Proton-Number Fluctuation as a Signal of the QCD Critical End Point

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We argue that the event-by-event fluctuation of the proton number is a meaningful and promising observable for the purpose of detecting the QCD critical end point in heavy-ion collision experiments. The long range fluctuation of the order parameter induces a characteristic correlation between protons which can be measured. The proton fluctuation also manifests itself as anomalous enhancement of charge fluctuations near the end point, which might be already seen in existing data.

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The event-by-event fluctuations in heavy-ion collisions carry information about the degrees of freedom of the created system and their correlations [1]. In particular, thermodynamic properties of QCD can be inferred from event-by-event fluctuation measurements [2–7].

Of particular interest are fluctuations originating from the QCD critical end point [3,4,8–12]. Since the fluctuation of the order parameter induces characteristic correlations among particles, in particular, pions, it is expected that the end point affects the event-by-event fluctuations of certain observables in a nontrivial way [3,4,13].

Here we discuss a new observable which may serve as a signal of the end point; the event-by-event fluctuation of the net proton number, i.e., the number of the protons minus the number of antiprotons observed [14].

Our starting point is the fact that the baryon number susceptibility χ_B [15–20] diverges at the critical end point [3,9,12,21]. χ_B is related to the average magnitude of the fluctuation δB of the baryon number:

$$\chi_B = \frac{1}{VT}\langle(\delta B)^2\rangle, \tag{1}$$

where V and T are the volume and the temperature. The divergence of χ_B is a consequence of the fact that the critical point is the end point of a line of first order phase transitions, which are characterized, in particular, by a jump in the baryon number density B/V [22]..

If, in a heavy-ion collision experiment, we could measure all the baryons, the enhancement of the event-by-event fluctuation of the baryon number in a given subvolume would be a signature of the end point. However, about one half of the emitted baryons are undetected neutrons which certainly contribute to the fluctuation of the baryon number. To what extent does the proton number fluctuation alone reflect the divergence of χ_B?

This Letter is devoted to clarifying where, in the observed quantities, the divergence occurs and advocating the proton number fluctuation as a sensible and promising observable for the search of the critical point in the heavy-ion experiments.

In this work, we confine ourselves to equilibrium thermodynamic fluctuations. Various important issues such as the nonequilibrium evolution of the fluctuations will be (and some already have been) studied separately.

For simplicity and clarity we shall work in QCD with exact isospin invariance. The relevant corrections due to isospin breaking are small as we discuss below. Let us first show that in this case the isospin number susceptibility, χ_I, is finite at the end point. The proof is based on the fact that the singular behavior of thermodynamic quantities near the critical point is due to the divergence of a certain correlation length. It is the correlation length in the σ channel, the channel with quantum numbers of the chiral condensate $\langle\bar{\psi}\psi\rangle$ [3,9]. A density-density correlator, such as $\chi_I = (1/T)\int d^3x\langle V_0(x)V_0(0)\rangle$ can diverge only if the density can mix with the σ field. For the isospin density this mixing is strictly forbidden by the $SU(2)_V$ (isospin) symmetry. The isospin density, $V_0(x)$, transforms as a triplet, **3**. On the other hand, σ is a singlet. The mixing is forbidden and there is no singular contribution in χ_I [23].

Small explicit breaking of the $SU(2)_V$ symmetry by the quark mass difference $m_u - m_d$ or the isospin chemical potential μ_I will induce singularity in χ_I, since $m_u - m_d$ and μ_I are $SU(2)_V$ triplets [25], and can produce $V_0\sigma$ mixing. In the context of heavy-ion collisions corresponding singular contributions are negligible.

We summarize by writing the singular parts of the baryon and isospin number susceptibilities:

$$\chi_B \sim \xi^{\gamma/\nu}, \qquad \chi_I = 0 \quad \text{(singular parts only)}, \tag{2}$$

where ξ is the divergent correlation length of the sigma field: $\xi = 1/m_\sigma$. The zero in Eq. (2) neglects small isospin breaking terms as well as finite terms. The universal values of the exponents are given by $\gamma \approx 1.2$, $\nu \approx 0.63$, $\alpha \approx 0.12 \ll \gamma$. Note that $\gamma/\nu = 2 - \eta$, where $\eta \approx 0.04$.

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Let us discuss the implications of (2). In particular, let us consider charge susceptibility. Study of charge fluctuations in heavy-ion collisions has attracted much attention recently. It was proposed that these fluctuations might reflect thermodynamic conditions earlier in the collision history, due to charge conservation [6,7]. The charge fluctuation magnitude per entropy is a measure of the charge per particle or degree of freedom. In the quark-gluon plasma (QGP) the charge per degree of freedom is smaller. If the charge fluctuation is equilibrating too slowly, the observed value will be smaller than the equilibrium magnitude which can be calculated using the resonance gas [5]. However, the estimates of the charge diffusion [27] suggest that for the rapidity windows achievable in present experiments charge diffusion is very efficient in equilibrating charge fluctuations, thus practically washing out the "history" effects. Experimentally, the QGP suppression of the charge fluctuation is not seen [28,29], which is consistent with the diffusion estimates [27]. The effect of the critical fluctuations we are discussing here is crucially different from the QGP suppression. While the latter is the history effect, the critical fluctuations are the equilibrium fluctuations pertaining to the freeze-out point, and the diffusion is necessary to establish them.

The measure of charge fluctuations, the charge number susceptibility, χ_Q, can be expressed in terms of χ_B and χ_I using the relation $Q = B/2 + I_3$ and the fact that isospin symmetry requires $\langle \delta B\, \delta I_3 \rangle = 0$:

$$\chi_Q = \frac{1}{VT}\langle (\delta Q)^2 \rangle = \frac{1}{4}\chi_B + \chi_I. \tag{3}$$

Equation (2) then implies that the charge susceptibility diverges at the critical point, due to the divergence of χ_B.

We now wish to relate the susceptibilities χ_B, χ_I, and χ_Q to observable particle number fluctuations. For simplicity, we shall limit our discussion by considering only protons, neutrons, and pions. Accounting for other particles will not alter our conclusions. In the hadron language, the susceptibilities may be written as

$$\chi_B = \frac{1}{VT}\langle (\delta N_{p-\bar{p}} + \delta N_{n-\bar{n}})^2 \rangle,$$
$$\chi_I = \frac{1}{VT}\left\langle \left(\frac{1}{2}\delta N_{p-\bar{p}} - \frac{1}{2}\delta N_{n-\bar{n}} + \delta N_{\pi^+-\pi^-}\right)^2 \right\rangle, \tag{4}$$

and

$$\chi_Q = \frac{1}{VT}\langle (\delta N_{p-\bar{p}} + \delta N_{\pi^+-\pi^-})^2 \rangle, \tag{5}$$

where we introduced notation $N_{p-\bar{p}} \equiv N_p - N_{\bar{p}}$ for the net proton number fluctuation, with δ denoting event-by-event deviation from the equilibrium value. Similar notations are used for neutrons and pions.

Now we concentrate on singular parts of the susceptibilities and ask a question: what does (2) imply for the individual particle number fluctuations? It is easy to check that the following set of relations between singular parts of the particle correlators reproduces the correct singular behavior given in (2):

$$\begin{gathered}\langle \delta N_{p-\bar{p}}\, \delta N_{p-\bar{p}} \rangle = \langle \delta N_{n-\bar{n}} \delta N_{n-\bar{n}} \rangle = \langle \delta N_{p-\bar{p}}\, \delta N_{n-\bar{n}} \rangle; \qquad \langle \delta N_{\pi^+-\pi^-}\, \delta N_{\pi^+-\pi^-} \rangle = 0 \\ \langle \delta N_{p-\bar{p}}\, \delta N_{\pi^+-\pi^-} \rangle = \langle \delta N_{n-\bar{n}} \delta N_{\pi^+-\pi^-} \rangle = 0 \qquad \text{(singular parts only).}\end{gathered} \tag{6}$$

Some of these equations follow trivially from isospin invariance, but some, for instance, the last equation on the first line and that on the second line, require a stronger condition. Such relations occur naturally if we attribute the divergences to the exchange of a sigma meson, which is an isospin singlet. Using Eqs. (6) we obtain

$$\chi_B = \frac{4}{VT}\langle \delta N_{p-\bar{p}}\, \delta N_{p-\bar{p}} \rangle, \qquad \chi_I = 0,$$
$$\chi_Q = \frac{1}{VT}\langle \delta N_{p-\bar{p}}\, \delta N_{p-\bar{p}} \rangle \qquad \text{(singular parts only).} \tag{7}$$

Remarkably, the singular part of the charge fluctuation comes from the protons. In other words, had we considered only contributions from charged pions in χ_Q, the singular parts of $\pi^+\pi^+$, $\pi^-\pi^-$, $\pi^+\pi^-$ correlators (all are singular at the critical point [4]) would have canceled each other. We see also that the proton number fluctuation completely reflects the singularity of the baryon number susceptibility, which justifies its use as a sensible probe of the QCD critical end point.

To provide a simple estimate of how large the net proton number fluctuation can become near the critical point, we begin by calculating the correlator

$$\langle \delta n_{\mathbf{p}} \delta n_{\mathbf{k}} \rangle, \tag{8}$$

where $n_{\mathbf{p}}$ is the net proton number in the momentum bin labeled by the value $\mathbf{p}$. In addition to the usual statistical fluctuation, the correlator (8) receives a contribution from the effective interaction with the sigma field σ, $\mathcal{L}_{\sigma pp} = g\sigma\bar{P}P$, where g is the dimensionless sigma-nucleon coupling and P is the Dirac field of a proton. All fluctuation observables of the protons can be constructed from (8) [3,13].

Near the critical point, the singular term in (8) is represented by a diagram of forward proton-proton scattering. A straightforward calculation following [13] gives,

$$V\langle \delta n_{\mathbf{p}} \delta n_{\mathbf{k}} \rangle = \frac{g^2}{m_\sigma^2 T}\frac{4m^2}{E_{\mathbf{p}}E_{\mathbf{k}}}[n^+_{\mathbf{p}}(1-n^+_{\mathbf{p}}) - n^-_{\mathbf{p}}(1-n^-_{\mathbf{p}})] \times [n^+_{\mathbf{k}}(1-n^+_{\mathbf{k}}) - n^-_{\mathbf{k}}(1-n^-_{\mathbf{k}})] \qquad \text{(singular parts only),} \tag{9}$$

where $m = 940$ MeV is the proton mass, $E_p = \sqrt{p^2 + m^2}$ and $n_p^{\pm} = [\exp\{(E_p \mp \mu_B)/T\} + 1]^{-1}$, while $m_\sigma = 1/\xi$ is the sigma meson (screening) mass.

Let us compare the singularity in (9) to the singularity in (2). The exponent $\gamma/\nu = 2 - \eta$ in (2) is very close to 2(the anomalous dimension η of the σ field is small) and is equal to two in the mean field approximation ($\eta = 0$), which is the same as the power of $1/m_\sigma$ in (9).

In a realistic heavy-ion collision environment finiteness of the space-time volume severely prevents m_σ from vanishing exactly [4,10]. The smallest achievable value is estimated to be around $(3 \text{ fm})^{-1}$.

One possible concern is that the rescattering in the final hadronic stage washes out critical point fluctuations. In this respect, one should bear in mind that the rescattering in question includes the exchange of the σ quanta, which, near the critical point, is the source of the fluctuations we consider. The critical fluctuations are washed out if the final (kinetic) freeze-out occurs sufficiently far from the critical point. In order to see the effect, one should dial control parameters (e.g., reduce the size of the ions to raise the freeze-out temperature) to bring the freeze-out closer to the critical point.

In (9), g is taken at zero momentum transfer, i.e., off the sigma mass shell. In vacuum, $g \simeq m/f_\pi$ ($f_\pi = 93$ MeV is the pion decay constant) is quite large ~ 10. For the quantitative estimate below, we assume that g does not change appreciably from its vacuum value near the chiral phase transition. (See, however, [30].)

First, let us assume that Au-Au collisions at RHIC at $\sqrt{s} = 130$ GeV froze out in the vicinity of the end point and estimate the effect of (9) in terms of the unknown mass m_σ, which is the measure of the proximity of the end point. At the chemical freeze-out, $T = 174$ MeV and $\mu_B = 46$ MeV [31]. Integrating over $\boldsymbol{p}$ and $\boldsymbol{k}$, we obtain the net proton number fluctuation $\langle(\delta N_{p-\bar{p}})^2\rangle$ and divide it by the sum of proton and antiproton numbers $\langle N_{p+\bar{p}}\rangle$:

$$\frac{\langle(\delta N_{p-\bar{p}})^2\rangle}{\langle N_{p+\bar{p}}\rangle}\bigg|_{\text{RHIC}} \approx 1.0 + 0.062\left(\frac{g}{10}\right)^2\left(\frac{200 \text{ MeV}}{m_\sigma}\right)^2. \tag{10}$$

The unity on the righ-hand side (rhs) is the trivial statistical contribution. Taking $g = 10$ and $m_\sigma = 60 \text{ MeV} \approx (3 \text{ fm})^{-1}$ [4,10], we find $\langle(\delta N_{p-\bar{p}})^2\rangle/\langle N_{p+\bar{p}}\rangle \approx 1.7$.

On the other hand, if the end point were located at a value of μ_B of order of a few hundred MeV, as inferred from simple model estimates [9] and suggested by the recent lattice simulation [11], it is possible that the SPS freeze-out is in the proximity of the critical point. Using the freeze-out parameters $(T, \mu_B) = (168 \text{ MeV}, 266 \text{ MeV})$ at SPS [32] we obtain

$$\frac{\langle(\delta N_{p-\bar{p}})^2\rangle}{\langle N_{p+\bar{p}}\rangle}\bigg|_{\text{SPS}} \approx 1.0 + 1.5\left(\frac{g}{10}\right)^2\left(\frac{200 \text{ MeV}}{m_\sigma}\right)^2. \tag{11}$$

Note that the coefficient of the second term has a much bigger value than in (10). This is because the singular term given by (9) grows as the square of the net proton number and also because there is a partial cancellation between protons and antiprotons at RHIC. We stress that the main feature in (10) and (11) is the singular dependence on m_σ, which makes the effect large when the freeze-out occurs near the critical point. There are other effects, which contribute to the rhs of (10) and (11), but which are not singular near the critical point (e.g., initial volume fluctuations caused by impact parameter fluctuations).

Experimentally, separating protons and measuring proton fluctuations is a feasible task in the RHIC as well as the SPS detectors. We hope that such data analysis will be available soon.

In order to test our ideas on the existing data we can, using (10) and (11), estimate the contribution of the proton fluctuation to the total charge fluctuation characterized by $D \equiv 4\langle(\delta Q)^2\rangle/\langle N_{\text{tot}}\rangle$ [7]. As (7) shows, pions do not contribute to the singular part of the charge fluctuation, but they dilute such a contribution of the protons. Using $\langle N_{\pi^+ + \pi^-}\rangle \approx 10\langle N_{p+\bar{p}}\rangle$ and $\langle(\delta N_{\pi^+ - \pi^-})^2\rangle/\langle N_{\pi^+ + \pi^-}\rangle \approx 1 - 0.3 = 0.7$, where the negative contribution -0.3 is due to the resonance decays [5], we obtain

$$\frac{D}{4} \equiv \frac{\langle(\delta Q)^2\rangle}{\langle N_{\text{tot}}\rangle} \approx \frac{\langle(\delta N_{p-\bar{p}})^2\rangle + \langle(\delta N_{\pi^+ - \pi^-})^2\rangle}{\langle N_{p+\bar{p}}\rangle + \langle N_{\pi^+ + \pi^-}\rangle} \approx 0.8, \tag{12}$$

where we have neglected the cross terms between δN_p and $\delta N_{\pi^\pm}$. We see that the fluctuation anomaly in the proton sector can result in a larger charge fluctuation than the resonance gas value ≈ 0.7 by about 10%. At SPS this effect is even stronger. At SPS, using (11), $g = 10$ and $m_\sigma = 200$ MeV we get $\langle(\delta N_{p-\bar{p}})^2\rangle/\langle N_{p+\bar{p}}\rangle \approx 2.5$, and with $\langle N_{\pi^+ + \pi^-}\rangle \approx 5\langle N_{p+\bar{p}}\rangle$ we obtain $D/4 \approx 1$.

Before these estimates can be compared to experiment, one must take into account the effect of limited acceptance of a given detector. It is easy to see that this effect reduces deviations from $D = 4$. Its estimates range from few percent corrections [7] to almost complete elimination of deviations from $D = 4$ [33], depending on the assumptions on the rapidity correlator of fluctuations and the width of the acceptance window. We do not discuss these issues here, and refer the reader to the literature.

Experimentally, the data from RHIC suggest that the magnitude of the fluctuation is slightly larger than a thermodynamical fluctuation in a resonance gas [28]. This effect is even more pronounced at SPS [29]. There are, of course, a number of possible explanations, for example, (i) acceptance, as we have just discussed [33]; (ii) remnant initial state correlations; (iii) decay of multiply charge clusters; (iv) other nonequilibrium fluctuations (e.g., by a mechanism similar to [21]); In this Letter we wish to point out that, quite independently of these other effects, an equilibrium critical fluctuation due to the proximity of the end point could explain the enhancement of charge fluctuations observed at RHIC and SPS. The independent measurement of proton fluctuations that we suggested would be necessary to confirm and sufficient to rule out this effect.

Is it possible that the light sigma effect is seen in both RHIC and SPS experiments? If this happens, the region of μ_B where m_σ is small (< 200 MeV) is rather wide — of the order of 100 MeV. Although unlikely, this might not be completely unnatural if one takes into account the fact that m_σ is suppressed on the crossover line stretching from the end point to $\mu_B = 0$ axis, even though it vanishes only at the end point (see, e.g., Fig. 5 in [12]).

As it should be clear from our discussion, measuring the charge fluctuations is not the most efficient way to search for the end point, although the effect may be seen in such observables too. A direct measurement of the proton number fluctuation as a function of the $\sqrt{s}$ of the collision is both feasible and is less afflicted by other effects. Correlation of such a measurement with other proposed signatures of the critical point (such as p_t fluctuations [3]) would affirm the discovery of the QCD critical point.

In conclusion, protons carry both the baryon and the electric charges. They are sensitive to the fluctuation of the order parameter. Because of the peculiar nature of the end-point — isospin blindness of the sigma field — the singularity of the baryon number susceptibility is completely reflected in the proton number fluctuation. Thus the net proton number fluctuation is a very useful observable. By studying the μ_B dependence of this fluctuation one may discover and determine the location of the critical point on the phase diagram of QCD.

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