2×10^{11} cm^{-2}. We note that this difference is important only for finding the magnitude of ρ, whereas the slope of the $\rho(T)$ dependence in the ballistic regime is not sensitive to such small variations in ρ_D. The measured and calculated resistivities are in good agreement for all samples over broad intervals of T and n. For sample Si43, the agreement with the theory holds up to such high temperatures ($T \sim 0.3E_F$) that $\delta\rho/\rho \sim 1$ (see Fig. 2 of Ref. [21]). In this case, which is beyond the applicability of the theory [12], we still calculated the corrections to the resistivity according to $\delta\rho = -\delta\sigma\rho_D^2$. For much more disordered sample Si46, the agreement with the theory is worse: the theoretical $\rho(T)$ curves are consistent with the data only at $T < 10$ K, which correspond for this sample mostly to the diffusive regime.

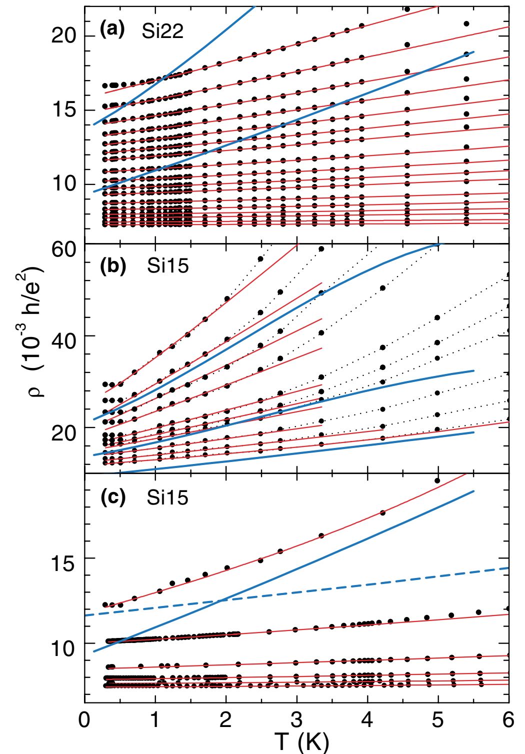

FIG. 1 (color). ρ vs T for two samples. Dots show the data; thin red solid lines correspond to Eq. (1) with $\Delta_v = 0$ and F_0^a from Ref. [14]. The densities, from top to bottom, are (in units of 10^{11} cm^{-2}): in (a) 5.7, 6.3, 6.9, 7.5, 8.1, 8.7, 9.3, 10.5, 11.7, 12.9, 14.1, 16.5, 18.9, 21.3, 23.7, 28.5, 35.7; in (b) 2.23, 2.46, 2.68, 2.90, 3.34, 3.56, 3.78, 4.33, 4.88, 5.44; and in (c) 5.44, 10.45, 15.95, 21.4, 26.9, 32.4. Thick blue solid lines reproduce calculated $\rho(T)$ dependences for sample Si-15 from Fig. 1b of Ref. [9], dashed blue line — full RPA calculations for $n = 5.6$ from Ref. [20].

For comparison, Fig. 1 also shows the $\rho(T)$ dependences, calculated for Si-MOSFETs within the model of the temperature dependent screening: (i) according to Ref. [9], taking into account the density-dependent collisional broadening, and (ii) according to the full RPA results [20].

The temperature range, in which ρ varies quasilinearly with T extends for approximately a decade up to $T \approx 0.1E_F$; it shrinks, however, towards low densities, $n \sim 1 \times 10^{11}$ cm^{-2}, and high densities, $n \sim 4 \times 10^{12}$ cm^{-2}. The linear $\rho(T)$ dependence is only a part of the overall nonlinear $\rho(T)$ dependence [11,22]; beyond this temperature range, the $\rho(T)$ data depart from the theory (see Fig. 1). Manifestly, the numerical calculations to all orders in T [9,20] do not provide a better fit than the linear T corrections [12]. At $T \to E_F$, the deviation from the theories might be caused by thermal activation of the interface localized states, which are ignored in the theory.

Weakening of the $\rho(T)$ dependence at low temperatures might be caused by a nonzero valley splitting Δ_v. Indeed, for samples Si22, Si15b, and Si6-14, the temperature of the $\rho(T)$ "saturation" (0.2–0.5 K, see Figs. 1) is of the order of valley splitting estimated from SdH measurements, $\sim$(0.6–0.8) K. However, for sample Si43, the saturation temperature is too high, (1–8) K depending on the density (see Fig. 2 of Ref. [21]), which makes this interpretation of the saturation dubious. The saturation at such high temperatures cannot be caused by electron overheating. One of the reasons for diminishing the interaction contribution might be strong (and sample-specific) intervalley scattering. A theory which takes the intervalley scattering into account is currently unavailable.

We now turn to the magnetoresistance (MR) data in the in-plane field $B_\parallel$. In contrast to the temperature dependences of ρ, the magnetoresistance agrees with the theory [12] only qualitatively. To quantify deviations from the theory, we treated F_0^a as an adjustable parameter in fitting the MR data. Figure 2 shows that the F_0^a values, found for sample Si6-14 from fitting at low ($E_Z/2T < 1$) and high ($E_Z/2T \gg 1$) fields, agree with each other within 10%; at the same time, these values differ by 30% from the values determined in SdH studies [14]. Similar situation is observed for Si43 in weak [Figs. 3(c) and 3(f)] and moderate [Figs. 3(b) and 3(e)] fields.

Fitting the weak-field MR at different temperatures provides an apparent T-dependent F_0^a [Fig. 2(a)]. On the other hand, our SdH data for the same sample do not confirm such a dependence: g^*m^* is constant within 2% over the same temperature range [14]. This discrepancy is consistent with our observation (see also Ref. [3]) that the experimental low-field dependence, $\rho(B_\parallel, T) \propto B^2/T^\alpha$ with $\alpha = 0.7$–1.2, differs from the theoretical one, Eq. (1).

The discrepancy between the theory and the MR data is much more pronounced for sample Si43 in strong fields $E_Z/2T \gg 1$, where even the sign of deviations becomes density dependent [compare Figs. 3(a) and 3(e)]. The nonuniversal behavior of the MR has been reported earlier [16,17]. It might be caused by the interaction of mobile electrons with field-dependent and sample-specific localized electron states. The low lying localized states, which are expected to be singly occupied and carry a nonzero spin, exhibit a substantial T-dependent magnetization [23]; it is natural therefore to expect the effect of localized states to be most pronounced in the MR.

The $F_0^a(n)$ values obtained from fitting the low-field MR for three samples are summarized in Fig. 4(a). The nonmonotonic density dependence of F_0^a and scattering of data for different samples indicate that the MR is more susceptible to the sample-specific effects than $\rho(T)$. For comparison and for testing our analysis, we performed an additional fit of the $\rho(T)$ data for three samples where we also treated F_0^a as an adjustable parameter. The corresponding F_0^a values obtained from $\rho(T)$ fitting are presented in Fig. 4(b). In contrast to Fig. 4(a), there is an excellent agreement between the F_0^a values extracted from SdH measurements and from fitting the $\rho(T)$ dependences. The agreement is observed over a wide density range $n = (1.5$–$40) \times 10^{11}$ cm^{-2} and confirms our conclusion that the theory [12] with FL parameters determined from SdH [14] agrees with the $\rho(T)$ data.

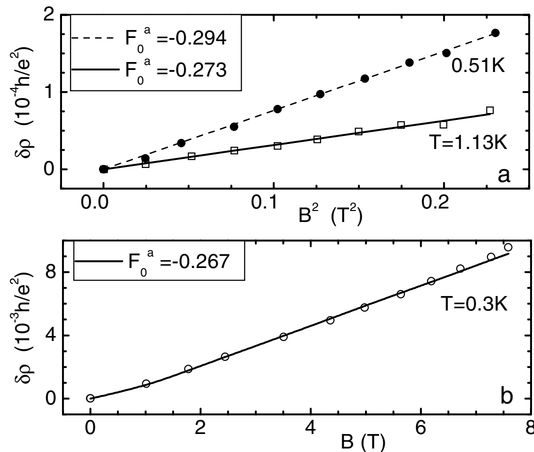

FIG. 2. Magnetoresistivity for sample Si6-14 versus $B_\parallel^2$ at low fields $E_Z/T < 1$ (a), and versus $B_\parallel$ at high fields $E_Z/T \gg 1$ (b). The electron density $n = 4.94 \times 10^{11}$ cm^{-2} is the same for both panels. Lines are the best fits with the F_0^a values shown in the panels.

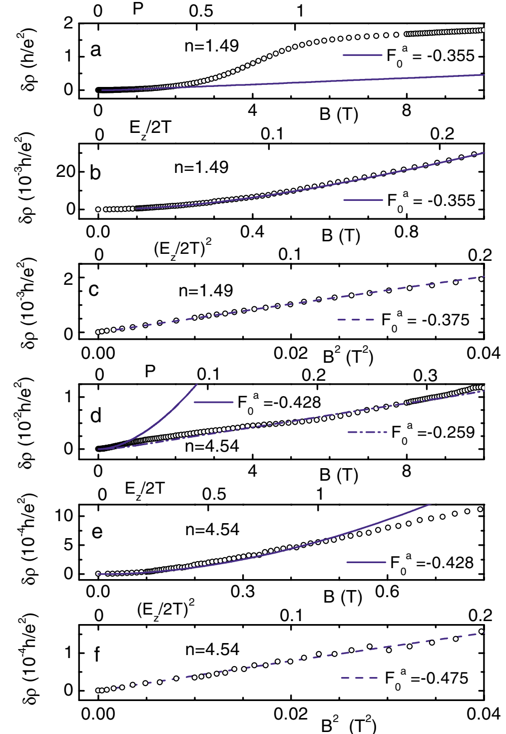

FIG. 3 (color online). Magnetoresistivity for sample Si43 vs $B_\parallel$ and $B_\parallel^2$ for two densities: $n = 1.49 \times 10^{11}$ cm^{-2} [(a),(b),(c)] and 4.54×10^{11} cm^{-2} [(d),(e),(f)]. The upper horizontal scales show $P \equiv g^*\mu_B B/2E_F$ in (a) and (d), and $E_Z/2T$ (b), (c), (e), and (f). $T = 0.31$ K.

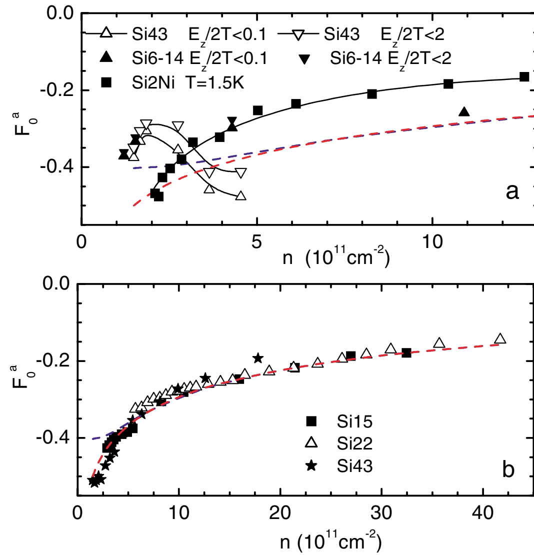

FIG. 4 (color online). Comparison of the $F_0^a(n)$ values determined from (a) fitting $\rho(B_\parallel)$ for three samples, and (b) fitting $\rho(T)$ for three samples. Dashed lines depict the upper and lower limits for F_0^a from SdH measurements [14].

In summary, we performed a rigorous experimental test of the applicability of the theory [12] to electron transport in Si inversion layers. For high-mobility samples, we found an excellent agreement between $\rho(T)$ and the theory in the ballistic regime over a wide range of temperatures and electron densities $n = (1.5\text{–}40) \times 10^{11}$ cm^{-2}; for the comparison, we used independently measured renormalized effective mass and g factor. Our experiments strongly support the theory attributing the anomalous metallic behavior of high-mobility Si-MOSFETs [6] to the interaction effects in the intermediate (ballistic) temperature regime [24]. The existing numerical RPA calculations to all orders in temperature [9,20] do not fit well the nonlinear $\rho(T)$ dependences, especially at high $T > 0.1E_F$ and over a broad density range. Sample-dependent deviations from the theory [12] have been observed in the slope $d\rho/dT$ for both the lowest temperatures and high temperatures ($T \lesssim E_F$). The deviations are more pronounced in the in-plane magnetoresistance, especially in high fields ($2\mu_B B/T > 1$). We attribute this nonuniversality to interaction of the mobile electrons with the field-affected interface localized electron states, which are ignored in the existing theories.

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