Driven Depinning of Strongly Disordered Media and Anisotropic Mean-Field Limits

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Extended systems driven through strong disorder are modeled generically using coarse-grained degrees of freedom that interact elastically in the directions parallel to the drive and slip along at least one of the directions transverse to the motion. In the limit of infinite-range elastic and viscous coupling this model has a tricritical point separating a region where the depinning is continuous, in the universality class of elastic depinning, from a region where depinning is hysteretic. Many of the collective transport models discussed in the literature are special cases of the generic model.

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Nonequilibrium transitions from stuck to moving states underlie the physics of a wide range of phenomena, from fracture and earthquake rupture to flux flow in type-II superconductors [1]. Various dynamical models have been proposed in different contexts. One class of models, overdamped *elastic* media pulled by an applied force F, has been studied extensively. These predict a nonequilibrium phase transition from a pinned state to a sliding state at a critical value F_T of the driving force. Generically, this depinning transition displays critical behavior as in equilibrium *continuous* transitions [2], with the medium's mean velocity v acting as the order parameter. In overdamped elastic media, the sliding state is unique and no hysteresis can occur [4]. Universality classes have been identified, which are distinguished, for example, by the range of interactions or by the periodicity (or nonperiodicity) of the pinning forces.

The elastic medium model is often inadequate to describe many systems which exhibit plasticity (e.g., due to topological defects in the medium) or inertial effects (underdamping). The dynamics in plastic systems can be inhomogeneous, with coexisting pinned and moving regions. The depinning transition may be discontinuous (first order), possibly with macroscopic hysteresis. Several mean-field models of driven extended systems with locally underdamped relaxation or phase slip have been proposed in the literature [1,5–10].

In this Letter, we present a model of driven disordered systems that encompasses several of the models discussed in the literature. This model incorporates the anisotropy of the sliding state in the plastic flow region that results either from flow along coupled channels oriented in the direction of drive (e.g., as in the moving smectic phase [11]) or in layered materials such as the high-T_c cuprate superconductors. We restrict ourselves to systems with a periodic structure along the direction of motion, such as charge density waves (CDWs), 2D colloidal arrays, and vortex lattices in type-II superconductors and consider only the dynamics of a scalar displacement field. This model uses coarse-grained degrees of freedom that are solidlike regions. These volumes slip relative to each other in the transverse dimensions, due to the presence of small scale defects (phase slips, dislocations, etc.) at their boundaries, but remain elastically coupled in the longitudinal dimensions. Our current results are for the mean-field limits, in the transverse dimensions at least, with transverse viscous interactions. The studies carried out so far of these types of models for finite range interactions [12,13] suggest that the mean-field approximation described here may give the correct topology for the phase diagram, although there will be corrections to the critical behavior in finite dimensions.

Anisotropic slip model.—Elastic models have their limits in describing real materials. Coppersmith argued that elastic models with weak disorder have arbitrarily large strains, as large rare regions with atypically low pinning result in large displacement gradients [14,15]. Though the elastic model breaks down, the low density of such regions may preclude its easy observation. Simulations, imaging, and noise experiments for moderate and strong disorder also indicate the breakdown of the elastic model [16–21]. These calculations and observations often show a sliding state that consists of coherent regions moving at different average velocities, with extended defect structures along their boundaries. The dynamics is strongly anisotropic, with slip occurring preferentially where the shear deformations are largest, at the borders of channels that are on average aligned with the direction of motion. Guided by this work, we propose an anisotropic description of the inhomogeneous dynamics.

We consider a $d = d_{\|} + d_{\perp}$-dimensional medium composed of $d_{\|}$-dimensional *elastic channels* coupled via interactions that allow for *slip* of the channels in the remaining $d_{\perp}$ directions. The system is driven by a uniform force F along one of the $d_{\|}$ directions. A cartoon of a $1 + 1$-dimensional system is shown in Fig. 1. We choose to discretize space in both transverse and longitudinal directions, using integer vectors i for the $d_{\|}$-dimensional intralayer index and ℓ for the $d_{\perp}$-dimensional layer index. The local displacement along the direction of motion is $\phi_{\ell}^{i}(t)$. Assuming overdamped dynamics, the equation of motion in the laboratory frame [22] is

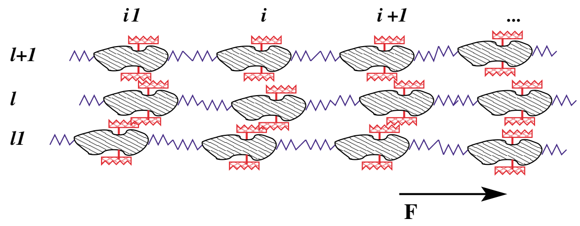

FIG. 1 (color online). A two-dimensional realization of the anisotropic driven medium. Degrees of freedom are labeled by discrete indices ℓ (parallel to driving force F) and i (transverse to F). Each degree of freedom interacts with its neighbors via elastic couplings in the longitudinal direction and via viscous or similar slip couplings in the transverse direction.

$$\dot{\phi}^{i}_{\ell}(t) = K\sum_{\langle j\rangle}(\phi^{j}_{\ell} - \phi^{i}_{\ell}) + F + h^{i}_{\ell}Y(\phi^{i}_{\ell} - \psi^{i}_{\ell}) + \sigma^{\alpha}_{\ell,i}, \tag{1}$$

where the dot denotes a time derivative (time is scaled so that the damping constant is unity) and $\langle j\rangle$ are the nearest neighbors to i. The first term on the right-hand side represents an elastic intrachannel coupling of strength K. The third term is the pinning force. The function $Y(x)$ has period 1, and the ψ^{i}_{ℓ} are random phases chosen independently and uniformly in $[0, 1)$. The random pinning strengths h^{i}_{ℓ} are chosen from a probability distribution $\rho(h)$. Finally, $\sigma^{\alpha}_{\ell,i}$ represents an interaction (of type α) that allows for slips of neighboring channels. In this Letter, we assume a linear viscoelastic (V) stress-strain relation,

$$\sigma^{V}_{\ell,i} = \int_{-\infty}^{t} dt'\sum_{\langle m\rangle}J^{\alpha}_{m\ell}(t - t')[\phi^{i}_{m}(t') - \phi^{i}_{\ell}(t)], \tag{2}$$

where $\langle m\rangle$ indexes the layers neighboring ℓ. The stress transfer function [1,5] $J^{\alpha}_{m\ell}$ is generally nonlocal in space and/or time. We focus on a simplified version of the viscoelastic model where the local slip force is a purely viscous coupling of strength η : $\sigma^{V}_{\ell,i} = \eta\sum_{\langle m\rangle}(\dot{\phi}^{i}_{m} - \dot{\phi}^{i}_{\ell})$. With this coupling, Eq. (1) is a simplified "viscous slip" form of the hydrodynamics of a driven viscoelastic medium that flows in response to large-scale shear, but responds elastically within the layers to long-wavelength compressions [23]. Gradients of the displacement *transverse* to the channels or layers correspond to *shear* deformations arising from defect structures between the channels. Some justification for modeling the slip that results from moving defects as viscous couplings comes from work [24] showing that that the hydrodynamics of a two-dimensional crystal with free dislocations is identical to that of a viscoelastic fluid. In contrast, gradients of displacement *longitudinal* to the drive mainly yield *compressional* deformations (exactly so when $d_{\parallel} = 1$). The elastic response to long-wavelength compressions in fluids is intimately related to the fact that compressions are associated with fluctuations in the conserved density, suggesting that the inclusion of compressional forces is necessary to describe the dynamics of systems with a conserved number of particles, such as vortex lattices. The coarse-grained model, Eq. (1), allows us to investigate the competition between elastic interaction and plastic flow in controlling the topology of the nonequilibrium phase diagram.

Mean-field treatment.—One mean-field approximation for the viscous slip model is obtained by taking the interactions to be infinite ranged in both the transverse and longitudinal directions. Each displacement then couples to others only through the mean velocity, $v = N^{-1}\sum\dot{\phi}^{i}_{\ell}$, and the mean displacement, $\overline{\phi} = N^{-1}\sum\phi^{i}_{\ell}$. We look for solutions with stationary velocity: $\overline{\phi} = vt$. As all displacements ϕ are coupled, they can be indexed by their disorder parameters ψ and h, rather than the spatial indices. The mean-field dynamics is governed by the equation

$$(1 + \eta)\dot{\phi}(\psi, h) = K(vt - \phi) + F + \eta v + hY(\phi - \psi). \tag{3}$$

When $K = 0$, the mean-field velocity is determined by the self-consistency condition $\langle\dot{\phi}(h)\rangle_{h} = v$, where the subscript h indicates an average over the distribution of pinning strengths $\rho(h)$. When $K \neq 0$, the mean-field velocity is found by imposing $\langle\phi(h, \psi) - vt\rangle_{h,\psi} = 0$.

It is useful to review the case where $\eta = 0$ and $K \neq 0$. In this limit, Eq. (3) reduces to the mean-field theory of a driven elastic medium worked out by Fisher and collaborators [25]. For the piecewise harmonic pinning $Y(\phi) = 1/2 - \phi$ for $0 \leq \phi \leq 1$, no moving solution exists for $F < F_{T} = \langle h^{2}/[2(K + h)]\rangle_{h}$. Just above threshold the mean velocity has a universal dependence on the driving force, with $v \sim (F - F_{T})^{\beta}$. The critical exponent β depends on the shape of the pinning force: $\beta = 1$ for the piecewise harmonic force and $\beta = 3/2$ for generic smooth forces. Using a functional RG expansion in $4 - \epsilon$ dimensions, Narayan and Fisher [25] showed that the discontinuous force captures a crucial intrinsic discontinuity of the large-scale, low-frequency dynamics, giving the general result $\beta = 1 - \epsilon/6 + \mathcal{O}(\epsilon^{2})$, in accord with numerical simulations for $d = 2, 3$ [3,26]. For simplicity and to reflect the "jerkiness" of the motion in finite-dimensional systems at low velocities, we use piecewise harmonic pinning.

When $\eta > 0$, the nature of the depinning differs qualitatively from the $\eta = 0$ case in that hysteresis in the dynamics can take place. This can be shown starting from the $\eta = 0$ solutions $v_{\eta=0}(F)$, with $v_{\eta=0}$ a well-defined function of F. The solution for general η can be found by substituting the effective drive force $G = F + \eta v$ for F in the $v_{\eta=0}(F)$ relation and scaling velocity down by $1 + \eta$. The linear transformation $F = G - \eta v$ then gives the general $v(F)$ curve. One result is that when $1 + \eta^{-1} < \max_{F}(dv_{\eta=0}/dF)$, the v vs F curve is multivalued. Two cases can now be considered. The viscous-elastic case $K \neq 0$ has quite different behavior from the purely viscous case $K = 0$, due to the distinct self-consistency conditions.

For the purely viscous case $K = 0$ (the $\tau = 0$ limit of the model in Ref. [5]), periodic solutions of period $T(h, G)$ are found for a single particle moving in a pinning force of strength h under the effective drive G. Self-consistency implies that $v = \langle [T(h, G)]^{-1} \rangle_h$, giving

$$v = \frac{1}{1+\eta} \int_0^{2G} dh\, \rho(h) h \left[\ln\left(\frac{G + h/2}{G - h/2} \right) \right]^{-1}. \quad (4)$$

The behavior now depends on the shape of $\rho(h)$. For distributions that vanish below a value $h_{\min} > 0$, no moving solutions exist for $F < h_{\min}/2$. In the extreme case of uniform pinning, $\rho(h) = \delta(h - h_0)$, the depinning is hysteretic for all nonzero values of η, with a jump from $v = 0$ to finite v at $F = h_0/2$. For the more general broad distribution with nonvanishing support at $h = 0$, no pinned solution exists for $F > 0$: while the bulk of the degrees of freedom are pinned at small F, weakly pinned ϕ can respond to small drives. Typical v vs F curves for $\rho(h) = e^{-h}$ are shown in Fig. 2(a). The lines in Fig. 2(b) represent the critical forces where there is a macroscopic jump in the average velocity (dynamic hysteresis). For $\eta > \eta_c$ and $F_\uparrow > F > F_\downarrow$ "fast" and "slow" sliding states coexist. The critical point (η_c, F_c) in Fig. 2(b) is in the universality class of the liquid-gas transition (and of the field-driven random field Ising model [12]). Near the critical point the mean velocity has universal scaling, $v - v_c \sim (\eta - \eta_c)^{1/\delta} \sim (F - F_c)^\beta$, with $v_c = v(F_c, \eta_c)$, and $\beta_{MF} = 1/2$, $\delta_{MF} = 3$. The point (η_c, F_c) survives in finite dimensions [12], with exponents distinct from their mean-field values.

For finite long-time elasticity, i.e., when $K \neq 0$, the behavior changes dramatically. The elastic forces or particle conservation enforce a uniform time-averaged velocity for all degrees of freedom. In this case the long-time uniform-v solution to Eq. (3) is $\phi(h, \psi, t) = vt + \tilde{\phi}(h, \psi, t)$ with $\tilde{\phi}(h, \psi, t)$ of period $1/v$ in t. The self-consistency condition is $\langle \tilde{\phi}(\psi, t) \rangle_{\psi,h} = 0$. The explicit solution can be obtained for the piecewise linear pinning force. (In the mean-field limit the nature of the phase diagram depends some on the shape of the pinning potential.) There is now a pinned phase that is stable [27] for $F < F_T = \langle h^2/[2(K + h)] \rangle_h$. One then solves

$$\delta F = \frac{\eta_c - \eta}{1 + \eta_c} v + \left\langle \frac{h^2}{(K+h)} \frac{1}{e^{(K+h)/[(1+\eta)v]} - 1} \right\rangle_h, \quad (5)$$

to find the mean velocity in the sliding state ($0 < \delta F \equiv F - F_T$), where $(\eta_c + 1)^{-1} = \langle h^2/[(K + h)^2] \rangle_h$. The v-Fcurves and a phase diagram are shown in Figs. 2(c) and 2(d) for $\rho(h) = e^{-h}$. There is now a *tricritical point* at $(\eta_c, F_c = F_T)$. For $\eta < \eta_c$, a continuous depinning transition at F_T separates a pinned state from a sliding state with *unique* velocity $v \sim (1 + \eta_c)[(F - F_T)/(\eta_c - \eta)]$, giving $\beta = 1$ in MFT. In finite dimensions, this transition is likely to remain in the same universality class as the depinning of an elastic medium ($\eta = 0$): numerical studies and analysis by Schwarz and Fisher [13] of a model with local viscouslike terms show that there is no hysteresis for small slip coupling and that the depinning transition exponents are the same as without slip coupling. In our mean-field example, the linear response diverges at η_c, $v(\eta = \eta_c) \sim 1/\ln(F - F_T)$. For $\eta > \eta_c$ there is hysteresis with coexistence of stuck and sliding states.

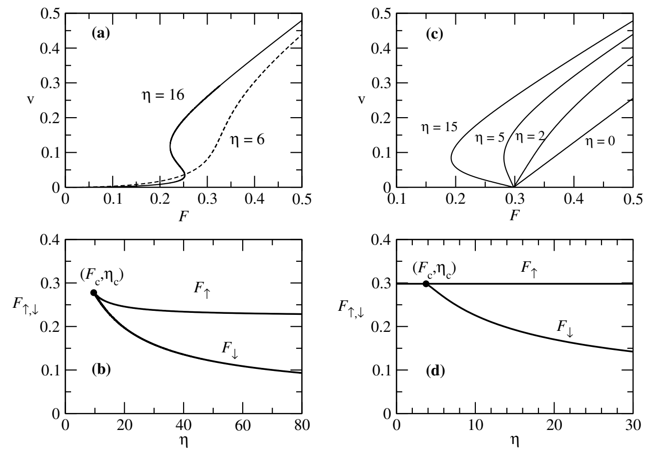

FIG. 2. Velocity curves and phase diagrams for the mean-field slip model. Part (a) shows v vs F curves for the purely viscous case and $\rho(h) = e^{-h}$, for two values of viscous coupling η. Part (b) shows the phase diagram from (a), with curves indicating jumps in the adiabatic response (i.e., $dv/dF \to \infty$) when the force F is increased from $F = 0$ $[F^\uparrow(\eta)]$ and decreased from a sliding state $[F^\downarrow(\eta)]$. The critical point $(\eta_c, F_c) \approx (9.61, 0.278)$. Part (c) shows example v vs F curves when there is elasticity, $K = 1$, for piecewise linear pinning forces and $\rho(h) = e^{-h}$. The curves are again continuous for $\eta < \eta_c$, but have a depinning transition at F_T, with critical behavior identical to the $\eta = 0$ case. When $\eta > \eta_c$, the depinning is abrupt and the v-F curves are hysteretic. Part (d) summarizes the $K = 1$, $\rho(h) = e^{-h}$ results in a phase diagram, indicating depinning from an initial pinned state at $F^\uparrow = F_T$ and repinning from a sliding state at a lower force $F^\downarrow(\eta)$. The point $(\eta_c, F_T) \approx (3.74, 0.298)$ is a tricritical point.

Relations to other models.—In the absence of viscous coupling ($\eta = 0$), Eq. (1) reduces to the conventional $d_\parallel$-dimensional phase-only model of driven CDWs. When $K = 0$ and η is finite, the model describes a viscous fluid in a disordered background [5].

The mean-field limit of the anisotropic slip model for finite K maps onto a periodic-pinning version of the infinite-range limit of the stress overshoot model studied by Schwarz and Fisher [6,13]. The dynamics of crack fronts in brittle materials can be dominated by local inertial effects, in which the motion of a crack segment creates a transient stress on other segments. In the infinite-range limit the stress transfer yields a global coupling of strength M between the mean displacements at two different times, analog to the effective force per unit velocity η in our model. The behavior we conjecture for finite dimensions is identical to that obtained by Schwarz and Fisher.

The viscous model ($\eta \neq 0$) with finite K is also closely related to a model of sliding CDWs that incorporates the coupling of the CDW to normal carriers by adding a global velocity coupling [9,10,28] to the conventional Fukuyama-Lee-Rice model. The precise relationship is obtained by considering a mean-field limit $d_\perp \to \infty$ with fixed $d_\parallel$. In this limit, each degree of freedom is coupled equally by slip to all other layers, but elastically coupled only to its nearest neighbors in the longitudinal direction. The equation of motion is then

$$(1+\eta)\dot{\phi}^i\ell = K\sum_{\langle j\rangle}(\phi\ell^j - \phi\ell^i) + F + \eta v + h\ell^i Y(\phi\ell^i - \psi\ell^i). \quad (6)$$

Equation (6) is identical to the equation of motion of a purely elastic CDW with friction $1+\eta$ and an effective driving force $F + \eta v$. One can then obtain the velocity-force characteristics consistent with Ref. [10] simply by translating the analytic results [25] for CDWs. Near threshold, we obtain $v \sim (F + \eta v - F_T)^\beta$, with $\beta = 1 - (4-d_\parallel)/6 + \mathcal{O}[(4-d_\parallel)^2]$. The transition is always hysteretic: the medium depins at $F^\uparrow = F_T$ when the force is ramped up and repins at a lower value, $F^\downarrow \approx F_T - [(1/\beta) - 1][\beta\eta/(1+\eta)]^{1/(1-\beta)}$, when the force is ramped down from the sliding state. This hysteresis for all $\eta > 0$ appears to be a consequence of the global nature of the coupling, in contrast with the results of Schwarz and Fisher for local slip coupling [13].

One can include phase-slip couplings σ^{PS} periodic in ϕ. The anisotropic model can describe the driven dynamics of intrinsically layered systems, such as vortex lattices in cuprate materials. Hysteresis depends on the strength of the correlations in the (weak-coupling) layered direction, as seen in simulations [18]. An anisotropic CDW model was studied in this context by Nattermann and Vinokur [7]. These authors modeled the transverse slip as a nonlinear coupling that is periodic in the local CDW phase differences; this allows for phase slips, i.e., the collapse of the CDW amplitude between coherent regions [8].

Summary.—We have proposed that the plastic flow of extended systems in a disordered background can be modeled using a general anisotropic approach that includes many specific physical systems as special cases. We have discussed the behavior of this model in a mean-field limit. When the degrees of freedom are allowed to move at distinct velocities, a pinned phase is generally not present, though there is hysteresis in the current-drive relation. Even when fluidlike shear takes place, particle conservation gives a sharp depinning transition in flow that takes place along channels. Our future goals include studying the model in finite dimensions and establishing the connection between the parameters of the model and physical parameters of a given experiment.

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