Comment on "Left-Handed Materials Do Not Make a Perfect Lens"

In a recent Letter [1] Garcia and Nieto-Vesperinas (GNV) dispute the claim of perfect lensing made in [2]. GNV claim that the solutions proposed in [2] imply infinite energy density and are therefore inadmissible. They also claim that finite absorption leads to catastrophic collapse of the amplifying solutions vital to focusing. GNV calculate in Eq. (6) of their Letter the electric fields for S-polarized light, transverse wave vector k_y^i, incident on a slab of negatively refracting material. (The surfaces of the slab lie in the xy plane and the electric field is assumed to lie along the x axis.)

Note that Eq. (6) contains several errors. I give two examples. First, the parallel electric field is discontinuous at each surface of the slab. Second, GNV calculate the field in the vacuum,

$$\mathbf{E}(z_0 \leq z \leq 0) = [A^{(i)} \exp(-K_i z_0), \quad 0, \quad 0]\left[\exp(ik_y^i y - K_i z) - \frac{\varepsilon_i + 4i}{\varepsilon_i}\exp(ik_y^i y - K_i z)\right],$$

where z_0 is the location of the source, $K_i = \sqrt{k_y^{i2} - k_0^2}$, and the dielectric function $\varepsilon = -1 + i\varepsilon_i$, which implies that fields in this region decay monotonically towards the interface. This solution is not consistent with causality which requires that the reflected wave decays in the opposite direction to the incident wave. Causality requires that we always consider a small positive imaginary part to *both* ε and μ and take the limit, $\lim \varepsilon \to -1$, $\lim \mu \to -1$. See, for example, Newton's book [3], p. 105.

Finally, Eq. (6) is derived under the assumption that $n_2 \exp(K_i d) \gg 2$ and therefore cannot in any case be used to take the limit $n_2 = \frac{1}{2}\varepsilon_i \to 0$. Equation (6) is singular in this limit, whereas the correct formula is not.

How does the causal theory of [2] avoid the divergences which GNV find? First consider the wave field transmitted through the slab in the limit $k_y^i \to \infty$. Clearly even infinitesimally small absorption prevents any divergence in this limit:

$$\lim_{k_y^i \to \infty} \mathbf{E}(z > d) = \lim_{k_y^i \to \infty} [A^{(i)} e^{-K_i z_0} \quad 0 \quad 0] e^{ik_y^i y - K_i(z-d)} \frac{-4e^{-K_i d}}{[i\mu_i + i\frac{k_0^2}{2K_i^2}(\varepsilon_i + \mu_i)]^2 - 4e^{-2K_i d}} = 0.$$

What of the limit $\lim \varepsilon \to -1$, $\lim \mu \to -1$ taken at finite k_y^i? Equation (6) of GNV shows a divergence in this limit but, in contrast, the correct result is finite:

$$\lim_{\substack{\varepsilon_i \to 0 \\ \mu_i \to 0}} \mathbf{E}(z > d) = \lim_{\substack{\varepsilon_i \to 0 \\ \mu_i \to 0}} [A^{(i)} e^{-K_i z_0} \quad 0 \quad 0] e^{ik_y^i y - K_i(z-d)} \frac{-4e^{-K_i d}}{[i\mu_i + i\frac{k_0^2}{2K_i^2}(\varepsilon_i + \mu_i)]^2 - 4e^{-2K_i d}}$$

$$= [A^{(i)} e^{-K_i z_0} \quad 0 \quad 0] e^{ik_y^i y - K_i(z-d)} e^{+K_i d}.$$

Clearly this gives the desired lensing solution with amplification of the incident wave field. This solution is valid provided that

$$4e^{-2K_i d} \gg \left(i\mu_i - i\frac{k_0^2}{2K_i^2}(\varepsilon_i + \mu_i)\right)^2,$$

which sets a natural limit to the largest value of k_y^i giving rise to an amplified solution (see [4]) and hence a limit to the resolution. In principle, by making absorption sufficiently small the resolution can be increased to be as large as desired without causing any divergences in the wave field. The disagreement with GNV arises from a combination of algebraic error, and neglect of causality.

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