Enhanced Nonperturbative Effects in Jet Distributions

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(Received 20 December 2002; revised manuscript received 12 March 2003; published 15 September 2003)

We consider the triple differential distribution $d\Gamma/dE_J dm_J^2 d\Omega_J$ for two-jet events at center of mass energy M, smeared over the end-point region $m_J^2 \ll M^2$, $|2E_J - M| \sim \Delta$, $\Lambda_{\rm QCD} \ll \Delta \ll M$. The leading nonperturbative correction, suppressed by $\Lambda_{\rm QCD}/\Delta$, is given by the matrix element of a single operator. A similar analysis is performed for three-jet events, and the generalization to any number of jets is discussed. At order $\Lambda_{\rm QCD}/\Delta$, nonperturbative effects in four or more jet events are completely determined in terms of two matrix elements which can be measured in two- and three-jet events.

DOI: 10.1103/PhysRevLett.91.122001 PACS numbers: 12.38.–t

This paper studies the semi-inclusive two-jet distribution $d\Gamma/dE_J dm_J^2 d\Omega_J$ in high energy processes such as $Z \to q\bar{q}$, where E_J and m_J are the energy and invariant mass of one of the jets, J. No restriction is placed on the kinematics of the second jet, J'. We are interested in nonperturbative effects that are enhanced in the end-point region $|2E_J - M| \ll M$, $m_J^2 \ll M^2$. In the end-point region, the jet J is narrow, and has energy close to $M/2$. At leading order in the parton model description, the quark and antiquark each have energy $M/2$, and $m_J^2 = 0$. Gluon radiation and nonperturbative effects such as hadronization can cause E_J and m_J^2 to deviate from their leading order parton model values. The aim of this Letter is to characterize these nonperturbative effects in QCD in terms of operator matrix elements and study the relationship between nonperturbative effects in two jets, and events with three or more jets. These nonperturbative effects do not depend on the precise jet definition; for example, a suitable choice is the Sterman-Weinberg prescription [1].

In the end-point region, the decay distribution for two-jet events can be written as

$$\frac{d\Gamma}{dE_J dm_J^2 d\Omega_J} = T(M, \Omega_J) J_1(M, m_J^2/M) \times \int dk^+ J_2(M, k^+) \times S[2E_J - M - k^+ - m_J^2/(2E_J)]. \quad (1)$$

Here the hard function T and the jet functions J_i are calculable perturbatively, and S is a nonperturbative shape function. The precise form of the perturbative expansions of these functions depends on the jet definition used. In the end-point region, this distribution must be smeared over a suitable region of E_J and m_J^2 to be physically meaningful. This general form for the decay distribution has been obtained [2] previously using standard "diagrammatic" factorization methods [3]. We begin this paper by reviewing the derivation of Eq. (1) using soft collinear effective field theory (SCET).

The effective field theory SCET [4–6] is appropriate for the kinematic region of interest. SCET describes the interaction of collinear and ultrasoft (usoft) degrees of freedom with momenta scaling as $p_c = (n \cdot p, \bar{n} \cdot p, p^\perp) \sim M(\lambda^2, 1, \lambda)$ and $p_{us} \sim M(\lambda^2, \lambda^2, \lambda^2)$. The lightlike vectors n and $\bar{n}$ satisfy $n^0 = \bar{n}^0 = 1$ and $\mathbf{n} = -\bar{\mathbf{n}}$, and the perpendicular components of any four-vector V are defined by $V_\perp^\mu = V^\mu - (n \cdot V)\bar{n}^\mu/2 - (\bar{n} \cdot V)n^\mu/2$. For our analysis, $\lambda \sim \Delta/M \ll 1$. The effective theory provides a simple method for the factorization of hard, collinear, and usoft degrees of freedom at the operator level. For example, the factorization of usoft degrees of freedom arises because they can be decoupled from the collinear degrees of freedom using a simple field redefinition. SCET gives field theoretical definitions of the various ingredients in Eq. (1).

The first step in the SCET derivation of Eq. (1) is matching the full theory current j^μ onto SCET. The current in the effective theory at leading order in λ is

$$j^\mu = [\bar{\xi}_{\bar{n}} W_{\bar{n}}] \Gamma^\mu C(\mathcal{P}^\dagger, \bar{\mathcal{P}}) [W_n^\dagger \xi_n], \quad (2)$$

where $\Gamma^\mu = g_V \gamma_\perp^\mu + g_A \gamma_\perp^\mu \gamma_5$, $g_{V,A}$ are the vector and axial couplings of the quarks to the Z boson, and $C(\mathcal{P}^\dagger, \bar{\mathcal{P}})$ is the matching coefficient which is one at tree level. The field ξ_n denotes a collinear fermion in the n direction, and we have used the convention

$$\xi_n(x) = \sum_{\tilde{p}} e^{-i\tilde{p}\cdot x} \xi_{n,\tilde{p}}(x), \quad (3)$$

where $\tilde{p}$ is the label momentum which contains components of order 1 and order λ. The order λ^2 components are associated with the spacetime dependence of the fields.

The label operators $\mathcal{P}$, $\bar{\mathcal{P}}$ pick out the order one momenta of the collinear fields and $W_n(x)$ denotes a Wilson line of collinear gluons along the path in the $\bar{n}$ direction. The Wilson lines $W_{n,\bar{n}}$ are required to ensure gauge invariance of the current in the effective theory [5]. The Lagrangian of the effective theory does not contain any direct coupling of collinear particles moving in the two separate directions defined by n and $\bar{n}$ [7]; however, they can still interact with one another via the emission of usoft gluons. The coupling of collinear and usoft degrees of freedom in the Lagrangian can be eliminated via the BPS field redefinition [6]:

$$\xi_n \to Y_n^\dagger \xi_n, \qquad A_n \to Y_n^\dagger A_n Y_n, \tag{4}$$

where

$$Y_n(z) = \exp\left[ig \int ds\, n \cdot A_{us}(ns + z) \right] \tag{5}$$

denotes a path-ordered Wilson line of usoft gluons in the n direction from $s = 0$ to $s = \infty$, since we are dealing with final state collinear fields. (For annihilation, Y_n is from $s = -\infty$ to $s = 0$.) It is well known from the diagrammatic approach to factorization that the usoft degrees of freedom couple to collinear degrees of freedom via a Wilson line [8].

To derive the factorized form in Eq. (1), we start from the general expression

$$\frac{d\Gamma}{d^4r} = \frac{1}{2M} \sum_{JX} (2\pi)^4 \delta^4(M - p_J - p_X)\delta^4(r - p_J) \times |\epsilon_\mu \langle J, X | j^\mu |0\rangle|^2, \tag{6}$$

where the sum includes the phase space integrations over all the particles in the final state, j^μ denotes the current producing the $q\bar{q}$ pair, and ϵ_μ is the polarization of the decaying particle. The final state hadrons have been divided into those in the quark jet J, with total momentum r and the remaining hadrons (including the antiquark jet) which form X.

Using these definitions, the matrix element of the current in Eq. (6) becomes

$$\langle JX | j^\mu |0\rangle = \sum_{\tilde{q}_1 \tilde{q}_2} C(q_1^+, q_2^-) \langle J | T\{[\bar{\xi}_{\bar{n}} W_{\bar{n}}]^a_{\tilde{q}_1 \alpha}\} |0\rangle (\Gamma^\mu)_{\alpha\beta} \times \langle X | T\{[Y_{\bar{n}} Y_n^\dagger]_a{}^b [W_n^\dagger \xi_n]_{\tilde{q}_2 \beta b}\} |0\rangle, \tag{7}$$

where α, β (a, b) denote spin (color) indices, and the subscript $[\cdot]_{\tilde{q}}$ denotes the total label momentum of the operator inside. In Eq. (7), and in the subsequent equations, we will use the notation $n \cdot p \equiv p^+$, $\bar{n} \cdot p \equiv p^-$ for any four-momentum p.

The operators inside the matrix elements in Eq. (7) are time ordered. The operator $T\{Y_n\}$ is a Wilson line where the operator time ordering agrees with the path ordering of matrix multiplication, $T\{Y_n\} = Y_n$. The operator $T\{Y_n^\dagger\}$ has operator time ordering in the opposite order as the path ordering of matrix multiplication. The two orderings can be made to coincide by taking the transpose of all the matrix indices, so that $T\{Y_{na}^{\dagger b}\}$ is the Wilson line in the $\bar{\mathbf{3}}$ representation, i.e., $T\{Y_{na}^{\dagger b}\} = \bar{Y}_{na}^b$. Similar results hold for the anti-time-ordered products.

Using Eq. (7), the differential distribution becomes

$$\frac{d\Gamma}{d^4r} = \frac{1}{2M} \int d^4s (2\pi)^4\, \delta^4(M - r - s) \times \epsilon_\mu \epsilon^{\nu *} (\Gamma^\mu)_{\alpha\beta} (\bar{\Gamma}_\nu)_{\rho\sigma} |C(r^+, s^-)|^2 \times \left[\sum_X \delta^4(s - p_X) |\langle X | T\{[Y_{\bar{n}} Y_n^\dagger][W_n^\dagger \xi_n]_{\tilde{s}} |0\rangle\}|^2 \right]_{\beta\rho a}^{\;\;c} \times \left[\sum_J \delta^4(r - p_J) |\langle J | T\{[\bar{\xi}_{\bar{n}} W_{\bar{n}}]_{\tilde{r}} |0\rangle\}|^2 \right]_{\sigma\alpha c}^{\;\;a}. \tag{8}$$

The two terms in square bracket can be simplified, following Ref. [6]. For the second term, we have

$$\sum_J \int d^4z\, e^{i(r-p_J)z} [|\langle J | T\{[\bar{\xi}_{\bar{n}} W_{\bar{n}}]_{\tilde{r}} |0\rangle\}|^2]_{\sigma\alpha c}{}^a = \left(\frac{\not{\bar{n}}}{2}\right)_{\sigma\alpha} \delta_c^a J_{1\tilde{r}}(r^-), \tag{9}$$

where the last equality defines $J_{1\tilde{r}}(r^-)$.

Combining the first term with the δ_c^a in Eq. (9) gives

$$\sum_X \int d^4z\, e^{i(s-p_X)z} [|\langle X | T\{[Y_{\bar{n}} Y_n^\dagger][W_n^\dagger \xi_n]_{\tilde{s}}\} |0\rangle|^2]_{\beta\rho a}{}^a = 3 \int dk^+ J_{2\,\tilde{s}}(k^+) S(-s^+ - k^+) \left(\frac{\not{n}}{2}\right)_{\beta\rho}. \tag{10}$$

The functions J_i depend on the jet definition. The shape function $S(k^+)$ is defined by [2]

$$S(k) = \frac{1}{3} \int \frac{du}{2\pi} e^{iku} \langle 0 | \mathrm{Tr}\, \bar{T}\{[Y_n Y_{\bar{n}}^\dagger](nu)\} T\{[Y_{\bar{n}} Y_n^\dagger](0)\} |0\rangle = \frac{1}{3} \langle 0 | \bar{Y}_n^{\dagger c}{}_a Y_{\bar{n}c}^{\dagger\;\, b} \delta(k - in \cdot \partial) Y_{\bar{n}b}{}^e \bar{Y}_{n\;\,e}^a |0\rangle. \tag{11}$$

The Wilson loop, shown in Fig. 1, is regulated to preserve reparametrization invariance [9], and is a function of $\bar{n} \cdot nu$. $S(k)$ is normalized so that $\int_{-\infty}^{\infty} dk S(k) = 1$.

The jet function in Eq. (9) depends on $\tilde{r}$, which has both $+$ and $\perp$ components. One can align $\bar{n}$ with the jet axis, i.e., choose $\mathbf{r}_\perp = \mathbf{0}$. With this choice, we write

$$J_i(r^+, r^-) \equiv J_{ir^+, \mathbf{0}_\perp}(r^-). \tag{12}$$

Combining Eqs. (8)–(12) and using $d^4r = (M/4)dE_J dm_J^2 d\Omega_J$ gives Eq. (1) with the hard function defined by

$$T(M, \Omega_J) = \frac{3}{512\pi^4} |C(M, M)|^2 \epsilon_\mu \epsilon^{\nu *} \mathrm{Tr}[\Gamma^\mu \not{n} \bar{\Gamma}_\nu \not{\bar{n}}]. \tag{13}$$

We can simplify the discussion by restricting ourselves to the leading order in perturbation theory, where

$$C(M, M) = 1, \qquad J_i(M, k^+) = 2\pi\delta(k^+). \tag{14}$$

Perturbative corrections to this result can be included by

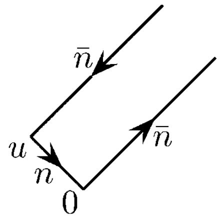

FIG. 1. Pictorial representation of the Wilson lines occurring in the shape function $S(k)$ defined in Eq. (11).

calculating the two-jet functions J_i and the Wilson coefficient T to higher orders in perturbation theory. For Sterman-Weinberg jets [1], the perturbative corrections to the J_i contain logarithms of the cone angle δ and the minimum energy cut E_c. We take E_c/M and δ to be $\mathcal{O}(\lambda)$. Therefore S, which involves only usoft degrees of freedom, does not depend on δ and E_c.

Integrating Eq. (1) over m_J^2 using Eq. (14) gives

$$\frac{d\Gamma}{dE_J d\Omega_J} = 2\frac{d\Gamma^{(0)}}{d\Omega_J} S(2E_J - M) + \cdots, \qquad (15)$$

where $d\Gamma^{(0)}/d\Omega_J$ is the parton model differential decay rate and the ellipsis denotes subdominant perturbative and power corrections. To this order, the shape of the jet energy spectrum is determined entirely by the nonperturbative shape function. Nonperturbative usoft radiation will reduce the energy associated with the jets. Thus, $S(k) = 0$ for $k > 0$.

To proceed further, we consider observables in which we smear the jet energy distribution over a region $|2E_J - M| \sim \Delta \ll M$, with $\Delta \gg \Lambda_{\rm QCD}$ [10]. In this region there are enhanced nonperturbative corrections suppressed only by powers of $\Lambda_{\rm QCD}/\Delta$. Note that for the total rate the nonperturbative corrections are much smaller since they are suppressed by powers of $\Lambda_{\rm QCD}/M$. To be more precise, consider a smooth window function $w_\Delta(E_J)$ which when integrated over E_J is normalized to unity and has support concentrated in the region $|E_J - M/2| < \Delta$. Then,

$$\left(\frac{d\Gamma}{d\Omega_J}\right)_\Delta = \int dE_J\, w_\Delta(E_J) \frac{d\Gamma}{dE_J d\Omega_J}, \qquad (16)$$

is such an observable. For it, we can expand the shape function in a power series, given by

$$S(k) = \delta(k) - \delta'(k)\langle 0|O_1|0\rangle + \tfrac{1}{2}\delta''(k)\langle 0|O_2|0\rangle + \cdots, \qquad (17)$$

where

$$O_m = \tfrac{1}{3}{\rm Tr}[Y_{\bar n}^\dagger (in\cdot D)^m Y_{\bar n}]. \qquad (18)$$

We can rewrite O_1 as

$$O_1 = O_1^q = \frac{1}{3}{\rm Tr}\int_0^\infty ds\, \mathcal{G}_{n\bar n}(\bar n s), \qquad (19)$$

where

$$\mathcal{G}_{n_1 n_2}(\bar n s) = Y_{\bar n}(\bar n s, 0)^\dagger n_1^\mu n_2^\nu G_{\mu\nu} Y_{\bar n}(\bar n s, 0). \qquad (20)$$

Here, $G^{\mu\nu}$ is the gluon field strength (with a factor of the strong coupling absorbed into it) and $Y_{\bar n}(\bar n s, 0)$ denotes a usoft Wilson line along the $\bar n$ direction from 0 to $\bar n s$. Note that the Fourier transform of $S(k)$ is a function only of the combination $(\bar n \cdot n)u$, so the matrix element of O_m must have the form

$$\langle 0|O_m|0\rangle = (\bar n\cdot n)^m \mathcal{A}_m^q, \qquad (21)$$

where $\mathcal{A}_m^q$ is a number of order $\Lambda_{\rm QCD}^m$. If the observed jet is the antiquark jet, then $in\cdot\partial$ in Eq. (11) is replaced by $i\bar n\cdot\partial$, and the leading correction is given by the vacuum matrix element of the antiquark operator

$$O_m^{\bar q} = \tfrac{1}{3}{\rm Tr}[\bar Y_n^\dagger (i\bar n\cdot D)^m \bar Y_n], \qquad (22)$$

which is $(\bar n\cdot n)\mathcal{A}_m^{\bar q}$. By charge conjugation, $\mathcal{A}_m^{\bar q} = \mathcal{A}_m^q$.

The nonperturbative corrections to the differential decay rate in Eq. (15) are singular at $E_J = M/2$. This is also the case for the perturbative corrections. However, for the smeared observable in Eq. (16), the expansion of the shape function gives

$$\left(\frac{d\Gamma}{d\Omega_J}\right)_\Delta = \left(\frac{d\Gamma^{(0)}}{d\Omega_J}\right)[w_\Delta(M/2) + w'_\Delta(M/2)\mathcal{A}_1^q + \cdots]. \qquad (23)$$

Since the matrix elements of the operators scale as $\mathcal{A}_m^q \sim \Lambda_{\rm QCD}^m$ and the mth derivative of the window function (evaluated at $E_J = M/2$) scales as $1/\Delta^{m+1}$, the square brackets on the right-hand side of Eq. (23) contain an expansion in powers of $\Lambda_{\rm QCD}/\Delta$.

A similar calculation also applies for events with more than two jets in the final state. For example, three-jet events contain an energetic gluon radiated off one of the quarks at a large angle. The subscripts 1, 2, and 3 refer to the quark, gluon, and antiquark jets, respectively. One of the jets is unobserved, and is summed over. Consider the case where the antiquark jet (jet 3) is unobserved, and the quark and gluon jet are observed. The lightlike vectors $n_{1,2}$ are defined by $n_1 = (1, \mathbf{n}_1)$, $n_2 = (1, \mathbf{n}_2)$, where $\mathbf{n}_{1,2}$ are unit vectors in the direction of the two observed jets. One can then construct the third vector,

$$n_3^0 = 1, \qquad \mathbf{n}_3 = -\frac{E_1\mathbf{n}_1 + E_2\mathbf{n}_2}{|E_1\mathbf{n}_1 + E_2\mathbf{n}_2|}, \qquad (24)$$

using only information from the two observed jets. A similar argument holds if the quark jet, or the gluon jet, is the unobserved jet.

Let i and j denote the two observed jets, and r denote the unobserved jet, where (i, j, r) is some permutation of $(1, 2, 3)$. We find at tree level that

$$\frac{d\Gamma_{ij}}{dE_i d\Omega_i dE_j d\Omega_j} = \frac{d\Gamma_{ij}^{(0)}}{dE_i d\Omega_i d\Omega_j} n_r \cdot n_j \times S_{ij}(n_r \cdot n_i E_i + n_r \cdot n_j E_j - M), \tag{25}$$

for the differential decay distribution for the observed jets i and j in terms of a nonperturbative shape function S_{ij}.

In the three-jet case, the current in SCET contains the gluon field A_{n_2}. Factoring the usoft degrees of freedom results in the shape function

$$S_{ij}(k) = \frac{1}{4}\int \frac{du}{2\pi} e^{iku} \operatorname{Tr}\langle 0|\bar{T}\{[Y_{n_3} Y_{n_2}^\dagger T^A Y_{n_2} Y_{n_1}^\dagger](un_r)\} \times T\{[Y_{n_1} Y_{n_2}^\dagger T^A Y_{n_2} Y_{n_3}^\dagger](0)\}|0\rangle. \tag{26}$$

Thus, a different shape function determines the usoft physics in three-jet events, and in general a new nonperturbative function is required for each additional jet. This is not surprising since the color structure is very different for events with different numbers of jets. The time-ordered product $T\{Y_{na}^{\dagger}{}^{b} Y_{nc}{}^{d}\}$ is equal to $(1/3)\delta_a^d \delta_c^b + 2\mathcal{Y}_{nAB}(T^A)_c^b(T^B)_a^d$, where $\mathcal{Y}_n$ is the adjoint Wilson line from 0 to ∞ in the n direction.

In the kinematic region $|M - E_i n_r \cdot n_i - E_j n_r \cdot n_j| \sim \Delta$, an expansion of $S_{ij}(k)$ analogous to the one in Eq. (17) can be performed, and the first correction to the smeared rate is determined by the operator

$$O_1^{(3)} = \tfrac{1}{3}\operatorname{Tr} Y_{n_1}^\dagger (in_r \cdot D) Y_{n_1} + \tfrac{1}{3}\operatorname{Tr} \bar{Y}_{n_3}^\dagger (in_r \cdot D)\bar{Y}_{n_3} + \tfrac{1}{8}\operatorname{Tr} \mathcal{Y}_{n_2}^\dagger (in_r \cdot D)\mathcal{Y}_{n_2}. \tag{27}$$

The matrix element of this operator is given by

$$\langle 0|O_1^{(3)}|0\rangle = n_r \cdot (n_1 + n_3)\,\mathcal{A}_1^q + n_r \cdot n_2\,\mathcal{A}_1^g. \tag{28}$$

where $\mathcal{A}_1^q$ is the same number that occurs in two-jet events and is defined by Eq. (21), and A_1^g is a new number of order $\Lambda_{\rm QCD}$.

In three-jet events with an unobserved gluon jet, $n_r = n_2$ and the vacuum matrix element of $O_1^{(3)}$ is $n_2 \cdot (n_1 + n_3)A_1^q$, and is completely determined by the two-jet case. If the unobserved jet is the quark jet, then $n_r = n_1$, and the vacuum matrix element of $O_1^{(3)}$ is $(n_1 \cdot n_2)A_1^g + (n_1 \cdot n_3)A_1^q$ and, if the unobserved jet is the antiquark jet, the vacuum matrix element of $O_1^{(3)}$ is $(n_3 \cdot n_2)A_1^g + (n_1 \cdot n_3)A_1^q$. If one cannot distinguish quark and gluon jets, then the three-jet distributions are given by averaging over the unobserved jet being the quark, gluon, or antiquark jet.

One can repeat the above analysis for events with four or more jets. In the end-point regions, the generalized shape functions can be expanded as in Eq. (17), and the leading nonperturbative corrections are expressed in terms of $\mathcal{A}_1^q$ and $\mathcal{A}_1^g$ which characterize two- and three-jet events. An important feature is that, even though the shape functions for events with different numbers of jets are very different, at order $\Delta/\Lambda_{\rm QCD}$ no new nonperturbative quantities enter for four or more jets *after smearing* by an amount $M \gg \Delta \gg \Lambda_{\rm QCD}$ over the end-point region. The reason is that the only Wilson lines that can arise from hard quark antiquark and gluon radiation are in the **3**, $\bar{\mathbf{3}}$, and **8** representations, and these already occur in the two- and three-jet cases. One can also predict the distributions for two jets plus a hard photon in terms of $\mathcal{A}_1^q$ measured in two-jet events, since the adjoint Wilson line from gluon emission does not enter.

$\mathcal{A}_1^q$ and $\mathcal{A}_1^g$ can be determined by fitting to the experimental data on two- and three-jet events. They can also be computed numerically by light-cone lattice gauge theory methods [11], since they involve only Wilson lines along lightlike directions.

In this Letter, we have focused on the nonperturbative effects. For comparison with experiment, it is important to include the perturbative corrections. At order α_s, they can be obtained similar to the computation in Ref. [1].

This work was supported in part by the Department of Energy under Grants No. DOE-FG03-97ER40546 and No. DOE-FG03-92ER40701. We would like to thank S. Ellis, G. Korchemsky, G. Sterman, and I. W. Stewart for helpful discussions.

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