Constraining Hadronic Superfluidity with Neutron Star Precession

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I show that the standard picture of the neutron star core containing coexisting neutron and proton superfluids, with the proton component forming a type II superconductor threaded by flux tubes, is inconsistent with observations of long-period (~ 1 yr) precession in isolated pulsars. I conclude that either the two superfluids coexist nowhere in the stellar core, or the core is a type I superconductor rather than type II. Either possibility would have interesting implications for neutron star cooling and theories of spin jumps (glitches).

Neutron superfluidity and proton superconductivity in neutron stars could have a number of interesting consequences for observed spin behavior and thermal evolution. Interaction of superfluid vorticity with the nuclei of the inner crust or superconducting flux tubes in the core could lead to the jumps in spin rate, *glitches*, seen in many neutron stars [1]. The specific heat of the stellar interior is determined by the state of the matter, while neutrino emission processes which cool a young neutron star are strongly suppressed in the presence of hadronic superfluids (see, e.g., [2]). The properties of condensed hadronic systems are also of interest in studies of heavy nuclei near the neutron drip line [3] and light halo nuclei [4]. The properties of hadronic systems in beta equilibrium are therefore a central problem in both nuclear astrophysics and nuclear physics.

Reliable predictions of the pairing states of the neutron star core are not yet possible as they require extrapolation of nucleon-nucleon potentials well above nuclear saturation density, $\rho_s \equiv 2.8 \times 10^{14}$ g cm^{-3}. The current picture of the neutron star interior posits that the outer core consists of mostly $^3P_2$ or $^3P_2$-$^3F_2$ superfluid neutrons, with about 5% of the mass in type II $^1S_0$ superconducting protons, normal electrons, and fewer muons [5–11]. Above a density $\simeq 1.7\rho_s$, the pairing situation is essentially unknown [11].

The compelling evidence for precession in isolated pulsars [12–14] provides new probes of the state of a neutron star's exotic interior. The periodic timing behavior of PSR B1828-11 and correlated changes in beam profile have been interpreted as due to precession with a period of ~1 yr and an amplitude of $\simeq 3°$ [15–17]. The measured precession period implies a fractional distortion of the star (in addition to its rotational distortion) of $\epsilon \simeq 10^{-8}$. This deformation could be sustained by magnetic stresses [18,19], crust stresses [20], or a combination of the two.

The picture of the outer core I will consider is as follows. The neutron fluid rotates by establishing a triangular array of quantized vortex lines, parallel to the axis of the angular momentum of the superfluid and with an areal density of $n_v = 2m_n\Omega_n/\pi\hbar \simeq 10^4 P(\text{s})^{-1}$ cm^{-2}, where m_n is the neutron mass, Ω_n is the angular velocity of the superfluid, and P is the spin period. The average vortex spacing is $l_v \equiv n_v^{-1/2} \simeq 10^{-2}P(\text{s})$ cm. If magnetic flux penetrates the superconducting core, it is organized into quantized flux tubes, with an areal density $n_\Phi \equiv B/\Phi_0 \sim 10^{19}B_{12}$ cm^{-2}, where $B_{12} \equiv 10^{12}B$, B is the average core field in Gauss, and $\Phi_0 \equiv \pi\hbar c/e = 2 \times 10^{-7}$ G cm^{-2} is the flux quantum. The average spacing between flux tubes is $l_\Phi \equiv n_\Phi^{-1/2} \simeq 3000B_{12}^{-1/2}$ fm. The magnetic field in the core of a flux tube is approximately the lower critical field for the superconducting transition, $H_{c1}(\simeq 10^{15}$ G). Unlike the vortex array, which is expected to be nearly rectilinear, the flux tube array is likely to have a very complicated and twisted structure [21]. Hence, the vortices are entangled in the far more numerous flux tubes. In this Letter, I show that this entanglement restricts the precession to be of very high frequency and low amplitude, in conflict with observations.

A flux tube has a core of normal protons; the radius of this region is of the order of the proton coherence length $\xi_p \simeq 30$ fm. Outside the flux tube, the magnetic field falls off exponentially over a distance equal to the London length: $\Lambda_p \simeq 80$ fm. (Type II superconductivity occurs when $\sqrt{2}\Lambda_p > \xi_p$.) A vortex has a core of normal neutrons, of characteristic radius the neutron coherence length, $\xi_n \sim 10$ fm. Entrainment of protons in the neutron flow about a vortex magnetizes the vortex [22]. The length scale over which the vortex's magnetic field decays is $\Lambda_n \simeq 10$ fm. These length scales are for typical parameters of the core: a superconducting transition temperature of 10^9 K, an effective proton mass of half the bare mass, a proton mass fraction of 0.05, and a total mass density of 3×10^{14} g cm^{-3} [23]. The protons do not rotate by establishing an array of vortices, but corotate with the crust and electron fluid approximately as a rigid body by adjusting the London current [22].

For the angular momentum of the neutron fluid to change, the vortices must move. The flux tubes in which they are entangled, however, impede their motion. As a vortex segment approaches a flux tube segment, the total

magnetic energy increases (decreases) if the vortex and flux tube are aligned (antialigned). The energy per intersection is $E_{int} \sim 5$ MeV, with a range $\sim \Lambda_p$ [21,24]. The vortices are effectively pinned against the flux array, unless they can push the flux tubes through the star, or cut through them by surmounting the numerous energy barriers. The pinning force is $F_p \equiv E_{int}/\Lambda_p \sim 0.1$ MeV fm^{-1} per intersection, corresponding to a force per unit length of vortex of $f_p \sim F_p n_\Phi^{1/2} = 3 \times 10^{15} B_{12}^{1/2}$ dyn cm^{-1}. Because the core is highly conductive, vortices can push the flux tubes only very slowly [21]. Hence, unless the vortices cut through the flux tubes, the core flux tubes and vortices move together, and the crust, which is frozen to the strong field emerging from the core, approximately follows the motion of the charged fluid.

The charged fluid of the core responds to changes in its rotation rate approximately as a rigid body if magnetic stresses are sufficient to enforce corotation. In the superconducting core, magnetic stresses propagate as cyclotron-vortex waves of frequency $\omega_{cv} = (H_{c1}B/4\pi\rho_p)^{1/2}k$, where k is the excitation wave number [25]. Taking $k = \pi/R$, where R is the stellar radius, $\rho_p = 1.5 \times 10^{13}$ g cm^{-3}, and $B = 5 \times 10^{12}$ G gives the characteristic frequency at which magnetic stresses are communicated through the core, $\omega_{cv,0} \simeq 10$ rad s^{-1}. The frequency $\omega_{cv,0}$ represents an approximate upper limit to the precession frequency of the star as a whole.

To calculate the precession dynamics, I assume that pressure gradients and magnetic stresses force the crust and the charged fluid to move together, and refer to the charged core fluid plus crust, whose spin rate we observe, as the "body", even though it contains $\lesssim 10\%$ of the star's mass. By the arguments given above, the core neutron fluid, which accounts for $\gtrsim 90\%$ of the stellar mass, has its angular momentum fixed to this body, through pinning of the vortices to the flux tubes. I assume that the $^1S_0$ vortices of the inner crust are not pinned to nuclei and nearly follow the body's rotation axis so that they have a negligible effect on the precession dynamics [26]. With these idealizations, let us write the inertia tensor of the charged fluid as the sum of a spherical piece, a centrifugal bulge that follows the instantaneous angular velocity and an oblate, biaxial deformation bulge aligned with the body's principal axis:

$$\boldsymbol{I}_c = I_{0,c}\boldsymbol{\delta} + \Delta I_\Omega\left(\boldsymbol{n}_\Omega\boldsymbol{n}_\Omega - \frac{1}{3}\boldsymbol{\delta}\right) + \Delta I_d\left(\boldsymbol{n}_d\boldsymbol{n}_d - \frac{1}{3}\boldsymbol{\delta}\right).$$

(1)

Here $I_{0,c}$ is the moment of inertia of the charged fluid (plus any components tightly coupled to it) when nonrotating and spherical, $\boldsymbol{\delta}$ is the unit tensor, $\boldsymbol{n}_\Omega$ is a unit vector along the body's angular velocity $\boldsymbol{\Omega}_c$, $\boldsymbol{n}_d$ is a unit vector along the principal axis of inertia, ΔI_Ω is the increase in oblateness about $\boldsymbol{\Omega}_c$ due to rotation, and ΔI_d is the portion of the body's deformation that is frozen in the body. Let the neutron fluid's angular momentum

vector $\boldsymbol{L}_n$ be perfectly tied to the core flux tube array, so that $\boldsymbol{L}_n$ is fixed with respect to the body. The total angular momentum is $\boldsymbol{L} = \boldsymbol{L}_n + \boldsymbol{L}_c$, where $\boldsymbol{L}_c$ is the total angular momentum of the body. The Euler equations in the body frame are $\boldsymbol{I}_c \cdot \dot{\boldsymbol{\Omega}}_c + \boldsymbol{\Omega}_c \times (\boldsymbol{L}_c + \boldsymbol{L}_n) = 0$. Define principal axes in the body (x_1, x_2, x_3), where x_3 is along the major principal axis ($\hat{\boldsymbol{x}}_3 = \boldsymbol{n}_d$). The principal moments of inertia are $I_1 = I_0 + 2\Delta I_\Omega/3 - \Delta I_d/3 = I_2 = I_3(1 + \epsilon)^{-1}$, where $\epsilon \equiv \Delta I_d/I_1 > 0$. Let the angle between x_3 and $\boldsymbol{L}_n$ be α. If $\boldsymbol{L}_c$, $\boldsymbol{L}_n$, and $\boldsymbol{\Omega}_c$ are all aligned, the star is in a state of minimum energy for a given angular momentum and does not precess. A likely precessional state is one in which $\boldsymbol{L}_c$ and $\boldsymbol{L}_n$ are perturbed slightly about this stationary point. To define angles, let x_3, $\boldsymbol{L}_n$, $\boldsymbol{L}_c$, and $\boldsymbol{\Omega}_c$ all lie in a plane at $t = 0$, with θ the angle between x_3 and $\boldsymbol{L}$, and θ' the angle between $\boldsymbol{\Omega}_c$ and $\boldsymbol{L}$ (see Fig. 1). Linearizing Euler's equations in Ω_{c1}, Ω_{c2}, and α, gives the solutions

$$\Omega_{c1}(t) = A\cos\Omega_p t - \theta_0\Omega_c, \qquad \Omega_{c2}(t) = A\sin\Omega_p t,$$

(2)

where $\Omega_p \equiv \epsilon\Omega_c + L_n/I_1$ is the body-frame precession frequency, and

$$\frac{A}{\Omega_c} = \alpha\frac{L_n}{I_1}\left[\frac{1}{\Omega_c} + \frac{1}{\Omega_p}\right] - \theta\left[1 + \frac{\Omega_p}{\Omega_c}\right],$$
$$\theta_0 = \alpha\frac{L_n}{I_1\Omega_p}.$$

(3)

The motion of $\boldsymbol{\Omega}_c$ is a circle of angular radius $|A|/\Omega_c$, about an axis that takes an angle θ_0 with respect to x_3 in the x_1-x_3 plane, completing one revolution in a time $2\pi/\Omega_p$. Removing the pinned component ($L_n = 0$) gives the familiar result of $\Omega_p = \epsilon\Omega_c$, $A/\Omega_c \simeq \theta$. Restoring the pinned neutron fluid, and taking $L_n \simeq I_n\Omega_c$ gives $\Omega_p \simeq (I_n/I_1)\Omega_c \simeq 10\Omega_c$, independent of α and θ. When Ω_p exceeds $\omega_{cv,0}$, as for PSR B 1828-11, the precession frequency is likely to be closer to $\omega_{cv,0} \simeq 10$ rad s^{-1}, still very high. *As long as $\boldsymbol{L}_n$ is pinned to the body, the star precesses at high frequency for any finite α or θ.*

First consider a state in which $\boldsymbol{L}_n$ and $\boldsymbol{L}_c$ are both aligned at $t = 0$, but $\boldsymbol{\Omega}_c$ is not along $\boldsymbol{L}$. In this case, $\alpha = \theta$, giving

$$\frac{A}{\Omega_c} = -\epsilon\theta\left(1 + \frac{\Omega_c}{\Omega_p}\right), \qquad \theta_0 = \theta\left(1 - \epsilon\frac{\Omega_c}{\Omega_p}\right). \quad (4)$$

The angular velocity vector of the body takes a tiny circle of (angular) radius $\simeq \epsilon\theta \ll \theta$ about axis o in Fig. 1, nearly coincident with $\boldsymbol{L}$. Since $\boldsymbol{L}_n$ is fixed in the body, it too goes around o. For the wobble angle to be $\sim 3°$ with $\boldsymbol{L}_n$ fixed in the body, $\alpha \ll \theta$ is obviously required. Suppose, for example, that $\alpha = 0$; that is, $\boldsymbol{L}_n$ is aligned with x_3. Then

$$\frac{A}{\Omega_c} = -\theta\left(1 + \frac{\Omega_p}{\Omega_c}\right) = -(\theta + \theta') \equiv -\beta, \qquad \theta_0 = 0.$$

(5)
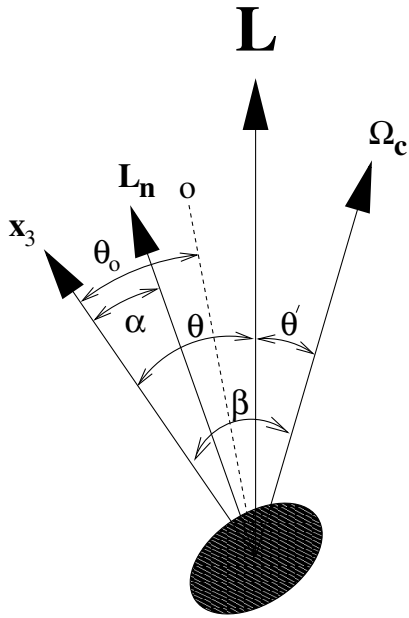
Now the angular velocity of the body takes a circle about

FIG. 1. The angles defined in the text. $\boldsymbol{\Omega}_c$ takes a circular path about axis o, the dashed line.

x_3, with radius $\beta \sim \theta$. Relatively large-amplitude precession occurs, but still at very high frequency. For precession of large amplitude to occur at low frequency, $\boldsymbol{L}_n$ must be able to closely follow the rotation axis of the body, so that the $\boldsymbol{\Omega}_c \times \boldsymbol{L}_n$ term in Euler's equations becomes small compared to $\boldsymbol{\Omega}_c \times \boldsymbol{L}_c$. For this to happen, the neutron vortices must be able to cut through the flux tubes. For $\beta \simeq 1°$, this is likely. The flow of the neutron superfluid past a vortex pinned against a flux tube creates a Magnus force per unit length of vortex at location $\boldsymbol{r}$ of $\boldsymbol{f}_m = \rho_n \boldsymbol{\kappa} \times ([\boldsymbol{\Omega}_n - \boldsymbol{\Omega}_c] \times \boldsymbol{r})$, where ρ_n is the mass density of the neutron superfluid and $\boldsymbol{\kappa}$ is a vector in the direction of the neutron vorticity with magnitude $h/2m_n$ [27]. For simplicity, take $\boldsymbol{\kappa} = \kappa \hat{\boldsymbol{x}}_3$ and $\boldsymbol{\Omega}_n = \Omega_n \hat{\boldsymbol{x}}_3$ and $\Omega_n = \Omega_c \equiv \Omega$. At $t = 0$, when $\boldsymbol{\Omega}_n$, $\boldsymbol{\Omega}_c$, and $\boldsymbol{L}$ all lie in the x_1-x_3 plane, the angular velocity of the body is $\boldsymbol{\Omega}_c = \Omega(-\sin\beta\hat{\boldsymbol{x}}_1 + \cos\beta\hat{\boldsymbol{x}}_3)$. The instantaneous Magnus force per unit length of vortex as a function of position in the star is, for small angles, $\boldsymbol{f}_m = -\hat{\boldsymbol{x}}_1 \rho_n \kappa \Omega \beta x_3$. If f_m exceeds f_p, the pinning force per unit length on a typical vortex, the vortices will cut through the flux tubes that are in their way. This condition gives $|x_3| > f_p/\rho_n \kappa \Omega \beta$. For $\Omega = 16 \text{ rad s}^{-1}$ (PSR B1828-11), the inferred β of 3°, and a density $\rho_s = 3 \times 10^{14} \text{ g cm}^{-3}$, $|\boldsymbol{f}_m|$ exceeds $f_p = 10^{16} \text{ dyn cm}^{-1}$ for $|x_3| > 2 \times 10^{-2}R$; that is, the Magnus force will force the vortices through the flux tubes almost everywhere in the star. This process is highly dissipative.

As a vortex is forced through a flux tube, quantized vortex waves, *kelvons*, are excited, which propagate along the vortex and eventually dissipate as heat [28]. In the rest frame of a straight vortex along the $\hat{z}$ axis, suppose a straight flux tube in the y-z plane approaches at speed v. Since $\Lambda_n < \Lambda_p$, the finite (magnetic) radius of the vortex

can be ignored. Let the vortex and flux tube overlap at $t = 0$. The vector separation between a point at the center of the flux tube which will coincide with the vortex at $t = 0$ is $\mathbf{s}(t) = vt\hat{\boldsymbol{x}}$. As a simple model of the interaction force, consider $\mathbf{f}_{\text{int}}(\mathbf{s}(t)) = F_p(s/\Lambda_p)\exp[(1 - s^2/\Lambda_p^2)/2]\delta(z)\hat{\boldsymbol{x}}$, where $\delta(z)$, the Dirac-delta function, gives the distribution of the interaction force along the vortex (justified below).

The relative velocity between vortices and flux tubes is $v \simeq R\Omega_c\beta$ in the initial stage that flux tubes cut through vortices; taking β comparable to the observed wobble angle gives $v = 10^6 \text{ cm s}^{-1}$ for PSR B1828-11. As a flux tube passes through a vortex, it excites kelvons of characteristic frequency $\omega_0 = v/\Lambda_p$. Kelvons on a free vortex are circularly polarized waves. The frequency of a kelvon is related to its wave number by $\omega_k = \hbar k^2/2\mu$ where μ is the effective mass of a kelvon, given by $\mu = m_n/\pi\Lambda$. The dimensionless parameter Λ is $\simeq 0.116 - \ln(k\xi_n)$ for wave numbers in the range $l_v^{-1} \ll k \ll \xi_n^{-1}$ [29], which is easily satisfied for the characteristic wave number $k_0 = (2\mu\omega_0/\hbar)^{1/2}$ of interest. For $v = 10^6 \text{ cm s}^{-1}$ and $\xi_n = 10$ fm, these relationships give $k_0 = 5 \times 10^{-4}$ fm and $\mu = 0.06m_n$.

The total energy transferred to a vortex per scattering in a potential is given in first-order perturbation theory by [28]

$$\Delta E = \frac{\hbar}{4\pi\rho_n\kappa\mu} \int_{-\infty}^{\infty} dk k^2$$
$$\times \left| \int_{-\infty}^{\infty} dz e^{-ikz} \int_{-\infty}^{\infty} dt e^{i\omega_k t} f_+(z, t) \right|^2. \quad (6)$$

Here $f_+(z, t) \equiv \mathbf{f}_{\text{int}}(z, t) \cdot \boldsymbol{\lambda}^*$ where $\boldsymbol{\lambda}^* \equiv (\hat{\boldsymbol{x}} + i\hat{\boldsymbol{y}})/\sqrt{2}$ is the right-circularly polarized unit vector for a kelvon. Since $k_0\Lambda_p \ll 1$, the flux tube exerts a highly localized force along the length of the vortex, justifying the use of the δ function in model for the force. To estimate the total dissipation rate in the core, take $l_\Phi \equiv n_\Phi^{-1/2} = (\Phi_0/B)^{1/2}$ as the average distance between intersections of a vortex line with a flux tube. The total number of vortices in the core is $N = 2\pi R^2 \Omega_n/\kappa$. Taking a typical vortex length of R, gives a total dissipation rate in the core of $\dot{E} \gtrsim Nn_\Phi Rv\Delta E$, or

$$\frac{dE}{dt} \gtrsim 4\frac{F_p^2 R^3 \Omega_n B}{\rho_n \kappa^2 \Phi_0} \left(\frac{2\mu\Lambda_p v}{\hbar}\right)^{1/2}, \quad (7)$$

a lower limit, since the excitation of flux tubes, which is also dissipative, was ignored. Different choices for the dependence of the interaction force on $\mathbf{s}$ give the same scaling on the parameters appearing in Eq. (7), with slightly different numerical factors. The use of Eq. (6) assumes that cuttings of the vortex at different locations can be treated as separate events, with the excitations due to different cuttings adding incoherently. This will be the case as long as $k_0 l_\Phi \gg 1$. For $v = 10^6 \text{ cm s}^{-1}$ and $B_{12} = 1$, $k_0 l_\Phi$ is ~ 2, so the approximation of kelvons as distinct wave packets is a somewhat crude one, but the lower limit

in Eq. (7) should be a reasonable estimate for the velocities of interest. Taking $v = 10^6$ cm s^{-1}, $\mu = 0.06 m_n$, $R = 10$ km, $\Lambda_p = 80$ fm, and $F_p = 0.1$ MeV fm^{-1} gives a dissipation rate of $dE/dt \sim 10^{41}$ ergs s^{-1}. Now consider the rotational energy of the precessing star. The energy in the body frame $E_{\rm rot}$ is related to the energy in the inertial frame E_0 by $E_{\rm rot} = E_0 - \boldsymbol{L} \cdot \boldsymbol{\Omega}$. Most of the angular momentum is in the neutrons, so $\boldsymbol{L} \simeq \boldsymbol{L}_n$. The excess rotational energy is thus $\Delta E_{\rm rot} \simeq I_n \Omega_n^2 \beta^2/2 \simeq 2 \times 10^{44}$ ergs. The characteristic damping time is $\tau_d \equiv \Delta E_{\rm rot}(dE/dt)^{-1} \lesssim 1$ hr. Over this short time scale, the precession damps to small amplitude. When β is $\lesssim 0.06°$, the Magnus force cannot drive the vortices though the flux tubes anywhere in the star; $\boldsymbol{L}_n$ is now fixed in the body, and therefore cannot follow the total angular momentum, so the star precesses at frequency $\Omega_p \simeq \omega_{cv,0} \simeq 10$ rad s^{-1}. In general, then, *long-period precession is not possible.*

To summarize, these estimates show that a neutron star core containing coexisting neutron vortices and proton flux tubes cannot precess with a period of ~ 1 yr. Since $\Omega_p = \epsilon \Omega_c + L_n/I_1$, the fraction of the neutron component's moment of inertia that is pinned against flux tubes must be $\ll \epsilon \simeq 10^{-8}$. Hence, observations require that *neutron vortices and proton flux tubes coexist nowhere in the star.* Either the star's magnetic field does not penetrate any part of the core that is a type II superconductor, which seems highly unlikely, or at least one of the hadronic fluids is not superfluid. This latter possibility appears unlikely in the face of pairing calculations which predict coexisting neutron and proton superfluids in the outer core [5–11].

If the core is a type I superconductor, at least in those regions containing vortices, the magnetic flux could exist in macroscopic normal regions that surround superconducting regions that carry no flux. In this case, the magnetic field would not represent the impediment to the motion of vortices that flux tubes do, and the star could precess with a long period. Perhaps PSR B1828-11 and other precession candidates are giving us the first clue that neutron stars contain a type I superconductor. Another, strange possibility is that "neutron stars" are in fact composed of strange quark matter [30].

The possibilities discussed above have interesting implications for models of neutron star spin and thermal evolution. Glitch models that rely on vortex-flux tube interactions, e.g., [21], would no longer apply, leaving the inner crust superfluid as a possible origin of glitches [31]. The URCA reactions, which are strongly suppressed in regions where both neutrons and protons are superfluid, could be significantly increased if macroscopic regions of the core are normal, affecting the thermal evolution of young neutron stars.

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