Granular Medium under Vertical Tapping: Change of Compaction and Convection Dynamics around the Liftoff Threshold

P. Philippe and D. Bideau

G.M.C.M., Batiment 11A, Campus de Beaulieu, Université de Rennes I, F-35042 Rennes, France

(Received 24 December 2002; published 5 September 2003)

A granular medium submitted to vertical tapping reveals simultaneously compaction and convection. The two phenomena are directly coupled and their dynamics can be quantified by a characteristic compaction time and by an estimation of the convective downhill speed along the wall. A remarkable change of behavior is observed around the liftoff acceleration threshold of the whole packing, with a drastic slowing down of both dynamics below this threshold. Above it, a collective shock wave densifies the packing at each tap, whereas, below it, cumulative localized rearrangements will compact the entire system in the long time range.

DOI: 10.1103/PhysRevLett.91.104302 PACS numbers: 45.70.Cc, 45.70.Mg, 81.05.Rm

When continuously submitted to weak mechanical perturbations, a granular medium slowly compacts. This gentle compaction, which is principally ruled by the steric constraint between the grains, is of great theoretical interest in the context of statistical physics. Obviously classical thermodynamics does not apply to powders. First, granular matter is well known to be an athermal system since the energy $k_B T$ is insignificant compared to the gravitational energy of a macroscopic grain. Second, the interactions between grains are dissipative (frictional sliding, inelastic collisions) and, when a granular medium is excited, it quickly settles down in a metastable static configuration where it remains indefinitely trapped. But continuous energy supply, such as mechanical shaking, can "thermalize" the medium and allows one to study the relaxation of this atypical out-of-equilibrium system which is believed to be fairly similar to glass forming materials [1,2]. This analogy is based on the geometric hindrance which controls the dynamical evolution of the structure in both cases. However, some distinctions must be emphasized, especially concerning the nature of the energy injection [3]: whereas thermal agitation is isotropic and random in time, mechanical shaking has specific period and orientation, induces time-dependent anisotropic motion in the packing, and, often, produces internal shock waves.

In this Letter, we present experiments with a view to testing and better characterizing this analogy. The first experiments of this type have been carried out in Chicago [4]: an empirical law in the inverse of the logarithm of the number of taps was proposed to describe the evolution of the volume fraction and some typical glassy behaviors have also been observed. But, the very small horizontal gap between the lateral walls ($D/d \sim 10$) made boundary effects very strong, giving rise to ordering of the beads near the wall and to values of the volume fraction significantly above the random-close-packing limit of identical hard spheres (approximately 64%). To restrict the influence of ordering effect near the wall, we work with a larger horizontal gap between the lateral walls ($D/d \sim 100$). The resulting compaction is quite original and has been precisely quantified in a previous study [5]. Moreover, and contrary to the Chicago's experiments, convection is also observed in the whole packing and its strength is estimated through the displacement of dyed tracers near the wall. These new experimental results reveal quantitatively that the dynamics of both phenomena are directly coupled and appear extremely dependent on the exact way of supplying energy in the packing.

Nearly monodisperse glass beads of diameter $d = 1$ mm are filled to a height of about 10 cm in a glass cylinder (diameter $D \approx 10$ cm). The whole container is shaken at regular intervals ($\Delta t = 1$ s) by vertical taps. Each tap is controlled by an entire cycle of a sine wave at a fixed frequency $f = 30$ Hz: $V_{\mathrm{com}}(t) = V_0(1 - \cos 2\pi f t)$ for $0 < t < 1/f$ and $V_{\mathrm{com}}(t) = 0$ elsewhere. This applied voltage is connected to an electromagnetic exciter (LDS V406) which induces a vertical displacement to the moving part supporting the container and the beads. The resulting motion of the whole system, monitored by an accelerometer at the bottom of the container, is more complicated than a simple sine wave: at first the system undergoes a positive acceleration followed by a negative peak of acceleration with a minimum equal to $-\gamma_{\max}$. After the applied voltage stops, the system relaxes to its normal position. A simple mechanical system gives a very good approximation of this displacement: in addition to the applied electromagnetic force, the intrinsic suspension of the exciter is equivalent to a spring of stiffness $k = 12.3\ \mathrm{N\,m^{-1}}$ and a viscous friction force is used to model eddy-current braking. When $\gamma_{\max}$ is large enough, the beads packing lifts off from the bottom of the container and achieves flight until it crashes back to the bottom. As in previous studies [4], this negative peak acceleration is used to parametrize the tap intensity by the dimensionless acceleration $\Gamma = \gamma_{\max}/g$ (with $g = 9.81\ \mathrm{m\,s^{-2}}$). The threshold Γ^*, above which there is a collective liftoff of the packing, is slightly greater than 1 because the friction between the packing and the lateral wall tends to oppose this liftoff. In our setup, it is estimated to $\Gamma^* \approx 1.2$ and does not seem to depend significantly on Φ, the average volume fraction of the packing.

The experiments are realized as follows: starting from a reproducible loose packing ($\Phi_0 = 58.3\% \pm 0.3\%$), sequences of 10 000 to 100 000 taps are carried out with a given acceleration Γ. The average volume fraction in the bulk, Φ, is estimated by the transmission ratio of a γ-ray beam through the packing. It has been pointed out in a previous work [5] that, for $\Gamma > \Gamma^*$, the dynamics of the compaction is strongly consistent with a stretched exponential law, popularized years ago by Kohlrausch [6] and later by Williams and Watts [7], and which we will refer to as KWW law: $\Phi(t) = \Phi_{ss} - (\Phi_{ss} - \Phi_0)\exp[-(t/\tau)^\beta]$. The characteristic time τ of this law gives a precise quantification of the compaction dynamics. Its dependence on the tapping acceleration Γ is plotted in Fig. 1. The part of this curve corresponding to the highest values of Γ comes from experiments already presented in an earlier work [5]. An Arrhenius's type law describes reasonably well the experimental dynamics with the parameters $\Gamma_0^\tau \approx 7.2$ and $\ln\tau_0 \approx 0.15$:

$$\tau(\Gamma) = \tau_0 \exp\left(\frac{\Gamma_0^\tau}{\Gamma}\right). \tag{1}$$

Note that Arrhenius-like forms have already been reported in a previous study on density fluctuations [4].

In Fig. 1, we also present new experimental points obtained in the domain $\Gamma < \Gamma^*$. In this regime, compaction is still observed but with a much smaller dynamics. For most of the experiments, only the beginning of the relaxation is accessible, and there is no evidence that a stretched exponential law still describes the whole compaction dynamics. Nevertheless, we can reasonably postulate the validity of the KWW law and thus obtain the characteristic time τ. In this case, Φ_{ss} cannot be directly measured from the final values of the volume fraction. But, according to the dependence of Φ_{ss} on Γ presented in the inset of Fig. 1, we can extrapolate Φ_{ss} between 62.5% and 63%. The new behavior obtained for the smallest values of Γ is still possibly compatible with an Arrhenius law but with very different values of the two parameters of Eq. (1): $\Gamma_0^\tau \sim 24$ and $\ln\tau_0 \sim -14$.

During the compaction process, convection is also observed and points out two different regimes. The crossover threshold appears at $\Gamma_c \approx 2$ and corresponds notably to a significant change in the dependence of Φ_{ss} on Γ (see the inset of Fig. 1). Above Γ_c, the free surface heaps up moderately and finally takes on a flat conical shape, probably brought about by a nearly toroidal convection roll which is the expected form of the convective flow in a cylindrical geometry with high enough accelerations [8]. Below Γ_c, the free surface of the packing progressively slopes along a preferred orientation which indicates a spontaneous breaking of the cylindrical symmetry on behalf of a plane symmetry (see Fig. 2). The steady-state slope of the inclined plane tends to increase when Γ is decreased but the time required for this to happen becomes extremely long. These sorts of free surface instabilities have been observed previously; see, for instance, Ref. [9]. As shown in Fig. 2, the convective flow, observed through the displacements of dyed tracers near the wall, is asymmetrical with a fastest zone situated below the bottom of the inclined free surface. The same experiments above Γ_c show a much less asymmetrical flow. Briefly, we believe that a slight vertical bias (less than 0.5°), inherent to the setup, gives rise to an asymmetrical mass flow and to an inclination of the free surface which, at low accelerations, cannot be counterbalanced by any surface avalanche induced by the taps.

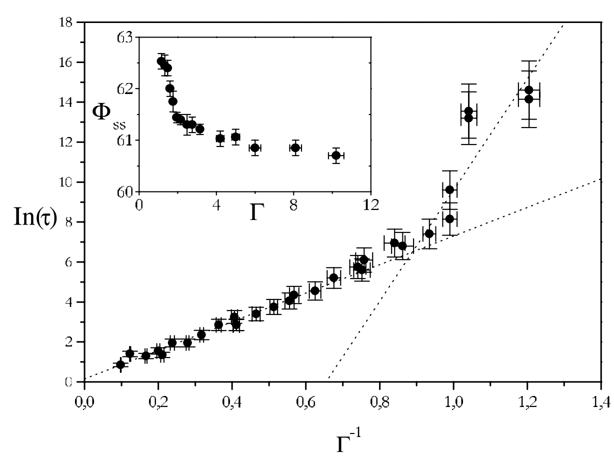

FIG. 1. Dependence of the compaction time τ on the inverse of the acceleration Γ. The dotted lines represent the two Arrhenius laws. Inset: variation of the final volume fraction Φ_{ss} with Γ in cases where a steady state is actually reached.

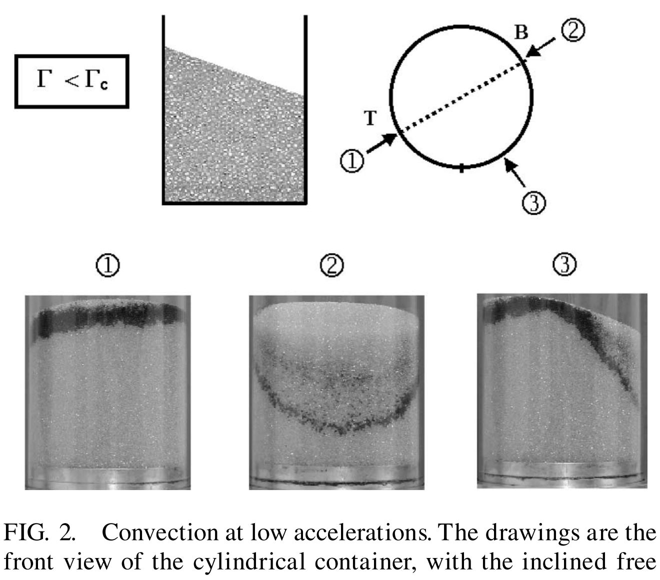

FIG. 2. Convection at low accelerations. The drawings are the front view of the cylindrical container, with the inclined free surface, and the above view, with the symmetrical plane in dotted line (T and B denote the top and the bottom of the surface). The photographs correspond to the three different views indicated on the upper drawing.

The convection strength is evaluated through the displacement of a ring of dyed tracers initially placed at the periphery of the free surface. The downward velocity near the wall is deduced from the maximum cumulative displacement of the tracers. It progressively decreases as the packing compacts and finally reaches a steady-state value V_{ss}. But, in the range $\Gamma < \Gamma^*$, a significant displacement of the tracers is obtained only after a huge number of taps and the final steady state is absolutely not reached. In this case, it is only possible to roughly estimate the downward velocity by the ratio $V = D_{\max}/N$, where $D_{\max}$ is the maximal cumulative displacement in bead diameters and N the number of taps. Moreover, to save time, we also worked with continuous vibration instead of tapping. For each Γ, the convective intensity was estimated by V with appropriate error bars taking into account the dependence of V on the different choices of N and $D_{\max}$. Although these errors can be quite large on the linear scale, they remain fairly reasonable on a logarithmic scale used in Fig. 3(a). For $\Gamma > \Gamma^*$, the values of V are obtained whether after vibration or tapping sequences without any appreciable change and are directly linked to the steady-state velocities V_{ss} which are easily measured in this case. It must be pointed out that the velocities V can become extremely small (down to displacement of about one diameter after more than 100 000 cycles). But, despite this very slight intensity, the convective flow can induce spectacular effects in the long time range such as the inclination of the free surface.

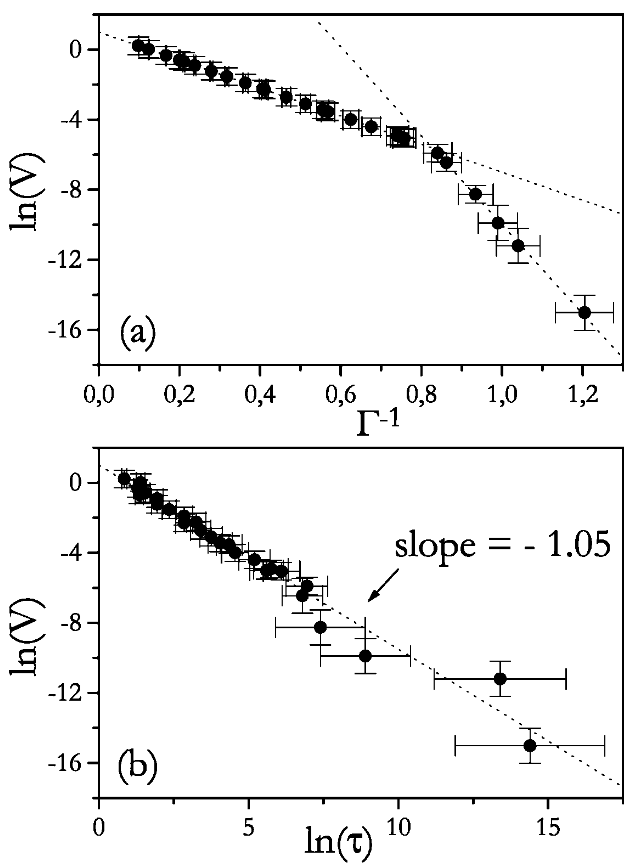

FIG. 3. (a) Dependence of the convection intensity V with the inverse of the acceleration parameter Γ (the two Arrhenius laws are plotted in dotted line). (b) Direct relation between the convection strength V and the compaction time τ.

As illustrated in Fig. 3(a), the dependence on Γ of the convection velocity V is quite consistent with two distinct Arrhenius type behaviors above and below Γ^*:

$$V(\Gamma) = V_0 \exp\left(-\frac{\Gamma_0^V}{\Gamma}\right). \tag{2}$$

Above Γ^*, the parameters are $\Gamma_0^V \approx 8.0$ and $\ln V_0 \approx 1.0$. Below Γ^*, there is a sharp deviation from the previous Arrhenius law with a significant slowing down; the corresponding parameters are $\Gamma_0^V \approx 25.4$ and $\ln V_0 \approx 15.4$.

In all these Arrhenius laws obtained for convection as well as for compaction dynamics, Γ appears to play a role directly equivalent to a temperature. But this is not systematic and, for instance, in the case of larger beads, Eqs. (1) and (2) are no longer applicable and must be modified.

The very similar dependence of compaction and convection on the nondimensional acceleration Γ points out a remarkable change of behavior around Γ^*, in the sense of a huge slowing down of both dynamics, as well as a strong coupling also illustrated by a power law with an exponent close to 1 between V and τ [see Fig. 3(b)].

As in glasses, where the structural relaxation time, or the viscosity, is exponentially dependent on the temperature, the compaction time and the inverse of the convection intensity are also exponentially dependent on the strength of the mechanical driving. This result confirms and reinforces the analogy between slightly driven granular media and glass forming materials. Nevertheless, whereas the use of temperature is obvious in glasses, the choice of a quantity able to characterize correctly the agitation of the grains is not clear and depends directly on the nature of the driving, that is to say, on the way of supplying energy to the grains.

Above Γ^*, each tap induces a flight of the whole packing as can be seen in Fig. 4 (pictures are extracted from a fast camera movie with larger beads to improve visualization without significant qualitative change). During its flight, the packing slightly dilates and its maximal altitude is quite small. It means that most of the contacts are maintained and explains why the packing keeps the structural memory of its previous configuration. The more significant structural modifications are likely to occur when the packing crashes back on the bottom of the container. Then we observe a shock wave which quickly propagates upward through the packing ($\Delta t \sim$ 10 ms) and which might densify collectively the grains in each horizontal layer (the horizontal dotted line in Fig. 4 highlights this overall compaction). Moreover, as the flight is almost symmetrical, the total effect of friction at the wall might be negligible during the flight and most of the convective flow is then also induced by the

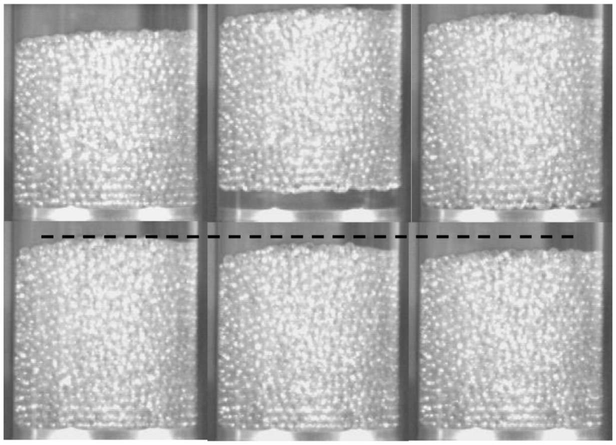

FIG. 4. Pictures of a 5 mm beads packing during a tap ($\Gamma = 5.95$ and $f = 30$ Hz): liftoff ($t = 0$), maximal altitude of the flight ($t = 28$ ms), beginning of the crash landing ($t = 60$ ms), intermediate positions ($t = 70$ ms and $t = 80$ ms), and end of the collision (the packing is completely at rest) ($t = 90$ ms).

shock wave. The kinetic energy of the packing just before its crash appears as the quantity able to characterize the amount of energy used to compact the grains during the propagation of the shock wave. Rather than the peak acceleration, the maximal velocity of the tap seems mostly linked to this kinetic energy and might be a more pertinent parameter than Γ.

Below Γ^*, a repetitive process of collective densification is no longer invoked since there is no coherent motion of the whole packing during a tap (or a vibration). The way of supplying energy to the grains is more subtle and difficult to observe. We think that local instabilities at some contacts are induced by the taps and give rise to a succession of small independent displacements (contact losses or little slip motions due to a local mobilization of the friction) which could be fairly similar to the "precursor" events observed before a surface avalanche [10]. This idea is in agreement with the diffusive agitation underscored in weakly vibrated granular media [11]. Moreover, this destabilization process might be activated near the wall. Therefore, in the long time range, these slight dynamical rearrangements can cumulatively develop a structural densification in the whole packing and induce a global convective flow. This energy supply is definitely less efficient than above Γ^* and explains the drastic slowing down of both compaction and convection dynamics.

The coupling between compaction and convection is hard to clarify experimentally. Nevertheless, we can propose the following interpretation in the case $\Gamma > \Gamma^*$. The dilation of the packing during its flight might be directly linked to its previous static volume fraction and it also controls the intensity of the bead displacements in the bulk when the packing crashes back, that is to say, the convection strength as well as the compaction rate. So, as the packing gets more compact, both phenomena progressively weaken until the compaction associated with the shock-wave dissipation simply counterbalances the flight dilation of the packing. Convection consequently reaches a final steady state with a given volume fraction Φ_{ss}.

In conclusion, the dynamics of a weakly excited granular medium was evaluated through the compaction time and through the intensity of the convective flow in the bulk. Both phenomena appear directly linked and reveals Arrhenius-like dependences on the driving strength Γ. Moreover, this glassylike behavior seems directly connected to the exact nature of the mechanical agitation. When the acceleration is high enough to impose a liftoff of the whole packing, energy is supplied to the grains during the crash landing of the packing after its flight: an upward shock wave induces a collective densification of the packing, layer by layer. Below the liftoff threshold, energy is much less efficiently supplied to the grains through small and localized instabilities. With the ambitious aim of developing a general description of dense granular media as a model for an out-of-equilibrium system, more quantitative characterizations following ours are strongly needed to further the understanding of the glassy dynamics in granular materials.

We are grateful to Y. Khidas, P. Richard, and S. Nagel for their remarks and suggestions on the manuscript, and to S. Bourles and P. Chasle for technical assistance.

[1] S. F. Edwards and R. B. S. Oakeshott, Physica (Amsterdam) **157A**, 1080 (1989); A. Mehta and S. F. Edwards, Physica (Amsterdam) **157A**, 1091 (1989).
[2] M. Nicodemi, A. Coniglio, and H. J. Herrmann, Phys. Rev. E **55**, 3962 (1997); J. Kurchan, J. Phys. Condens. Matter **12**, 6611 (2000); A. J. Liu and S. R. Nagel, Nature (London) **396**, 21 (1998).
[3] E. Caglioti, A. Coniglio, H. J. Herrmann, V. Loreto, and M. Nicodemi, Physica (Amsterdam) **265A**, 311 (1999).
[4] J. B. Knight, C. G. Fandrich, C. N. Lau, H. M. Jaeger, and S. R. Nagel, Phys. Rev. E **51**, 3957 (1995); E. R. Nowak, J. B. Knight, E. Ben-Naim, H. M. Jaeger, and S. R. Nagel, Phys. Rev. E **57**, 1971 (1998).
[5] P. Philippe and D. Bideau, Europhys. Lett. **60**, 677 (2002).
[6] R. Kohlrausch, Pogg. Ann. Phys. Chem. **91**, 179 (1854).
[7] G. Williams and D. C. Watts, Trans. Faraday Soc. **66**, 80 (1970).
[8] J. B. Knight, E. E. Ehrichs, V. Y. Kuperman, J. K. Flint, H. M. Jaeger, and S. R. Nagel, Phys. Rev. E **54**, 5726 (1996).
[9] P. Evesque and J. Rajchenbach, Phys. Rev. Lett. **62**, 44 (1989); C. Laroche, S. Douady, and S. Fauve, J. Phys. (Paris) **50**, 699 (1989).
[10] L. Staron, J. P. Vilotte, and F. Radjai, Phys. Rev. Lett. **89**, 204302 (2002).
[11] G. D'Anna and G. Gremaud, Phys. Rev. Lett. **87**, 254302 (2001); Nature (London) **413**, 407 (2001).