Robust Long-Distance Entanglement and a Loophole-Free Bell Test with Ions and Photons

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Two trapped ions that are kilometers apart can be entangled by the joint detection of two photons, each coming from one of the ions, in a basis of entangled states. Such a detection is possible with linear optical elements. The use of two-photon interference allows entanglement distribution free of interferometric sensitivity to the path length of the photons. The present method of creating entangled ions also opens up the possibility of a loophole-free test of Bell's inequalities.

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Two parties that share entanglement can use it to perform quantum cryptography [1,2] or other quantum communication tasks, such as quantum teleportation [3]. The creation of entanglement between very distant locations is thus an important goal. Owing to unavoidable transmission losses and errors, long-distance entanglement creation is likely to require a quantum repeater protocol, such as that of Ref. [4] which starts by establishing entanglement between nodes that are separated by some basic distance, followed by entanglement swapping and entanglement purification procedures performed at all nodes that lie between the two desired end points.

Photons are the optimal systems for entanglement distribution because they propagate fast and can preserve their coherence over long distances. For entanglement swapping and purification, it is advantageous to have systems that can be stored easily and between which quantum gates can be realized efficiently. The systems used should also have coherence times that exceed the times required for photons to propagate over long distances. Trapped ions fulfill all these requirements.

In a recent proposal for long-distance quantum communication [5], which was inspired by Ref. [6], entanglement between atomic ensembles is established by the detection of a single photon that could have been emitted by either ensemble. Here we propose a scheme where distant trapped ions are entangled by the joint detection of *two* photons, one coming from each ion. The two-photon detection, which requires only linear optical elements and single-photon detectors, realizes a partial Bell state analysis [7,8], such that for some outcomes the two distant ions are projected into entangled states. A practical advantage of using two-photon interference for the creation of long-distance entanglement is that it is insensitive to the phase accumulated by the photons on their way from the ions to the place where they are detected.

The violation of Bell's inequality [9] by entangled quantum systems shows that quantum physics is not compatible with local realism. For a rigorous experimental demonstration of the violation, first, the settings of the detection apparatus of two distant observers have to be changed randomly so fast that no information about the settings can travel from one observer to the other at or below the speed of light during the course of each measurement, and, second, the detection efficiency has to be so high that the results of the experiment cannot be explained in local realistic terms by systems selectively escaping detection depending on the apparatus settings. Experiments not meeting these two conditions are said to leave open the *locality* and *detection* loopholes, respectively. The first one was addressed in experiments with entangled photons [10,11], and the second one in an experiment with trapped ions [12], but closing both loopholes in a single experiment has so far remained an elusive goal, despite several proposals [13]. Quantum state detection of ions can be performed with almost perfect efficiency. With the proposed scheme for entanglement creation between distant ions a final Bell experiment seems within reach of current technology.

We now describe the proposed protocol in more detail, starting with the establishment of entanglement between a pair of ions shared by two parties, Alice and Bob. Each party starts out with an ion that contains a lambda system of levels. The excited state $|e\rangle$ can decay into two degenerate metastable states $|s_1\rangle$ and $|s_2\rangle$ by emitting a photon into one of the two orthogonal polarization modes a_1 or a_2. The degeneracy of s_1 and s_2 is important to ensure indistinguishability of the photons later on. For simplicity we assume that the transition probabilities for $e \rightarrow s_1$ and $e \rightarrow s_2$ are identical as well. If this is not the case, it can be compensated by selective attenuation of one polarization during transmission.

The protocol starts by exciting both ions to the state $|e\rangle_A|e\rangle_B$. Emission of a photon by each ion leads to the state

$$\tfrac{1}{2}[|s_1\rangle_A a_1^\dagger + |s_2\rangle_A a_2^\dagger][|s_1\rangle_B b_1^\dagger + |s_2\rangle_B b_2^\dagger]|0\rangle, \qquad (1)$$

so that each ion is maximally entangled with the photon it has emitted. The photons from A and B propagate to some intermediate location where a partial Bell state analysis is performed. Figure 1 shows the well-known method for

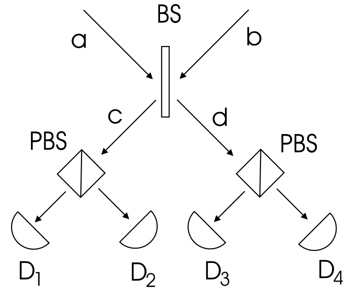

FIG. 1. Partial Bell state analyzer as implemented in Ref. [8]. One photon comes in through mode a, the other through mode b. Only if the two photons are in the antisymmetric Bell state $|\psi_-\rangle$ of Eq. (2), there will be one photon in each output mode of the first beam splitter (BS), c and d. Therefore a coincidence detection between D_1 and D_3 or D_2 and D_4 identifies this state. This was first pointed out in Ref. [7]. If the photons are in the state $|\psi_+\rangle$, they both go into c or both into d and are then split by the subsequent polarizing beam splitters (PBS), because they have orthogonal polarizations. Therefore coincidences between D_1 and D_2 or between D_3 and D_4 signify a state $|\psi_+\rangle$. For the two other Bell states $|\phi_+\rangle$ and $|\phi_-\rangle$ both photons go to the same detector.

detecting the two states $|\psi_\pm\rangle = (1/\sqrt{2})(a_1^\dagger b_2^\dagger \pm a_2^\dagger b_1^\dagger)|0\rangle$ using only linear optical elements.

Detection of the two photons in the state $|\psi_\pm\rangle$ will project the two distant ions into the corresponding $|\psi_\pm\rangle$ state, $(1/\sqrt{2})(|s_1\rangle_A|s_2\rangle_B \pm |s_2\rangle_A|s_1\rangle_B)$. The two remaining Bell states, $|\phi_\pm\rangle = (1/\sqrt{2})(a_1^\dagger b_1^\dagger \pm a_2^\dagger b_2^\dagger)|0\rangle$, cannot be distinguished at the same time using only linear optical elements. The phases $\phi_A = kL_A$ and $\phi_B = kL_B$, where k is the wave number and $L_{A,B}$ are the path lengths, that the photons acquire on their path from the ions to the Bell state analysis lead only to a multiplicative factor $e^{i(\phi_A+\phi_B)}$ in Eq. (1) and thus have no effect on the entanglement. The only condition on the path lengths is that the photon wave packets should overlap well in time at the detection station, since they have to be indistinguishable in order for the Bell state analysis to work.

The insensitivity to the phase should be contrasted with schemes that create entanglement between two distant two-level systems, consisting of levels e and g, by detecting, at some intermediate location, a single photon that could have been emitted by either of the two systems [5,6,14]. Such a detection projects the two systems into the state $(1/\sqrt{2})(e^{i\phi_A}|g\rangle_A|e\rangle_B + e^{i\phi_B}|e\rangle_A|g\rangle_B)$. The relative phase between the two terms depends on the difference in the path lengths from each two-level system to the intermediate location. If the relative phase fluctuates significantly, the entanglement is destroyed. Therefore the use of two-photon interference significantly improves the robustness of entanglement distribution. A recent proposal [15] to create entanglement between distant atoms via the *absorption* of entangled photons shares this basic advantage.

Once entangled ion states have been established between locations separated by a certain basic distance (which is chosen depending on the transmission losses), the length over which entangled states are shared can be extended following the quantum repeater protocol of Ref. [4]. All the required quantum operations can be performed locally at each station. Quantum gates between ions following the proposal of Ref. [16], where the ions in a trap are coupled by the phonons corresponding to their collective motion, have recently been demonstrated experimentally for $^{40}Ca^+$ ions [17]. A similar scheme was previously used in Ref. [18] to prepare an entangled state of two $^{9}Be^+$ ions. A robust two-ion gate has also recently been demonstrated for $^{9}Be^+$ ions [19]. Establishing long-distance entanglement following the above protocol should thus be a realistic goal.

The goal of performing a loophole-free Bell experiment puts constraints on the timing of the various operations (Fig. 2). First, there has to be no way for the measurement result on one side to be influenced by the choice of measurement basis on the other side (by signals at or below c). This implies that the random choice of basis on side B (C_B) has to lie outside the backward lightcone of the event D_A, which corresponds to the moment when the measurement result is finally established on side A. Analogously, C_A has to lie outside the backward lightcone of D_B. This means that, for a distance L between A and B, the time that passes from the choice of basis to the completion of detection may not be larger than L/c.

There is another constraint that is specific to the present scheme. Each run of the experiment starts with the excitation of both ions and the subsequent emission of two photons, corresponding to the events E_A and E_B in Fig. 2.

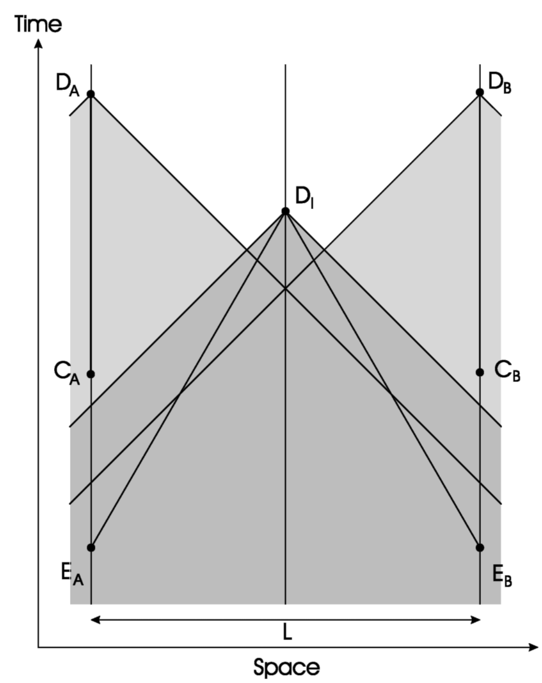

FIG. 2. Diagram illustrating the timing constraints for a loophole-free Bell experiment.

Note that the time delay between excitation and emission is negligible in the present context. The photons propagate inside optical fibers (with velocity $v < c$) to the intermediate location I. The detection of two coincident photons in I (event D_I) decides that a given run is going to contribute to the data for the Bell inequality test [20]. In order to avoid a detection-type loophole for the ion pairs, the timing has to be such that the detection or nondetection of a coincidence at I cannot have been influenced by the choices of measurement basis in A and B. That is, events C_A and C_B have to lie outside the backward light-cone of D_I. Figure 2 shows a situation where this constraint is tighter than the first one. However, since the coherence time of the ions is very long, it is always possible to increase the time delay between excitation and choice of basis, allowing the time between C_A (C_B) and D_A (D_B) to approach the limit of L/c.

We now discuss the practical implementation of the entanglement distribution and Bell test in more detail. To keep transmission losses as low as possible, it would be ideal to work with photons at the typical telecom wavelengths of 1.3 or 1.5 μm. Unfortunately it is difficult to find suitable ionic transitions at these wavelengths. However, suitable transitions in the visible and near-infrared range exist, where photon losses in optical fibers are still at an acceptable level, on the order of 1 dB/km [21]. This means that for a 10 km distance between Alice and Bob, both photons will reach the intermediate station in 10% of the cases. This distance corresponds to a maximum allowed time from choice of basis to completed detection of 33 μs.

Singly positively charged earth alkaline ions, which have only one electron outside a closed shell, are the most common in trapped ion experiments. For concreteness we focus on a possible implementation using $^{40}Ca^+$, whose usefulness in the quantum information context has been demonstrated in the recent experiments of Refs. [17,22–24]. However, the use of other ions should certainly not be excluded. The relevant levels of $^{40}Ca^+$ are shown in Fig. 3. The metastable $D_{5/2}$ and $D_{3/2}$ states have lifetimes of the order of 1 s. One could use one of the $P_{3/2}$ levels, say $m = -3/2$, to serve as the excited level e. This state can be prepared by first preparing the $|S_{1/2}, m = -1/2\rangle$ state by optical pumping [22], and then applying σ_- light at 393 nm. For s_1 and s_2 one could use the $m = -5/2$ and $m = -1/2$ sublevels of $D_{5/2}$, which are coupled to e by σ_- and σ_+ light at 854 nm (see Fig. 3). Decay from e to the $m = -3/2$ state of $D_{5/2}$ is impossible if the photon is emitted in the direction of the quantization axis. The rate for the $P_{3/2} \rightarrow D_{5/2}$ transition is of order 0.5×10^7/s [25]. Unwanted photons from $P_{3/2} \rightarrow S_{1/2}$ (393 nm) and from $P_{3/2} \rightarrow D_{3/2}$ (850 nm) can be avoided by spectral filtering with interference filters.

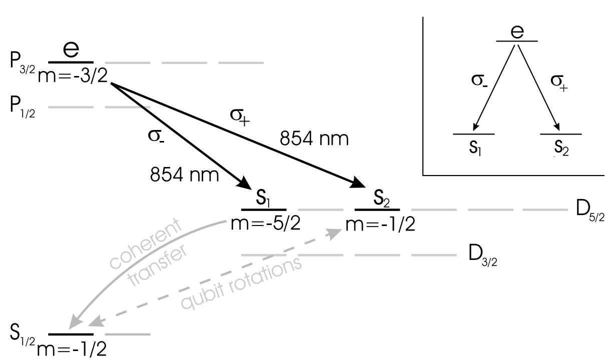

FIG. 3. Relevant levels of $^{40}Ca^+$ ions.

It is essential to be able to detect the states of the ions in different bases. The necessary rotations could be performed by first coherently transferring s_1 ($|D_{5/2}, m = -5/2\rangle$) to the $|S_{1/2}, m = -1/2\rangle$ state by applying a laser pulse at 729 nm as in Ref. [23] and then applying pulses of different length to the transition between that state and s_2 ($|D_{5/2}, m = -1/2\rangle$), corresponding to single-qubit rotations in the quantum computation scheme of Ref. [17]. These transformations can be performed in a few microseconds [23]. The detection could then proceed following Ref. [17], by using a cycling transition between the $S_{1/2}$ and $P_{1/2}$ levels (397 nm) [22]. The $P_{1/2} \rightarrow S_{1/2}$ transition rate is of the order of 1.3×10^8/s [22], which determines the number of photons that can be emitted during the cycling process. It does not seem unrealistic to detect 1% of these photons, if the collection efficiency is optimized. This would allow the detection of 30 photons in 23 μs, more than sufficient for unambiguous state discrimination. Sufficiently fast random switching between different measurement bases should not be difficult to implement using electro-optic or acousto-optic modulation [10,11]. The parameter that needs to be changed in the present context is the length of the pulses at 729 nm.

A cavity is likely to be required for the present scheme in order to achieve directional emission of the photons from the ions. To determine the expected enhanced emission into the cavity mode for the proposed $^{40}Ca^+$ implementation one has to take into account not only the coupling of the $P_{3/2} \rightarrow D_{5/2}$ transition to the cavity and the cavity decay rate, but also the fact that $P_{3/2}$ decays preferentially to $S_{1/2}$ [25]. A treatment following Ref. [28], but including this loss mechanism, gives the probability $p_{\rm cav}$ for a photon to be emitted into the cavity mode after excitation to e as

$$p_{\rm cav} = \frac{4\gamma\Omega^2}{(\gamma + \Gamma)(\gamma\Gamma + 4\Omega^2)}, \tag{2}$$

where $\gamma = 4\pi c/FL$ is the decay rate of the cavity, F is its finesse, L is its length, $\Omega = \frac{D}{\hbar}\sqrt{hc/2\epsilon_0\lambda V}$ is the coupling constant between the transition and the cavity mode, D is the dipole element, λ is the wavelength of the transition, V is the mode volume (which can be made as small as $L^2\lambda/4$ for a confocal cavity with waist $\sqrt{L\lambda/\pi}$), and Γ is the non-cavity-related loss rate, which is equal to 1.47×10^8/s in our case [25]. In order to maximize $p_{\rm cav}$, Ω should be as large as possible, and γ should be equal to

2Ω. The latter condition implies an optimal cavity finesse of $F = 19\,000$ for our system. Coupling of a trapped ion to a 2 cm cavity with higher finesse was recently demonstrated in Ref. [24]. The probability $p_{\rm cav}$ depends strongly on the cavity length. If one chooses $L = 3$ mm, which might still be compatible with the trap dimensions of Ref. [22], one finds $\gamma = 9.9 \times 10^6/\mathrm{s}$, leading to $p_{\rm cav} = 0.01$. For $L = 1$ mm one would get $p_{\rm cav} = 0.06$, while for $L = 1$ cm one gets $p_{\rm cav} = 0.001$. The above value for γ implies that the photon wave packets are about 100 ns long. Such a long coherence time makes it easy to achieve good overlap for the Bell state detection and also makes unwanted birefringence effects in the optical fibers negligible [21].

We now estimate the expected overall creation rate of two-ion entangled pairs. Following the above timing considerations, we assume that the ions are excited every 30 μs, corresponding to a rate of $33\,000/\mathrm{s}$. This has to be multiplied by $p_{\rm cav}^2$, by a factor close to 1 to describe coupling from the two cavities to fibers, by the probabilities for both photons to reach the intermediate station ($1/10$ for a 10 km distance) and to lead to a coincident detection ($\eta^2/2$, where we assume a detection efficiency $\eta = 0.7$). Multiplying all these factors, one finds an expected overall rate of the order of five pairs per minute, which would be sufficient for performing a Bell experiment in a few hours, and might also allow basic demonstrations of the quantum repeater protocol. Shorter cavities would be possible for microtraps [29], leading to greatly increased rates (e.g., three pairs per second for $L = 1$ mm).

In conclusion, a loophole-free test of Bell's inequalities should be possible with just a single entanglement creation step between two ions separated by a distance of order 10 km. Beyond that, the present scheme allows the creation of robust long-distance entanglement.

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Note added.—Since the submission of this work we have learned about recent related work on a loophole-free Bell test [30] and on entanglement creation by two-photon interference [31].

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