

## Hunting for Glueballs in Electron-Positron Annihilation

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We calculate the cross section for the exclusive production of  $J^{PC} = 0^{++}$  glueballs  $\mathcal{G}_0$  in association with the  $J/\psi$  in  $e^+e^-$  annihilation using the perturbative QCD factorization formalism. The required long-distance matrix element for the glueball is bounded by CUSB data from a search for resonances in radiative  $Y$  decay. The cross section for  $e^+e^- \rightarrow J/\psi + \mathcal{G}_0$  at  $\sqrt{s} = 10.6$  GeV is similar to exclusive charmonium-pair production  $e^+e^- \rightarrow J/\psi + h$  for  $h = \eta_c$  and  $\chi_{c0}$ , and is larger by a factor of 2 than that for  $h = \eta_c(2S)$ . As the subprocesses  $\gamma^* \rightarrow (c\bar{c})(c\bar{c})$  and  $\gamma^* \rightarrow (c\bar{c})(gg)$  are of the same nominal order in perturbative QCD, it is possible that some portion of the anomalously large signal observed by Belle in  $e^+e^- \rightarrow J/\psi X$  may actually be due to the production of charmonium-glueball  $J/\psi\mathcal{G}_J$  pairs.

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Bound states of gluons provide an explicit signature of the non-Abelian interactions of quantum chromodynamics. In fact, in a model universe without quarks, the hadronic spectrum of QCD would consist solely of color-singlet glueball states. In the physical world, the purely gluonic components mix with  $q\bar{q}$  pairs, leading to an enriched spectrum of isospin-zero states as well as  $q\bar{q}g$  hybrids. The existence of this exotic spectrum is as essential a prediction of QCD as the Higgs particle is for the electroweak theory.

Lattice gauge theory predicts the spectrum and quantum numbers of gluonic states. According to a recent calculation [1], the ground-state masses for the  $J^{PC} = 0^{++}$  and  $2^{++}$  glueballs  $\mathcal{G}_J$  are 1.73 and 2.40 GeV, respectively [1]. Thus far, the empirical evidence for glueballs is not decisive, probably because of complications from mixing with the quark degrees of freedom, but there are indications of an extra neutral scalar state perhaps due to a glueball of mass (before mixing) near 1.7 GeV [2].

An important mechanism for producing glueballs is the radiative decay of heavy quarkonium, particularly  $J/\psi \rightarrow \gamma\mathcal{G}_J$  and  $Y \rightarrow \gamma\mathcal{G}_J$  [3]. In these reactions, the quarkonium decays to an intermediate  $\gamma gg$  state which then can couple to any charge conjugation parity  $C = +$  isospin  $I = 0$  gluonic or hybrid state. For example, the BES Collaboration [4] has observed the radiative decays of the  $J/\psi$  and the  $\psi(2S)$  to  $\gamma f_0(1710)$ , a glueball candidate. In this Letter we focus on another optimal mechanism for the production of  $\mathcal{G}_0$  and  $\mathcal{G}_2$  at  $e^+e^-$  colliders, the reaction  $e^+e^- \rightarrow \gamma^* \rightarrow H\mathcal{G}_J$ ,  $H = J/\psi$ , or  $Y$  [5], in which a  $C = +$  glueball can be produced in association with a quarkonium state from the subprocess  $\gamma^* \rightarrow (Q\bar{Q})(gg)$ . Two-gluon components in  $\eta$  particles have been estimated recently [6]. One of the six Feynman diagrams for the subprocess is shown in Fig. 1; the remaining diagrams are permutations of the photon and

the two gluons. A related reaction  $\gamma^* \rightarrow \pi^0\mathcal{G}_J$  has been considered [7] as a source of pseudoscalar glueballs. We shall show that these reactions satisfy perturbative QCD (pQCD) factorization. Unlike radiative quarkonium decay, this channel imposes no *a priori* limit on the mass of the glueball.

The main background to charmonium-glueball production  $e^+e^- \rightarrow J/\psi\mathcal{G}_J$  is exclusive quarkonium pairs such as  $\gamma^* \rightarrow J/\psi\eta_c$ , arising from the subprocess  $\gamma^* \rightarrow (c\bar{c})(c\bar{c})$ . The exclusive production of charmonium pairs has in fact been observed recently with a substantial rate at Belle [8]. The rates for exclusive charmonium-pair production reported by Belle are significantly larger than predictions based on pQCD [9,10]. The Belle experiment identifies one member of the pair, the  $J/\psi$ , via its leptonic decay; the other quarkonium state is inferred by identifying the missing mass of the spectator system with the charmonium states  $\eta_c$ ,  $\chi_{c0}$ , and  $\eta_c(2S)$  which occur within the detector mass resolution. As noted in Refs. [11,12], some of the Belle signal for quarkonium pairs may be due to two-photon annihilation  $e^+e^- \rightarrow \gamma^*\gamma^* \rightarrow J/\psi J/\psi$ . Here we note that because the subprocesses  $\gamma^* \rightarrow (c\bar{c})(c\bar{c})$  and  $\gamma^* \rightarrow (c\bar{c})(gg)$  are of the same

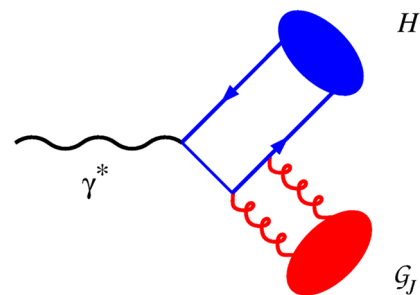


FIG. 1 (color online). Feynman diagram for  $\gamma^* \rightarrow H + \mathcal{G}_J$ .

nominal order in pQCD, it is possible that some portion of the signal observed by Belle in  $e^+e^- \rightarrow J/\psi X$  may actually be due to the production of  $J/\psi \mathcal{G}_J$  pairs.

In general, exclusive amplitudes can be computed in QCD by convoluting the light-front wave functions  $\psi_{n/H}(x_i, \mathbf{k}_{\perp i})$  of each hadron with the corresponding  $n$ -particle irreducible quark-gluon matrix elements, summed over  $n$  [13]. For hadronic amplitudes involving a hard momentum transfer  $Q$ , it is usually possible to expand the quark-gluon scattering amplitude as a function of  $\mathbf{k}_{\perp}^2/Q^2$ . The leading-twist contribution can then be computed from a hard-scattering amplitude  $T_H$  where the external quarks and gluons associated with each hadron are collinear. Furthermore, only the minimum number of quark and gluon quanta contribute at leading order in  $1/Q^2$ . In our case, the relevant hard-scattering amplitude is  $T_H(\gamma^* \rightarrow c\bar{c}gg)$  computed with collinear  $c$  and  $\bar{c}$  and collinear  $gg$ . As  $T_H$  at leading twist is independent of the constituent's relative transverse momentum  $\mathbf{k}_{\perp i}$ , the convolution with the light-front wave functions and the integration over the relative transverse momentum then project out the  $L_z = 0$  component of the light-front wave functions with minimal  $n$ —the hadron distribution amplitudes  $\phi_H(x, Q)$ .

In this Letter we shall calculate the cross section for  $e^+e^- \rightarrow H\mathcal{G}_{J=0,2}$  using pQCD factorization. The amplitude at leading twist can be expressed as a factorized product of the perturbative hard-scattering amplitude  $T_H(\gamma^* \rightarrow Q\bar{Q}gg)$  convoluted with the nonperturbative distribution amplitudes for the heavy quarkonium and

$$\phi_J(x, Q) = \frac{F_{\alpha\beta}^J}{\sqrt{2(N_c^2 - 1)}} \int \frac{d^2\mathbf{k}_{\perp} dz^- d^2\mathbf{z}_{\perp}}{(2\pi)^3 k^+ x(1-x)} e^{-i(xk^+z^- - \mathbf{k}_{\perp} \cdot \mathbf{z}_{\perp})} \langle \mathcal{G}_J | T G_a^{+\alpha}(0^+, z^-, \mathbf{z}_{\perp}) G_a^{+\beta}(0) | 0 \rangle, \quad (1)$$

where  $x$  and  $\mathbf{k}_{\perp}$  are the light-cone momentum fraction and transverse momentum of a gluon inside the  $\mathcal{G}_J$  with momentum  $k = (k^+ = n \cdot k, k^- = \bar{n} \cdot k, \mathbf{0}_{\perp})$  and mass  $M_{\mathcal{G}_J}$ . The  $S$ -wave component is projected out by integrating over  $\mathbf{k}_{\perp}$ . The lightlike vectors  $n$  and  $\bar{n}$  satisfy  $n^2 = \bar{n}^2 = 0$  and  $n \cdot \bar{n} = 2$ . The tensor  $F_{\alpha\beta}^J$  projects the massless spin- $J$  components; they are defined by  $F_{\alpha\beta}^0 = [-g_{\alpha\beta} + \frac{1}{2}(n_{\alpha}\bar{n}_{\beta} + n_{\beta}\bar{n}_{\alpha})]/\sqrt{2}$ , and  $F_{\alpha\beta}^2$  is the massless spin-2 polarization tensor  $\epsilon_{\alpha\beta}$ . The glueball distribution amplitude can also be defined from the two-gluon light-front wave functions  $\psi_{\mathcal{G}_J}(x, \mathbf{k}_{\perp}, \lambda_i)$  with gluon spin projection  $\lambda_i = S_i^z = \pm 1$ , integrated over transverse momentum in light-cone gauge  $A^+ = 0$ .

The relative rates for the production of heavy scalar glueballs with higher radial number  $N$  are determined by the normalization of the corresponding glueball distribution amplitudes. In effect, the integral of the distribution amplitude over  $x$  is the relativistic generalization of the Schrödinger wave function at the origin. Thus the distribution amplitudes for the  $0^{++}$  glueballs tend to scale inversely with their mean radius  $\langle r_N \rangle$ . According to bag models [16],  $\langle r_N \rangle \sim 0.6$  fm, independent of  $N$ , suggesting equal rates for the heavier glueballs. On the other hand,

glueball states. We shall find that  $\gamma^* \rightarrow J/\psi \mathcal{G}_0$  production dominates over that of  $J/\psi \mathcal{G}_2$  and show how the angular distribution of the final state can be used to determine the angular momentum  $J$  and projection  $J_z$  of the glueball. We shall show that only  $J_z = \pm 2$  tensor states are produced by the pQCD mechanism at leading twist. A bound on the normalization of the distribution amplitude for the glueball state can be extracted from a resonance search by CUSB in  $Y \rightarrow \gamma X$  [14]. We shall show that the rate for  $e^+e^- \rightarrow J/\psi \mathcal{G}_0$  production could be comparable to the corresponding nonrelativistic QCD (NRQCD) prediction for  $e^+e^- \rightarrow J/\psi \eta_c$  without exceeding the CUSB bound from radiative  $Y$  decay.

The distribution amplitude  $\phi_H(x, Q)$  required for the formation of the  $H$  in a hard process is directly related to the NRQCD matrix element for the leptonic decay rate of  $H$ . Its  $x$  dependence is peaked at  $x \sim 1/2$ . The key quantity which determines the normalization of the  $\gamma^* \rightarrow H\mathcal{G}_J$  processes is then the distribution amplitude  $\phi_J(x, Q)$  of the  $\mathcal{G}_J$ . The pQCD factorization picture provides a direct relation among the various glueball production processes, as they all involve the same process independent  $\phi_J(x, Q)$ . The  $\phi_J(x, Q)$  can be determined phenomenologically by fitting to the measured production rate of a glueball candidate. In leading-twist approximation the spin structure of the two-gluon system in hard-scattering amplitude becomes that of a massless spin- $J = 0, 2$  state. Therefore the field-theoretic definition of the  $\phi_J(x, Q)$  in light-cone gauge reduces to [15]

the virial theorem extended to the light-front formalism suggests that mean transverse momentum and  $1/\langle r_N \rangle$  increase monotonically with glueball mass. If this is the case, then the production rate in the  $\gamma^* \rightarrow H\mathcal{G}_J$  will tend to increase for heavier glueball states, assuming that the annihilation energy  $\sqrt{s}$  poses no phase-space restriction. Lattice gauge theory and light-front Hamiltonian methods should eventually determine the glueball distribution amplitudes, thus providing consistency checks on the production mechanisms considered here.

As noted above, the amplitude for  $\gamma^* \rightarrow H(p)\mathcal{G}_J(k)$  can be computed as the convolution of  $T_H(\gamma^* \rightarrow Q\bar{Q}gg)$  with  $\phi_J(x, Q)$  weighted by the NRQCD matrix element. In leading twist  $k^-$  is neglected, and thus the glueball momentum is approximated by  $k = k^+ \bar{n}/2$  in  $T_H(\gamma^* \rightarrow Q\bar{Q}gg)$ . The resulting effective vertex  $\mathcal{A}_J^{\mu}$  is [15]

$$\mathcal{A}_0^{\mu} = -\frac{8ig_s^2 ee_Q m_Q^2 \sqrt{N_c^2 - 1}}{N_c k \cdot n p \cdot \bar{n}} \left( \epsilon_H^{\mu} - \frac{\bar{n}^{\mu} k \cdot n \epsilon_H \cdot \bar{n}}{2p \cdot \bar{n}} \right) \times \sqrt{\frac{\langle O_1 \rangle_H}{m_Q^3}} I_0, \quad (2)$$

$$\mathcal{A}_2^\mu = -\frac{4ig_s^2 ee_Q m_Q^2 \sqrt{2(N_c^2 - 1)}}{3N_c k \cdot np \cdot \bar{n}} \epsilon_2^{\mu\nu}(\lambda_2) \epsilon_H^\nu \sqrt{\frac{\langle Q_1^1 \rangle_H}{m_Q^7}} I_2, \quad (3)$$

where  $\mu$  and  $\epsilon_H$  are the vector indices for the  $\gamma^*$  and polarization vector for the  $H$ , respectively. The mass and fractional charge of the heavy quark  $Q$  are expressed as  $m_Q$  and  $e_Q$ . Here  $\langle O_1 \rangle_H$  and  $\langle Q_1^1 \rangle_H$  are the vacuum-saturated analogs of NRQCD matrix elements  $\langle O_1(^3S_1) \rangle_H$  and  $\langle Q_1(^3S_1) \rangle_H$  for annihilation decays defined in Refs. [17,18], respectively. To leading order in the heavy-quark velocity  $v_Q$  in the quarkonium rest frame, the  $\langle O_1(^3S_1) \rangle_H$  is related to the radial wave function at the origin  $R(0)$  in the color-singlet model [19] and the decay constant  $f_H$ , which is defined by  $\langle 0 | J_{e.m.}^\mu | H \rangle = 2M_H e_Q f_H \epsilon_H^\mu$ , as  $\langle O_1 \rangle_H = (N_c/2\pi) |R(0)|^2 = 2M_H f_H^2$ . The nonperturbative factors for  $\mathcal{G}_J$  are written as  $I_0 = \int_0^1 dx \phi_0(x, Q)$  and  $I_2 = \int_0^1 dx \phi_2(x, Q)/[x(1-x)]$ . In leading twist the valence gluons are collinear and therefore the only allowed polarization states for  $\mathcal{G}_2$  are  $\lambda_2 = \pm 2$ . For  $\mathcal{G}_2$  production the longitudinal polarization is prohibited by Bose symmetry. This is true for any production process for  $\mathcal{G}_2$ , for which pQCD factorization is valid. The amplitude (3) for  $\mathcal{G}_2$  is proportional to the factor  $\langle Q_1^1 \rangle_H/m_Q^7$  which is suppressed to  $\langle O_1 \rangle_H/m_Q^3$  by  $v_Q^4$ . Therefore, in the remainder of this Letter we consider only  $\mathcal{G}_0$ ; the analysis for  $\mathcal{G}_2$  can be found in our forthcoming publication [15]. Using the vertex (2), we obtain the width  $\Gamma_0$  for radiative  $Y$  decay into  $\mathcal{G}_0$  as

$$\Gamma_0 = \frac{16\pi^2 \alpha_s^2 \alpha e_b^2 (N_c^2 - 1) \Phi_0^\gamma \langle O_1 \rangle_Y}{3N_c^2 m_b} |I_0|^2, \quad (4)$$

where  $\Phi_0^\gamma = 1 - M_{\mathcal{G}_0}^2/M_Y^2$ . In Ref. [20], the decay rate for the process  $Y \rightarrow \gamma f_0$  has been calculated treating  $f_0$  as a glueball candidate. The  $\Gamma_0$  agrees with Eq. (5) of Ref. [20], after including a missing factor 2/3 and neglecting  $M_{\mathcal{G}_0}$  [21].

Our result for the differential cross section for  $e^+e^- \rightarrow J/\psi \mathcal{G}_0$  normalized to  $\sigma_{\mu^+\mu^-} = 4\pi\alpha^2/(3s)$  is

$$\frac{dR_{J/\psi \mathcal{G}_0}}{d\cos\theta^*} = \frac{3\pi^2 \alpha_s^2 e_c^2 (N_c^2 - 1) r^2 \Phi_0^{ee} \langle O_1 \rangle_{J/\psi} |I_0|^2}{N_c^2 (1 - \frac{r^2}{4})^2 m_c^3 s} \times \left[ \sin^2\theta^* + \frac{r^2}{4} (1 + \cos^2\theta^*) \right], \quad (5)$$

where  $\theta^*$  is the scattering angle in the center-of-mass frame,  $r = 4m_c/\sqrt{s}$ , and the phase-space factor  $\Phi_0^{ee}$  is defined by

$$\Phi_0^{ee} = \frac{1}{s} \sqrt{[s - (M_{J/\psi} + M_{\mathcal{G}_0})^2][s - (M_{J/\psi} - M_{\mathcal{G}_0})^2]}. \quad (6)$$

The angular factors in the expression (5) can be understood physically. If the hadron pair is produced at  $\theta^* = 0$ , i.e., aligned with the lepton beams, then only final states with  $J_z = \pm 1$  can contribute, because the  $e^+$  and  $e^-$

annihilate with opposite chirality. Thus in the case of scalar glueballs, the  $J/\psi$  with helicity  $\pm 1$  is produced with a  $1 + \cos^2\theta^*$  distribution. If the  $J/\psi$  is longitudinally polarized, the cross section must vanish in the forward direction, and thus it has a  $\sin^2\theta^*$  distribution.

The rate integrated over angle is

$$R_{J/\psi \mathcal{G}_0} = \frac{32\pi^2 \alpha_s^2 e_c^2 r^2 (1 + \frac{r^2}{2}) \Phi_0^{ee} \langle O_1 \rangle_{J/\psi} |I_0|^2}{9(1 - \frac{r^2}{4})^2 m_c^3 s}. \quad (7)$$

The size of the cross section can be estimated using the asymptotic form of the ratio  $\mathcal{R} = R_{J/\psi \mathcal{G}_0}/R_{J/\psi \eta_c}$

$$\mathcal{R} \simeq \frac{9}{4} \left( \frac{\alpha_s^{\mathcal{G}_0}}{\alpha_s^{\eta_c}} \right)^2 \frac{1 + \frac{r^2}{2}}{r^2(1 - r^2)(1 - \frac{r^2}{4})^2} \frac{m_c |I_0|^2}{\langle O_1 \rangle_{\eta_c}}, \quad (8)$$

where we neglected QED contributions to  $R_{J/\psi \eta_c}$  given in Ref. [9]. In the ratio  $\mathcal{R}$  the phase-space factor  $\Phi_0^{ee}$  cancels the  $\sqrt{1 - r^2}$  for  $e^+e^- \rightarrow J/\psi \eta_c$ . The  $\alpha_s$ 's for the two processes are written distinctively because they have different effective scales. However, the main uncertainties from the choice of running coupling scale and scheme largely cancel in the ratio  $\mathcal{R}$ . Here  $m_c |I_0|^2 / \langle O_1 \rangle_{\eta_c}$  represents the ratio of the square of the wave function at the origin of the glueball compared to that of the  $\eta_c$ .

We next investigate whether some portion of the anomalously large signal for  $J/\psi + \eta_c$ ,  $\chi_{c0}$ , and  $\eta_c(2S)$  observed by the Belle Collaboration could actually be coming from the process  $e^+e^- \rightarrow J/\psi \mathcal{G}_0$ . We calculate the cross section assuming glueball mass  $M_{\mathcal{G}_0}$  the same as those for  $\eta_c$ ,  $\chi_{c0}$ , and  $\eta_c(2S)$ . In order to predict the production cross section  $\sigma_{J/\psi \mathcal{G}_0}$ , we need to know the nonperturbative factors  $\langle O_1 \rangle_{J/\psi}$  and  $I_0$ . The  $\langle O_1 \rangle_{J/\psi}$  is determined through the leptonic decay rate of  $J/\psi$ . As the glueball distribution amplitude is process independent, we can extract an upper bound to  $I_0$  from the CUSB data for the resonance search from  $Y \rightarrow \gamma X$ . We follow the method used in Ref. [22]. The branching fraction  $\text{Br}[\gamma \mathcal{G}_0]$  for the process  $Y \rightarrow \gamma \mathcal{G}_0$  is obtained by

$$\text{Br}[\gamma \mathcal{G}_0] = \frac{\Gamma_0}{\Gamma[e^+e^-]_{\text{NRQCD}}} \text{Br}[e^+e^-]_{\text{exp}}, \quad (9)$$

where  $\text{Br}[e^+e^-]_{\text{exp}} = 2.38\%$  and  $\Gamma[e^+e^-]_{\text{NRQCD}} = 2\pi e_b^2 \alpha^2 \langle O_1 \rangle_Y / (3m_b^2)$ . In the ratio (9)  $\langle O_1 \rangle_Y$  dependence cancels. The branching fraction must be less than allowed by the CUSB excluded region. In order to extract the bound, we note that the mass resolution of the CUSB data is 20 MeV. If the decay width  $\Gamma[\mathcal{G}_0]$  of the  $\mathcal{G}_0$  is larger than the resolution, one must rescale the boundary of the excluded region by the factor  $\Gamma[\mathcal{G}_0]/20$  MeV. The decay width  $\Gamma[\mathcal{G}_0]$  cannot be computed using perturbation theory because factorization is not valid for this nonperturbative quantity. However, if Belle's  $J/\psi \eta_c$  signal also contains  $J/\psi \mathcal{G}_0$ ,  $\Gamma[\mathcal{G}_0]$  must be less than 110 MeV, which is the full width at half maximum of the  $\eta_c$  peak in the Belle fit to the  $J/\psi$  momentum distribution. The first row in Table I gives the upper limits to  $|I_0|^2$  for  $m_b =$

TABLE I. Upper limits to the nonperturbative constant  $|I_0|^2$ , cross section  $\sigma_{J/\psi\mathcal{G}_0}$ , and the ratio  $\sigma_{J/\psi\mathcal{G}_0}/\sigma_{J/\psi h}$ , at  $\sqrt{s} = 10.6$  GeV, assuming  $M_{\mathcal{G}_0} = M_h$ , where  $h = \eta_c, \chi_{c0}$ , and  $\eta_c(2S)$ . The limits are determined by the  $Y \rightarrow \gamma X$  search of the CUSB Collaboration [14].

$M_{\mathcal{G}_0} = M_h$	$h = \eta_c$	$\chi_{c0}$	$\eta_c(2S)$
$ I_0 _{\max}^2 (10^{-3} \text{ GeV}^2)$	5.2	5.8	6.2
$\sigma_{J/\psi\mathcal{G}_0}^{\max}$	1.4 fb	1.5 fb	1.6 fb
$\sigma_{J/\psi\mathcal{G}_0}^{\max}/\sigma_{J/\psi h}$	0.63	0.72	1.9

4.73 GeV and  $M_{\mathcal{G}_0} = 2.98, 3.42,$  and  $3.65$  GeV corresponding to  $M_{\mathcal{G}_0} = M_{\eta_c}, M_{\chi_{c0}},$  and  $M_{\eta_c(2S)}$ , respectively. Values for  $|I_0|^2$  above the bound are excluded by 90% confidence level. We choose  $\alpha_s(\mu^2) = 0.26$  using the modified minimal subtraction  $\overline{\text{MS}}$  scheme and the scale  $\mu^2 = e^{-5/3}\langle|\mathbf{k}|^2\rangle$  [23] where  $\langle|\mathbf{k}|^2\rangle$  is the mean 3-momentum squared for a single gluon.

Now we are ready to find upper limits to  $\sigma_{J/\psi\mathcal{G}_0}$  at the  $B$  factories. Substituting  $|I_0|_{\max}^2$  to Eq. (7), we get the cross sections  $\sigma_{J/\psi\mathcal{G}_0} = R_{J/\psi\mathcal{G}_0} \sigma_{\mu^+\mu^-}$  in the second row in Table I. In order to make our prediction consistent with the previous analyses on exclusive charmonium-pair production, we use the same input parameters given in Refs. [9,11,12]:  $\langle O_1 \rangle_{J/\psi} = 0.335 \text{ GeV}^3$ ,  $m_c = 1.40$  GeV, and  $M_{J/\psi} = 3.10$  GeV. The strong coupling constant is chosen to be  $\alpha_s = 0.260, 0.264,$  and  $0.265$  for  $M_{\mathcal{G}_0} = M_{\eta_c}, M_{\chi_{c0}},$  and  $M_{\eta_c(2S)}$ , respectively, applying the same method used for the radiative  $Y$  decay. The ratios to the cross sections for exclusive charmonium-pair productions are given in the third row in Table I.

The cross sections for  $J/\psi + \eta_c, \chi_{c0},$  and  $\eta_c(2S)$  recently measured by the Belle Collaboration are not well understood within NRQCD. Based on the assumption that the measured signals at Belle include the  $J/\psi + \mathcal{G}_0$  signal within the mass region corresponding to  $\eta_c, \chi_{c0},$  and  $\eta_c(2S)$  we get the cross section for  $J/\psi\mathcal{G}_0$ . We thus find that the upper limit to the cross section  $\sigma_{J/\psi\mathcal{G}_0}$  is comparable to the NRQCD prediction of the cross sections for  $e^+e^- \rightarrow J/\psi + h$  for  $h = \eta_c$  and  $\chi_{c0}$ , and larger by a factor of 2 to that for  $h = \eta_c(2S)$ , suggesting the possibility that a significant fraction of the anomalously large cross section measured by Belle may be due to glueballs in association with  $J/\psi$  production. In fact, there is a possibility of a resonance signal in the Belle data for  $e^+e^- \rightarrow J/\psi X$  at the missing mass  $\mathcal{M}_X \sim 1.7$  GeV. A resonance search in the radiative  $Y(nS)$  decay by the CLEO Collaboration and an independent study by the BaBar Collaboration on charmonium-pair production in  $e^+e^-$  annihilation will provide stringent tests of this scenario.

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