Synergy of Anomalous Transport and Radiation in the Density Limit

M. Z. Tokar

Institut für Plasmaphysik, Forschungszentrum Jülich GmbH, Association FZJ-Euratom, 52425, Jülich, Germany

(Received 7 May 2003; published 26 August 2003)

The critical plasma density n_{cr} above which the edge anomalous transport in tokamaks is dominated by drift resistive ballooning instability is found analytically. In this transport regime, the drastic increase of particle losses and drop of the edge temperature provoke a strong increase in impurity radiation, and thermal equilibrium does not exist if the density is ramped up above the ultimate limit $n_{\max}$. Because of the nonlinear character of impurity radiation, this density limit $n_{\max}$ is very close to n_{cr} and practically does not change with the ion effective charge. The importance of the synergy between the anomalous transport and impurity radiation for the density limit phenomenon is confirmed by the results of numerical simulations.

DOI: 10.1103/PhysRevLett.91.095001 PACS numbers: 52.55.Fa, 52.35.Py

I. Introduction.—The density limit, i.e., the maximum plasma density attainable before disruption, is one of the most fundamental operational boundaries for fusion devices [1]. Understanding this phenomenon is of extreme importance for the construction of an economically profitable thermonuclear reactor [2] since the fusion power scales quadratically with the density. On the one hand, impurity radiation increases dramatically when the density limit is exceeded [1]. Therefore it is often interpreted as a breakdown of the plasma edge thermal balance caused by an uncontrollable growth of radiation losses (see numerous citations in Ref. [1]). On the other hand, the density limit is preceded by a significant reduction of the particle confinement and an increase in the cross-field convective power losses at the plasma edge [1,3,4]. This evolution is accompanied by a principle change in the edge turbulence, and, in particular, the amplitude of low-frequency perturbations attributed to drift resistive ballooning (DRB) instability [5] increases significantly [1]. These observations are in line with the results of edge turbulence modeling [6], which predict that the particle flux should grow fast with increasing plasma collisionality. Recent computations with the code BOUT [7] have demonstrated that by approaching the experimental density limit the anomalous diffusion produced by DRB modes can grow up to 10–30 m^2/s. In independent calculations with the transport code 2D UEDGE, such a huge diffusion led to the edge radiative instability and X-point multifaceted asymmetric radiation from the edge (MARFE) formation.

In the present Letter, a simple model for the plasma edge with prescribed dependence of the diffusion coefficient on plasma parameters is proposed. This allows one to take into account impurity radiation and convective energy losses *self-consistently*. It is demonstrated that the synergy of these processes is of crucial importance for the density limit phenomenon.

II. Basic equations.—Modeling of linear instabilities and turbulence [6–9] revealed different mechanisms, which could be responsible for the experimentally observed fluctuations of plasma parameters and anomalous transport of particles and energy at the plasma edge in fusion devices. "Normal" drift modes lead to a gyro-Bohm type of transport coefficients, $D_{gB} \sim [(c_s \rho_s^2)/L_n] g$, where $c_s = \sqrt{T/m_i}$ and $\rho_s = c_s/\omega_{iL}$ are the ion sound velocity and Larmor radius, respectively, $L_n = -\frac{d \ln n}{dr}$ is the radial e-folding length of the plasma density and the factor g takes into account the dependence on other parameters. Since $D_{gB} \sim T^{3/2}$, this channel is of importance at a high edge temperature T corresponding normally to a low enough plasma density.

With increasing density and decreasing temperature, DRB modes should become dominant in the edge transport. Indeed, $D_{\mathrm{DRB}} \sim (2\pi q)^2 \rho_e^2 \nu_{ei} \frac{R}{L_n}$ [5], where q is the safety factor, ρ_e the electron Larmor radius, ν_{ei} the electron-ion Coulomb collision frequency, and R the torus major radius. Therefore, $D_{\mathrm{DRB}} \sim n/\sqrt{T}$.

A transition between two transport regimes occurs when $D_{gB} \approx D_{\mathrm{DRB}}$. This criterion is similar to the requirement of Ref. [6]: For the dominance of DRB modes, the diamagnetic parameter $\alpha_d \sim \sqrt{D_{gB}/D_{\mathrm{DRB}}}$ should be smaller than 1. Henceforth, the following rule will be adopted by determining the particle diffusivity $D_\perp$ and plasma heat conductivity $\kappa_\perp$ at the plasma edge:

$$D_\perp = \max(D_{gB}, D_{\mathrm{DRB}}), \qquad \kappa_\perp = 3 n D_\perp. \tag{1}$$

The variation of the plasma temperature and density in time t and with the distance x from the last closed magnetic surface (LCMS) toward the plasma core is governed by one-dimensional equations for the particle and energy transport:

$$\frac{\partial n}{\partial t} + \frac{\partial \Gamma_\perp}{\partial x} = n n_a k_i, \tag{2}$$

$$3 \frac{\partial n T}{\partial t} + \frac{\partial q_\perp}{\partial x} = -E_i n n_a k_i - Q_{\mathrm{rad}}. \tag{3}$$

Here $\Gamma_\perp = -D_\perp \frac{\partial n}{\partial x}$ and $q_\perp = 3T\Gamma_\perp - \kappa_\perp \frac{\partial T}{\partial x}$ are the densities of particle and heat fluxes, respectively. The density of neutral particles generated by plasma recycling at the device walls, n_a, is determined in a diffusion approximation [10]:

$$\frac{\partial}{\partial x}\left[-\frac{T}{m_i(k_i + k_{cx})n}\frac{\partial n_a}{\partial x}\right] = -k_i n n_a, \qquad (4)$$

with k_i and k_{cx} being the rate coefficients of the ionization and charge-exchange processes; E_i is the energy lost by electrons on ionization of a neutral particle.

The edge radiation comes mostly from weakly ionized species of light impurities such as carbon and oxygen with charges up to that of Li-like particles, Z_{Li}. The power density of radiation losses, Q_{rad}, can be estimated as $\sum_{Z=1}^{Z_{\mathrm{Li}}} L_Z n_Z n$ with L_Z being the cooling rate of impurity ions of the charge Z. Recombination plays normally a small role in impurity particle balance at the plasma edge compared with their transport and ionization processes [11] and the densities of impurity ions, n_Z, are determined from continuity equations:

$$\frac{\partial n_Z}{\partial t} + \frac{\partial}{\partial x}\left(-D_\perp \frac{\partial n_Z}{\partial x}\right) = k_i^{Z-1} n n_{Z-1} - k_i^Z n n_Z, \qquad (5)$$

where k_i^Z is the ionization rate coefficient. For the density of impurity neutrals, n_0, we have

$$\frac{\partial n_0 V_I}{\partial x} = -k_i^0 n n_0, \qquad (6)$$

with the velocity V_I determined by the mechanism of impurity erosion.

The boundary conditions to Eqs. (2)–(6) at the LCMS, $x = 0$, imply (i) a given e-folding length δ for n, T, and n_Z determined by the transport in the scrape-off layer, (ii) recycling of the main particles, i.e., the balance of the neutral influx and plasma outflow, and (iii) a prescribed influx of impurity neutrals, Γ_I. At the interface of the edge region with the core, $x = x_{\mathrm{core}}$ (i), the plasma and heat flux densities, n_{core} and q_{core}, respectively, are fixed and (ii) the densities of neutrals and of low-Z radiant impurity species reduce to zero.

III. Analytical consideration.—For a qualitative analysis, we consider stationary states with $\frac{\partial\cdots}{\partial t} = 0$. An integral of Eq. (3) over the width of the edge layer gives a power balance for this region:

$$q_{\mathrm{core}} = 3\Gamma_p T_L + \Gamma_p E_i + \kappa_\perp \frac{T_L}{\delta} + q_{\mathrm{rad}}. \qquad (7)$$

Here the first term is the convective energy losses, the second one is losses on ionization of recycling neutrals, the third one is with plasma heat conduction, and the last one is with impurity radiation. The density of the charge particle flux through the LCMS, Γ_p, can be estimated as $D_\perp(n_{\mathrm{core}}/L_n)$, where the e-folding length of the density at the plasma edge is determined by the penetration depth of recycling neutrals. From Eq. (4), we obtain $L_n \approx 1/(\sigma_* n_{\mathrm{core}})$ with $\sigma_* = \sqrt{[m_i(k_i + k_{cx})k_i]/T}$ and thus

$$\Gamma_p \approx \alpha D_\perp(n_{\mathrm{core}}, T) n_{\mathrm{core}}^2 \sigma_*(T). \qquad (8)$$

The numerical factor α is due to the density dependence of the particle diffusivity $D_\perp$ [10]. For that given by Eq. (1), $\alpha \approx 0.1$ is a reasonable estimate. A numerical modeling (see Ref. [10] and the section below) predicts that the plasma temperature at the LCMS, T_L, is by a factor of 2 lower than the mean temperature in the edge region, T.

In order to assess the flux density of radiation losses, $q_{\mathrm{rad}} = \int Q_{\mathrm{rad}} dx$, we take into account that the cooling rates of all radiant species with $Z \le Z_{\mathrm{Li}}$ are close to each other [12] and one can introduce an effective L_{rad}. Therefore $Q_{\mathrm{rad}} \approx n n_{\mathrm{rad}} L_{\mathrm{rad}}$ with $n_{\mathrm{rad}} = \sum_{Z=1}^{Z_{\mathrm{Li}}} n_Z$. An equation for n_{rad} results from the summation of the continuity equations (5) over Z. By taking into account that in the region where Li-like particles are ionized they dominantly contribute to n_{rad} [11], one gets

$$\frac{\partial}{\partial x}\left(-D_\perp \frac{\partial n_{\mathrm{rad}}}{\partial x}\right) \simeq k_i^0 n n_0 - k_i^{Z_{Li}} n n_{\mathrm{rad}}. \qquad (9)$$

By replacing the plasma density and temperature profiles with their characteristic values in the edge region, Eqs. (6) and (9) can be integrated analytically [13]. With the solution found, we obtain

$$q_{\mathrm{rad}} = \Gamma_I \frac{L_{\mathrm{rad}}}{k_i^{Z_{\mathrm{Li}}}} \frac{l_0}{l_{\mathrm{rad}} + l_0}, \qquad (10)$$

where $l_0 = V_I/(k_i^0 n_{\mathrm{core}})$ and $l_{\mathrm{rad}} = \sqrt{D_\perp/(k_i^{Z_{\mathrm{Li}}} n_{\mathrm{core}})}$ are the penetration depths of impurity neutrals and radiant species, respectively.

Thus, for given heat flux and plasma densities at the core boundary, Eq. (7) provides a nonlinear algebraic equation for the edge temperature. As an example, Fig. 1 shows T versus n_{core} computed for the parameters of Texas Experimental Tokamak (TEXT) [4]: $R = 1$ m, $B_T = 2.8$ T, $q_{\mathrm{core}} = 3.5$ W/cm^2, $q = 3$. First, consider the curve 1 found without impurity radiation. One can see that at $n_{\mathrm{cr}} \approx 0.8 \times 10^{14}$ cm^{-3} there is a principle change in the density dependence of the temperature. This critical condition corresponds to the transition into the regime with the edge transport dominated by DRB modes, i.e., $D_{gB} = D_{\mathrm{DRB}}$. The convective loss term dominates in the right-hand side of Eq. (7) and, by making use of Eq. (8) with the estimate for L_n given above, one gets the dependence of n_{cr} on other parameters:

$$n_{\mathrm{cr}} \sim q_{\mathrm{core}}^{0.24} \frac{B_T^{0.48}}{q^{0.6}} \sim q_{\mathrm{core}}^{0.24} \frac{I_p^{0.6}}{B_T^{0.12}}, \qquad (11)$$

where I_p is the plasma current. For Ohmically heated plasmas with $q_{\mathrm{core}} \sim I_p$, the current dependence of n_{cr} is close to Greenwald scaling [3], $n_{\mathrm{core}} \sim I_p$. With additional

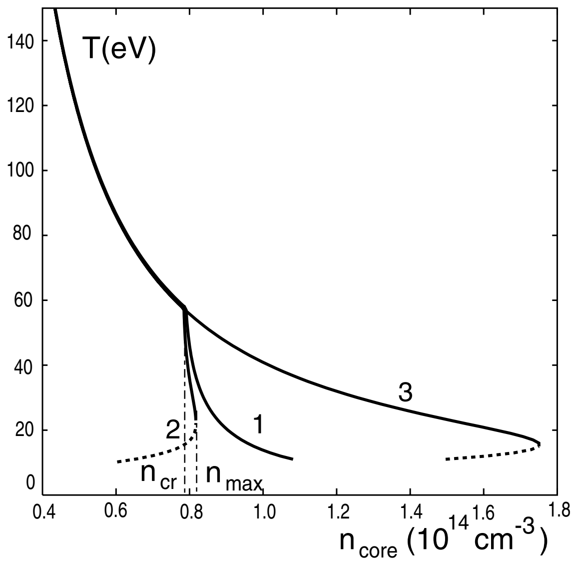

FIG. 1. Edge temperature versus core density computed without (curve 1), with (curve 2) impurity radiation, and under assumption $D_\perp = D_{gB}$ in the whole parameter range (curve 3). (The broken branches of curves 2 and 3 correspond to unstable states.)

heating, q_{core} is independent of the current and one gets $n_{\mathrm{max}} \sim I_p^{0.6}$. The deviation from the Greenwald scaling is probably due to neglected parametric dependence of the factor g in D_{gB}.

When n_{cr} is exceeded, T drops and particle flux grows very fast with the density. Indeed, σ_* changes weakly for $T > 15$ eV and $L_n \sim 1/n_{\mathrm{core}}$; besides $\rho_e^2 \sim T$ and $\nu_{ei} \sim n/T^{3/2}$. Thus, $D_{\mathrm{DRB}} \sim n^2/\sqrt{T}$, $\Gamma_p \sim n^4/\sqrt{T}$, and, by taking into account that the convective energy loss dominates in the total one, i.e., $q_{\mathrm{core}} \approx 3\Gamma_p T$, one gets $T \sim n_{\mathrm{core}}^{-8}$ and $\Gamma_p \sim n_{\mathrm{core}}^8$. As a result, impurity radiation can rise dramatically because (i) the influx of impurities is normally proportional to Γ_p and (ii) the lifetime of radiant particles increases when their ionization rate drops with decreasing temperature. The curve 2 in Fig. 1 shows T versus n_{core} calculated by taking into account the radiation of carbon impurity released into the plasma with $\Gamma_I = 0.02\Gamma_p$. The temperature independent erosion coefficient was assumed here by implying that at low T carbon physical sputtering [14] is replaced by chemical erosion [15]. One can see that no stationary states exist for $n_{\mathrm{core}} > n_{\mathrm{max}}$; i.e., the latter is the density limit set in by radiation. However, the extreme proximity of n_{max} to n_{cr} shows the principle importance of the transition to the DRB dominated transport regime for the level of n_{max}. The curve 3 in Fig. 1 demonstrates $T(n_{\mathrm{core}})$ computed by assuming $D_\perp \equiv D_{gB}$ in the whole parameter range. One can see that the maximum density is here significantly higher than with $D_\perp$ from Eq. (1). The density limit at n_{max} is close to an experimental one of 0.8×10^{14} cm^{-3} [4].

Normally, from the fact that the density limit is ultimately owing to radiation, one would guess that the absolute value of n_{max} should be sensitive to the ion effective charge Z_{eff}. This is, however, not the case. By increasing the carbon erosion coefficient by an order of magnitude from 0.02 to 0.2, Z_{eff} was changed in the range 1.1–2. That led to an alteration of n_{max} from 0.82 to 0.74×10^{14} cm^{-3}, i.e., by 10% only. The cause is a very nonlinear variation of q_{rad} due to $k_i^{Z_{\mathrm{Li}}} \sim \exp[-(I_{Z_{\mathrm{Li}}}/T)]$, where $I_{Z_{\mathrm{Li}}}$ is the ionization potential of Li-like impurity ions [11]. For $n_{\mathrm{core}} < n_{\mathrm{cr}}$ where T is high enough q_{rad} is less than 10% of q_{core}. In the DRB transport regime, T drops and q_{rad} increases very fast with n_{core}. Namely, the nonlinear character of q_{rad} behavior but not its absolute level is responsible for the bifurcation at n_{max}, where radiation losses are 20%–25% of the input power. Thus, similarly to the Greenwald limit [1,3], n_{max} is practically independent of Z_{eff} and radiation.

IV. Results of numerical modeling.—In order to validate the results of our analytical consideration, Eqs. (2)–(6) were integrated numerically. Figure 2(a) shows temperature profiles attained in stationary states for different n_{core} without impurity radiation. One can see the noticeable change in the profile and reduction of the temperature at the LCMS when n_{core} is ramped up from

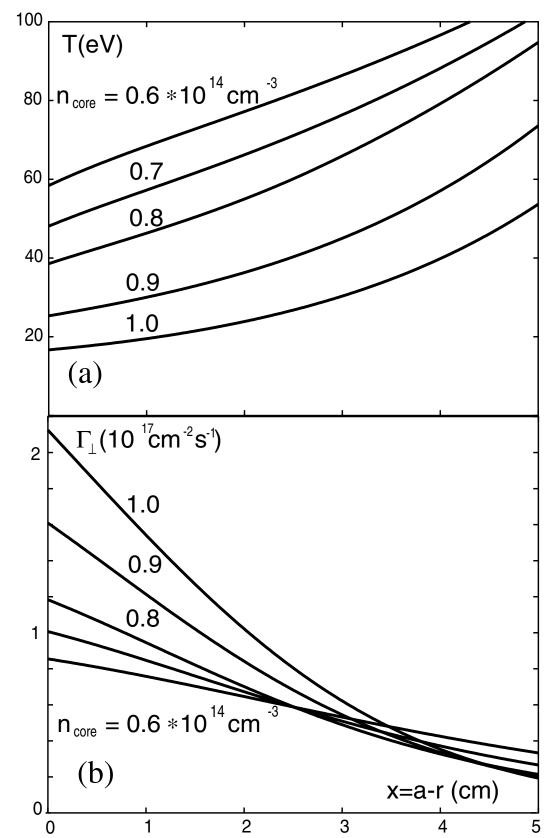

FIG. 2. Stationary profiles of the plasma temperature and particle flux density computed without impurity radiation.

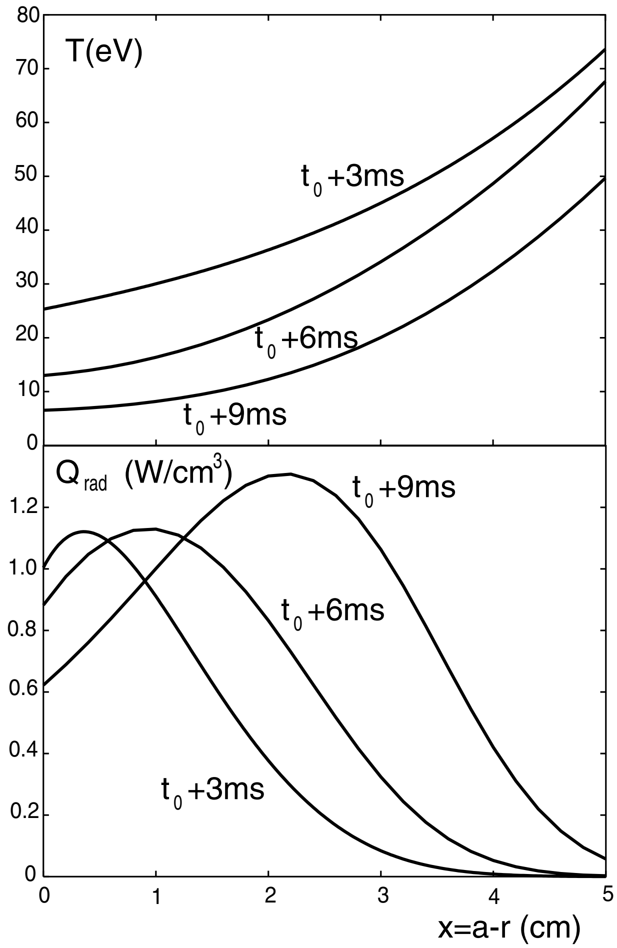

FIG. 3. Time evolution of the plasma temperature and radiation density profiles for $n_{\mathrm{core}} = 0.9 \times 10^{14}\ \mathrm{cm}^{-3} > n_{\mathrm{cr}}$ computed by taking impurity radiation into account (t_0 is the time when the release of carbon impurity is initiated).

0.8 to $0.9 \times 10^{14}\ \mathrm{cm}^{-3}$. This indicates a transition in the edge transport, which also manifests itself in the increase of the particle flux shown in Fig. 2(b).

The effect of radiation was simulated by starting the release of impurity neutrals after establishment of a stationary equilibrium. For $n_{\mathrm{core}} \leq 0.8 \times 10^{14}\ \mathrm{cm}^{-3} \lesssim n_{\mathrm{cr}}$, this did not lead to any significant change in the temperature profile even after 1 s. When n_{core} was increased up to $0.9 \times 10^{14}\ \mathrm{cm}^{-3} \gtrsim n_{\mathrm{cr}}$, the impurity release led to a very fast time evolution of the temperature and radiation power density profiles shown in Fig. 3. Finally, this evolution terminates into a radiative collapse of the plasma.

V. Conclusion.—The model proposed predicts that the edge transport becomes dominated by drift resistive ballooning modes when the plasma density exceeds the critical level n_{cr}. As a result, the particle losses increase and the edge temperature drops strongly, in agreement with observations done on many tokamaks in the vicinity of density limit [1,3,4]. Consequently, the radiation losses from impurities increase drastically and, according to the analytical consideration, stationary states do not exist if the plasma density is raised above the limit $n_{\max}$ slightly higher than n_{cr}. Numerical modeling shows that with usual impurities such as carbon a radiative collapse of the plasma occurs directly when n_{cr} is exceeded.

In this Letter, the radial radiative collapse has been considered as a density limiting phenomenon. In divertor machines, however, poloidally asymmetric X-point MARFE normally develops first. For a consideration of this case, a further development of the model is needed. Qualitatively, one could expect that $n_{\max}$ approaches even closer to n_{cr}. On the contrary, if the release of impurity diminishes strongly with decreasing temperature, e.g., in the case of pure physical sputtering, $n_{\max}$ noticeably exceeding n_{cr} can be expected. This would explain the increase of the density limit under conditions of wall boronization and siliconization [16].

[1] M. Greenwald, Plasma Phys. Controlled Fusion **44**, R27 (2002).
[2] Y. Shimomura *et al.*, Nucl. Fusion **41**, 309 (2001).
[3] M. Greenwald *et al.*, Nucl. Fusion **28**, 2199 (1988).
[4] D. L. Brower *et al.*, Phys. Rev. Lett. **67**, 200 (1991).
[5] P. N. Guzdar *et al.*, Phys. Fluids B **5**, 3712 (1993).
[6] B. N. Rogers *et al.*, Phys. Rev. Lett. **81**, 4396 (1998).
[7] X. Q. Xu *et al.*, Phys. Plasmas **10**, 1773 (2003).
[8] A. Zeiler *et al.*, Phys. Plasmas **5**, 2654 (1998).
[9] B. Scott, Plasma Phys. Controlled Fusion **39**, 471 (1997).
[10] M. Z. Tokar, Plasma Phys. Controlled Fusion **35**, 1119 (1993).
[11] M. Z. Tokar, Plasma Phys. Controlled Fusion **36**, 1819 (1994).
[12] X. Bonnin *et al.*, *Atomic and Plasma-Material Interaction Data for Fusion* [Supplement to Nucl. Fusion **2**, 117 (1992)].
[13] M. Z. Tokar, Plasma Phys. Controlled Fusion **39**, 569 (1997).
[14] W. Eckstein *et al.*, *Atomic and Plasma-Material Interaction Data for Fusion* [Supplement to Nucl. Fusion **1**, 51 (1991)].
[15] P. Bogen and D. Rusbüldt, Nucl. Fusion **32**, 1057 (1992).
[16] J. Rapp *et al.*, J. Nucl. Mater. **290–293**, 1148 (2001).