Specific Heat of Classical Disordered Elastic Systems

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We study the thermodynamics of disordered elastic systems, applied to vortex lattices in the Bragg glass phase. Using the replica variational method we compute the specific heat of pinned vortons in the classical limit. We find that the contribution of disorder is positive, linear at low temperature, and exhibits a maximum. It is found to be important compared to other contributions, e.g., core electrons, mean field, and nonlinear elasticity that we evaluate. The contribution of droplets is subdominant at weak disorder in $d = 3$.

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Understanding the temperature dependence of the specific heat in glasses remains puzzling, e.g., the linear low T behavior observed in structural glasses [1] and spin glasses [2]. In some systems the crossover temperature from quantum to classical behavior may be quite low. The phenomenological two levels system model [3] yields a linear behavior *both* in the classical and quantum regime. Although the classical problem appears simpler there are only few, and mostly mean field, solvable models of glassy systems where one can actually compute the specific heat [2,4]. Ordered systems with continous symmetry admit spin wave type excitations which yield a T-independent specific heat $C_v = C_{\rm eq}$ from the equipartition of the energy. Nonlinearities such as quenched disorder will cause a deviation which is interesting to characterize and compare with the linear contribution from the two level system arguments.

A class of glassy systems recently much studied is disordered elastic systems, ranging from vortex lattices [5–8], electron crystals [9], charge and spin density waves [10] to disordered liquid crystals [11]. In all of these systems the competition between disorder and elasticity leads to pinning and glassy behavior. Specific heat measurements in density waves gapped systems showed intermediate linear and sublinear regimes with nonequilibration effects [12,13]. In superconductors in a field, the H and T dependence of C_v relates both to the symmetry of the order parameter and to the thermodynamics of the vortex lattice. If the contribution of the normal electrons in the vortex cores dominates, the standard expectation is that C_v is linear in T with a linear in H dependence for s wave [14] and $H^{1/2}$ for d wave [15]. A specific heat linear in temperature has indeed been measured in various materials. A $H^{1/2}$ dependence has been observed and argued for d-wave superconductivity in YbaCuO [16–18]. However the nonlinear H dependence observed in other, *a priori* non-d-wave materials, is a well known puzzle, as discussed in [19,20]. On the other hand, the contribution of the phonons of the vortex lattice VL, the so-called "vortons," seems to be within experimental resolution [21], and may lead, within the full temperature range below VL melting, to more complex behaviors. These were analyzed in the absence of disorder, assuming a dissipative quantum dynamics with friction η arising from interactions between vortons and vortex core electrons [22,23]. It yields again $C_v \propto \eta T$ with different H dependence but only for $T < T_v^D$, the vorton Debye-like temperature which is poorly known (estimates in YBaCuO range from well below 1 K up to 10 K in the superclean limit [22]). Above T_v^D one recovers the equipartition value to which the specific heat anomaly was compared at melting [24–27]. These analyses, however, neglect disorder and other nonlinearities which for $T > T_v^D$ can be treated classically.

In this Letter, we compute the specific heat of an elastic system in the presence of pinning disorder in the classical regime. We show that disorder produces a substantial rise above equipartition, linear at low temperature and exhibiting a maximum at a characteristic depinning temperature. We show that in $d > 2$ the contribution from the two-well droplet arguments is subdominant at weak disorder. We find that the disorder contribution is quite sizable compared to other contributions, e.g., of the nonlinear elasticity that we also evaluate. These results hold for a periodic object, i.e., a Bragg glass, or for interfaces with continuous degrees of freedom. In a companion paper [28] an analysis of the quantum regime revealed, in the absence of dissipation, a $C_v \sim T^3$ behavior.

An elastic system, such as the vortex lattice (VL) with external field aligned with z axis, is described by a N-component vector displacement field $u_\alpha(R_i, z)$ ($N = 2$ for the VL in $d = 3$). The equilibrium positions R_i form a perfect N-dimensional lattice of spacing a. Interactions result in an elastic energy $H_{\rm el}$ associated to the phonons of the vortex lattices $H_{\rm el}[u] = \frac{1}{2}\int_q u_\alpha(q)\Phi_{\alpha\beta}(q)u_\beta(-q)$. Here $\int_q \equiv \int_{BZ}[d^2q_\perp/(2\pi)^2]\int_{-\pi/s}^{\pi/s}[dq_z/(2\pi)]$ denotes integration on the first Brillouin zone, $q = (q_\perp, q_z)$ a d dim-vector, and s is the distance between layers. For the triangular VL:

$$\Phi(q) = (c_{66}q_\perp^2 + c_{44}q_z^2)P^T(q_\perp) + (c_{11}q_\perp^2 + c_{44}q_z^2)P^L(q_\perp), \quad (1)$$

with $P^L_{\alpha,\beta}(k) = k_\alpha k_\beta/k^2$ and $P^T = \mathbb{I} - P^L$. The dispersion of elastic moduli is implicit whenever needed. Impurity disorder is modeled by a short range Gaussian random potential with in plane correlator $\Delta(r) = \delta s \epsilon_0^2 e^{-r^2/(4r_f^2)}$ interacting with the local vortex density. Here $2r_f = \xi$ the superconducting coherence length, $\epsilon_0 = (\Phi_0/4\pi\lambda)^2$ is the vortex energy scale per unit length along z, and δ is a (small) dimensionless disorder parameter [29]. The equilibrium Bragg glass phase (absence of dislocations, $a/R_a \ll 1$, R_a being the translational correlation length) is described by the replicated partition function $\overline{Z}^n = \mathrm{Tr} e^{-\beta H_{\rm eff}}$, $\beta = 1/T$, and $\overline{...}$ denotes disorder average. After standard manipulations [30] the replicated Hamiltonian becomes $H_{\rm eff}[u] = \sum_a H_{\rm el}[u^a] + H_{\rm dis}[u]$ with:

$$H_{\rm dis} = -\frac{\beta}{2}\int d^2r dz \sum_{ab} R(u_a(r,z) - u_b(r,z)). \quad (2)$$

Here $R(u) = \rho_0^2 \sum_K \Delta_K \cos(K \cdot u)$ in terms of ρ_0 the average vortex density and the disorder harmonics $\Delta_K = \int d^N u e^{iK\cdot u}\Delta(u)$ at the reciprocal lattice vectors. More generally, an elastic manifold (such as a directed polymer $d = 1$) in a N-dimensional embedding space in the presence of a random potential $W(u, x)$ is described by a similar model with $\overline{W(u,x)W(u',x)} = \delta^{(d)}(x - x') \times R(u - u')$, $R(u) = -NV(u^2/N)$. We compute the specific heat *per unit volume* $C_v(T) = -(T/\Omega)(\partial^2 \overline{F}/\partial T^2)$ where $\overline{F}$ is the free energy, $\Omega = L^d$ the volume. We present the method using an isotropic disorder and elasticity tensor $\Phi_{\alpha\beta}(q) = cq^2\delta_{\alpha\beta}$, and generalize to the vortex lattice later.

To obtain the low T behavior, a first approach would be to assume a *single minimum* of the energy $H[u]$ and expand around it. One then finds:

$$C_v(T) = C_{\rm eq} + AT + O(T^2). \quad (3)$$

The exact expression for the linear term $A = A_{1\rm min}$, given in [31], involves cubic and quartic anharmonicity in $H[u]$ in a given disorder realization. Disorder averaging is only easy to perform perturbatively in disorder, yielding:

$$A = A_{1\rm min,pert} = -\frac{1}{6}J_1^3(\nabla_u^2)^3 R(u)|_{u=0}, \quad (4)$$

with $J_1 = \int_q 1/(cq^2)$ [for the manifold problem it gives $A = 4J_1^3(N^2 + 6N + 8)V'''(0)/(3N)$]. Although such a single minimum low T expansion is useful for pure systems, such as the Sine-Gordon model, for disordered systems more than one minimum typically exists beyond the Larkin length R_c. The resulting contribution to the specific heat can then be estimated combining the droplet picture with the two levels argument [3,32]. At each length scale l each of the subsystems $i \in (L/l)^d$ may have a low lying secondary minimum (droplet) at excitation energy E_i independently distributed with probability $P(E)dE = (dE/E_c)(R_c/l)^\theta \mathcal{F}(ER_c^\theta/E_c l^\theta)$ where E_c is the typical pinning energy $E_c = cr_f^2 R_c^{d-2}$, θ the free energy exponent. Approximating the contribution from scale l to the specific heat as:

$$C_l = L^{-d}\sum_{i=1}^{(L/l)^d}\left(\frac{E_i}{T}\right)^2 \frac{e^{-\beta E_i}}{(1+e^{-\beta E_i})^2} \approx \frac{\pi^2}{6}\frac{T\mathcal{F}(0)}{E_c l^d}\left(\frac{R_c}{l}\right)^\theta,$$

treating as independent two level systems, $C_v = \int_{l>R_c}(dl/l)C_l$ is dominated by the smallest scales, yielding:

$$A_{\rm drop} \approx \frac{\pi^2 \mathcal{F}(0)}{6(d+\theta)}E_c^{-1}R_c^{-d}. \quad (5)$$

While the two wells-droplet argument estimates as $A_{\rm drop}$ the contribution only of scales larger than R_c, one can only hope to use $A_{1\rm min}$ to estimate the contributions of scales smaller than R_c. It is thus instructive to compare them. The perturbative expression (4) is infrared divergent for $d \le 2$ as thermal fluctuations diverge. If one restricts by hand the integral in (4) to $q > 1/R_c$ one finds a contribution of the same order as the droplet one $A_{\rm drop}$. In $d > 2$ the integral is instead controlled by small scales and the droplet contribution is then *subdominant*. The two levels droplet model of [3] can be improved by including anharmonicity in each well, for identical wells it simply adds $A = A_{1\rm min} + A_{\rm drop}$.

The variational method [30,33] extends these phenomenological considerations into a first principle quantitative calculation in which the Larkin length naturally appears in a self-consistent way. We introduce a Gaussian trial Hamiltonian $H_0 = \frac{1}{2}\int_q G^{-1}_{\alpha\beta,ab}(q)u^a_\alpha(q)u^b_\beta(-q)$ which minimizes the variational free energy $F_{\rm var} = F_0 + \langle H_{\rm eff} - H_0\rangle_{H_0}$, where F_0 denotes the free energy calculated with H_0. The specific heat $C_v = \lim_{n\to 0} -\partial_T \partial_\beta(\overline{Z}^n/n\Omega)$ can be reexpressed as

$$C_v = \partial_T \lim_{n\to 0}\frac{1}{n\Omega}\left(\left\langle\sum_a H_{\rm el}[u_a]\right\rangle_{H_{\rm eff}} + 2\langle H_{\rm dis}[u]\rangle_{H_{\rm eff}}\right). \quad (6)$$

Note the factor of 2 due to the β dependence of the disorder term. Here we evaluate these averages using the variational Hamiltonian H_0 instead of the exact one $H_{\rm eff}$. Thanks to the variational equations this is equivalent to $C_v(T) = -(T/\Omega)\partial^2 F_{\rm var}/\partial T^2$.

We applied this variational approach to some pure models and checked that it is quite accurate [31]. At low T it exactly matches the expansion around the minimum $A_{\rm var} = A_{1\rm min}$ and we checked, for the Sine-Gordon model in $d = 2$, that it is identical to the low T expansion of the exact result [34].

In the disordered case the solution of the variational equations requires replica symmetry breaking (RSB). One denotes $\tilde{G}(q) = G_{aa}(q)$ and parametrizes $G_{a\neq b}(q)$ by $G(q, u)$, where $0 < u < 1$ and similarly for $\mathcal{B}_{ab}(x = 0) = \mathcal{B}_{ab} = \langle[u^a_\alpha(x) - u^b_\alpha(x)]^2\rangle/N$ with $\tilde{\mathcal{B}} = 0$ and $\mathcal{B}(u)$. For $d > 2$ one finds [30,33] a continuous RSB with a breakpoint u_c and

$$\mathcal{B}(u > u_c) = \mathcal{B} = 2\gamma T J_1(\Sigma), \tag{7}$$

$$1 = -4\gamma \hat{V}''(\mathcal{B}) J_2(\Sigma), \tag{8}$$

$$J_n(z) = \int_q \frac{1}{(cq^2 + z)^n}, \quad \Sigma = cR_c^{-2}, \tag{9}$$

where $\gamma = 1$, $\hat{V}(\mathcal{B}) = -\frac{1}{N}\rho_0^2 \sum_K \Delta_K \exp(-\mathcal{B}K^2/2)$, and R_c is the Larkin length [35]. Equation (8) is the so-called marginality condition (MC) which also holds for the one step solution in $d = 2$. Starting from the expression:

$$\frac{1}{N\Omega}\overline{\langle H\rangle} = \int_q \frac{1}{2} q^2 \tilde{G}(q) + \frac{1}{T}\int_0^1 du[\hat{V}(0) - \hat{V}(\mathcal{B}(u))], \tag{10}$$

which, as $B(w) = \mathcal{B}(u)$ (setting $w = u/T$) and $w_c = u_c/T$ [36], turns out to depend implicitly on T only through Σ and $\mathcal{B}$, and using (6)–(9), one obtains for the specific heat

$$C_v(T) = C_{\rm eq} + \frac{N}{T^2} F(\mathcal{B}),$$
$$F(\mathcal{B}) = \hat{V}(\mathcal{B}) - \hat{V}(0) - \mathcal{B}\hat{V}'(\mathcal{B}) + \frac{1}{2}\mathcal{B}^2 \hat{V}''(\mathcal{B}), \tag{11}$$

where $C_{\rm eq} = \frac{1}{2}\mathcal{N}/\Omega = \frac{N}{2}\int_q$ is the equipartition value, i.e., half the total number of modes per unit volume [37]. Equation (11) is valid for any T, for periodic objects (Bragg glass) as well as manifolds, and independently of the replica structure of the solution (if broken, provided MC holds). One thus finds that disorder increases the specific heat which has now a maximum and decreases back to equipartition $C_{\rm eq}$ at high T. Expanding (11) at low T one finds again (3) with an amplitude $A_{\rm var} = \frac{8}{3!} N \hat{V}'''(0) J_1(\Sigma_{T=0})^3$. For weak disorder $R_c > a$, dispersionless elasticity and $r_f < a$, one finds from (7)–(9) $A_{\rm var} \approx \tilde{A}/(c r_f^2 R_c a^3)$, $\tilde{A} = [\pi N(N + 4)]/96$ indeed larger by a factor $(R_c/a)^3$ than (5). It also confirms the above discussion: thanks to RSB (i.e., $\Sigma \neq 0$), the problems (e.g., in $d = 2$) of single minimum (i.e., replica symmetric) perturbation theory are cured, the Larkin length being the natural scale. A plot of $C_v(T)$ is shown on Fig. 1

The generalization to the vortex lattice using (1) and $N = 2$ is straightforward. Equation (11) still holds, but in the formulas (7) and (8) which determine $\mathcal{B}$ and $\Sigma = c_{66} R_c^{-2}$ one sets $\gamma = \frac{1}{2}$ and replaces $J_n(\Sigma) = \int_q (c_{66} q_\perp^2 + c_{44} q_z^2 + \Sigma)^{-n}$ (we neglect compression modes, i.e., $c_{66}/c_{11} \ll 1$).

Taking dispersion in c_{44} and anisotropy into account yields several regimes analyzed in [31], and for simplicity we present here only results for the weak disorder regime $R_c > a$. The fluctuation of a vortex position is then measured from $\mathcal{B} = T J_1(\Sigma) \approx T J_1(0) = c_L^2 a^2 T/T_m$, where T_m is the melting temperature and $c_L \approx 0.1$–0.2 is the Lindemann number. Inserting in (11) this yields $C_v(T)$ which has a maximum for $T = T^*$, i.e., $\mathcal{B} = \mathcal{B}^*$ determined by solving $4F(\mathcal{B}^*) = (\mathcal{B}^*)^3 \hat{V}'''(\mathcal{B}^*)$. These fluctuations are estimated as $J_1(\Sigma) \approx 1/(4a\sqrt{c_{66}c_{44}})$ where here and below we denote $c_{44} = c_{44}(q_\perp = \pi/a)$ [below the dimensional crossover field, $c_{66}/(c_{44}a^2) \ll 1/s^2$, the integral over q_z can be extended up to infinity and the integral is dominated by $q_\perp = \pi/a$].

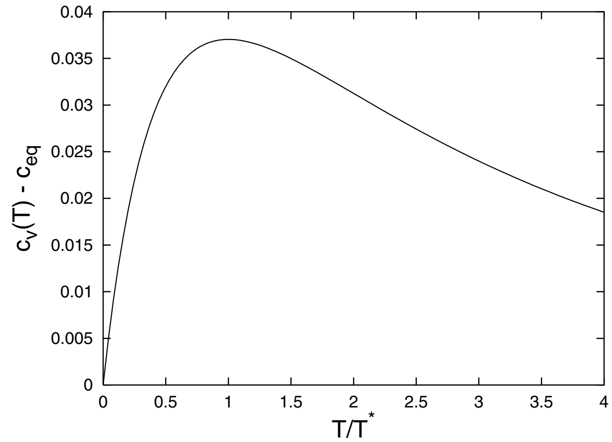

FIG. 1. Specific heat $C_v(T) - C_{\rm eq}$ in arbitrary units as a function of T/T^*.

We can now discuss in detail the behavior of $C_v(T)$ for $r_f < a$. In this limit $\hat{V}(\mathcal{B})$ takes the dependence $\hat{V}(\mathcal{B}) \approx -D/(2r_f^2 + \mathcal{B})$, with $D = \delta r_f^2 s \epsilon_0^2/a^2$, for $\mathcal{B} \ll a^2$ which holds up to melting. One finds:

$$C_v(T) = C_{\rm eq} + AT/[1 + T/(2T^*)]^3, \tag{12}$$

$$A \approx \frac{\delta}{128} c_L^2 \left(\frac{\epsilon_0^2}{c_{66}c_{44}a^4}\right) \frac{sa^2}{r_f^6 T_m}. \tag{13}$$

The maximum occurs for $\mathcal{B}^* = r_f^2$ and thus $T = T^* = T_m r_f^2/(c_L^2 a^2)$, i.e., the so-called depinning temperature [5], which can be below melting. The amplitude A (and large T tail) is independent of the detailed form of $\hat{V}(\mathcal{B})$ and is strongly enhanced by the anisotropy and dispersion of c_{44}. The value at the maximum is $C_v^* - C_{\rm eq} = 8AT^*/27$. Using $c_{44} \approx \epsilon_0 \epsilon^2/a^2$, $c_{66} \approx \epsilon_0/(4a^2)$ the calculation of $J_2(\Sigma)$ shows [31] that $R_c > a$ holds for $\delta < \delta_c = 4\pi^{3/2}(\epsilon r_f^4/sa^3)$. At this value of the disorder one obtains $A \approx c_L^2/(T_m r_f^2 a\epsilon)$ and $C_v^* - C_{\rm eq} \approx 1/(a^3\epsilon)$ ($\epsilon < 1$ the anisotropy parameter [5]). $C_v^* - C_{\rm eq}$ should be compared with the equipartition value, estimated as $C_{\rm eq} = 1/(sa^2)$. These become comparable around the dimensional crossover field $B = B_{\rm cr}$ such that $s \sim a\epsilon$.

The above classical contribution (12) will hold only above the vorton Debye temperature T_v^D, below which quantum effects become important. T_v^D depends on the vortex mass, the Hall force, and the friction force, which arise from the coupling of moving vortices with the normal electrons bath in the presence of scattering. Estimates for T_v^D range [22] from $10^{-3}T_c$ in dirty (friction dominated) materials to $10^{-1}T_c$ in the superclean limit (Hall dominated). There should thus exist a broad regime of temperature and field where the Bragg glass is stable and the result (12) holds.

To assess whether this contribution (12) is observable, let us compare it with other terms linear in T in C_v. First the normal electrons in the vortex core lead to [14]:

$$C_{v\mathrm{core}}(T) \approx \frac{T}{T_f k_f^{-2} s}\left(\frac{H}{H_{c2}}\right)^{\alpha}, \tag{14}$$

where T_f is the Fermi temperature of the normal metal, $\alpha = 1$ for s-wave superconductor [38] and $\alpha = 1/2$ for lines of nodes in the gap [15]. Given the large ratio $T_f/T_m = O(10^2)$ for, e.g., YBaCuO, one finds, comparing A_{core} and A_{dis} that the contribution from the cores can be comparable or smaller than the one from the disorder.

There are other contributions from the vortex lattice as well. The mean field specific heat [23] being $C_{mf} \approx \epsilon_0 T/(T_c^2 a^2)$, the ratio $A_{\mathrm{dis}}/A_{mf} \approx [c_L^2 a T_c/(r_f T_m)]^2$ can be large. We have also computed the contribution from nonlinear elasticity of the VL. Performing, as in [39], an expansion of the vortex interaction energy $\int dz \sum_{ij} \epsilon_0 K_0\{[R_i - R_j + u_i(z) - u_j(z)]/\lambda\}$, we obtain

$$A_{nl} = \rho_0 \epsilon_0 a^{-4}\left(\frac{c_L^2 a^2}{T_m}\right)^2\left[\gamma_4 + \gamma_3\sqrt{c_{44}}c_{66}\left(\frac{c_L^2 a^2}{T_m}\right)\frac{\epsilon_0}{a}\right] \tag{15}$$

up to $O((a/\lambda)^2)$ where the numerical prefactors γ_4 and γ_3 are complicated lattice sums, given in [31]. One can then estimate $A_{nl}/A_{\mathrm{dis}} \sim r_f^2/a^2$ and this contribution is likely to remain small until melting. None of these contributions [40] is expected to exhibit a maximum at scale distinct from T_m.

To conclude, we found by explicit calculation that the vortex lattice classical contribution to C_v due to disorder can be important at the very least in the range $10^{-1}T_c$ to T_m, and possibly more. The coefficient A_{dis} of the low T linear behavior is magnetic field dependent, $A_{\mathrm{dis}} \propto 1/(T_m a)$. It is convenient to express results using the melting temperature T_m, which is experimentally measured [24,25,27] and can also be estimated from the standard elastic expression of $T_m = 4a^3\sqrt{c_{66}c_{44}}c_L^2$ allowing to extract directly the magnetic field dependence. $T_m \sim 1/\sqrt{B}$ leads to $T^* \propto \sqrt{B}$ and $A_{\mathrm{dis}} \propto B$. $C_v(T)$ exhibits, as compared to other contributions, a distinct maximum around the depinning temperature scale, whenever smaller than melting. A crude estimate of T^* is $T^* \simeq T_m B/(c_L^2 H_{c2})$, which for a typical $c_L \sim 0.12$ gives a T^* which is a fraction of T_m for fields of about a Tesla (a fraction of order unity for YBaCuO around $B \simeq 10$ T the so-called tricritical point [27]). It would be very interesting to perform precise measurements of C_v to check the present proposal.

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