# Elliptic Flow at Large Transverse Momenta from Quark Coalescence

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We show that hadronization via quark coalescence enhances hadron elliptic flow at large $p_\perp$ relative to that of partons at the same transverse momentum. Therefore, compared to earlier results based on covariant parton transport theory, more moderate initial parton densities $dN/d\eta(b=0) \sim 1500$–$3000$ can explain the differential elliptic flow $v_2(p_\perp)$ data for Au + Au reactions at $\sqrt{s} = 130$ and 200$A$ GeV from BNL RHIC. In addition, $v_2(p_\perp)$ could saturate at about 50% higher values for baryons than for mesons. If strange quarks have weaker flow than light quarks, hadron $v_2$ at high $p_\perp$ decreases with relative strangeness content.

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*Introduction.*—The goal of relativistic heavy-ion collision experiments is to produce *macroscopic* amounts of deconfined partonic matter and study its collective behavior. One of the important experimental probes of collective dynamics in $A + A$ reactions is differential elliptic flow [1], $v_2(p_\perp) \equiv \langle \cos(2\phi) \rangle_{p_\perp}$, the second Fourier moment of the azimuthal momentum distribution for a given $p_\perp$. Measurements of elliptic flow at high transverse momentum provide important constraints about the density and effective energy loss of partons [3,4].

Recent data from the BNL Relativistic Heavy Ion Collider (RHIC) for Au + Au reactions at $\sqrt{s_{NN}} = 130$ and 200 GeV show a remarkable saturation property of elliptic flow in the region 2 GeV $< p_\perp <$ 6 GeV with $v_2$ reaching up to 0.2 [5–8]. The saturation pattern, which corresponds to a factor of 2 azimuthal angle asymmetry of high-$p_\perp$ particle production relative to the reaction plane, is still waiting for theoretical explanation.

The saturation and eventual decrease of $v_2$ at high $p_\perp$ has been demonstrated as a consequence of finite inelastic parton energy loss [3]. Though the qualitative features in the data were explained, for realistic diffuse nuclei the calculations show a rapid *decrease* of $v_2$ above $p_\perp >$ 3–4 GeV, contrary to the saturation out to $p_\perp \approx 6$ GeV seen in the data.

Calculations of elliptic flow based on ideal (nondissipative) hydrodynamics [9–12] can reproduce the low $p_\perp < 2$ GeV data at RHIC remarkably well; however, they overshoot the data above $p_\perp > 2$ GeV [5,13]. The lack of saturation is due to the assumption of zero mean free path and that local equilibrium can be maintained throughout the evolution [4,14].

Covariant parton transport theory [4,15–19] overcomes this problem via replacing the assumption of local equilibrium by that of a finite local mean free path $\lambda(s, x) \equiv 1/\sigma(s)n(x)$. The theory then naturally interpolates between free streaming ($\lambda = \infty$) and ideal hydrodynamics ($\lambda = 0$). Several studies confirm that initial parton densities [20] and elastic $2 \rightarrow 2$ parton cross sections estimated from perturbative QCD, $dN_g/d\eta(b=0) \sim 1000$ and $\sigma_{gg \rightarrow gg} \approx 3$ mb, generate too small collective effects at RHIC [4,16,17,19]. Nevertheless, quantitative agreement with the $v_2(p_\perp)$ data is possible, provided initial parton densities and/or cross sections are enhanced by an order of magnitude to $\sigma dN_g/d\eta(b=0) \sim 45\,000$ mb [4]. A similar enhancement is indicated by the pion interferometry data as well [19]. The origin of such an opaque parton environment is the RHIC "opacity puzzle."

To compare to the experiments, parton transport models also have to incorporate the hadronization process. The studies mentioned above considered two simple schemes: 1 parton $\rightarrow 1\pi$ hadronization, motivated by parton-hadron duality, and independent fragmentation. An alternative model of hadronization is quark coalescence, in which the relevant degrees of freedom are not free partons but massive (dressed) valence quarks. Gluons are assumed to have converted to quarks; therefore, there are no dynamical gluons considered.

Quark coalescence has been applied successfully in the algebraic coalescence rehadronization [21] and microscopic coalescence rehadronization [22] models to explain particle abundances and spectra in heavy-ion collisions. It was also suggested recently in Ref. [2] as an explanation for the anomalous meson/baryon ratio and features of the elliptic flow data at RHIC. In this Letter, we show that hadronization via quark coalescence can resolve most of the opacity puzzle because it leads to an amplification of elliptic flow at high $p_\perp$.

*Quark coalescence.*—The usual starting point of coalescence models is the statement that the invariant spectrum of produced particles is proportional to the *product* of the invariant spectra of constituents [23]. This means that (assuming that different quark and antiquark distributions are the same) the hadron spectra at midrapidity are given by those of partons via

  

$$\frac{dN_B}{d^2p_\perp}(\vec{p}_\perp) = C_B(p_\perp)\left[\frac{dN_q}{d^2p_\perp}(\vec{p}_\perp/3)\right]^3,$$
$$\frac{dN_M}{d^2p_\perp}(\vec{p}_\perp) = C_M(p_\perp)\left[\frac{dN_q}{d^2p_\perp}(\vec{p}_\perp/2)\right]^2, \qquad (1)$$

where the coefficients $C_M$ and $C_B$ are the probabilities for $q\bar{q} \to$ meson and $qqq \to$ baryon coalescence. We allow for $p_T$ dependent coalescence factors because more careful treatment of the coalescence problem [24] shows that such a dependence may arise, e.g., due to kinematic (energy) factors or strong radial flow. This, however, does not influence elliptic flow because it is a ratio from which the coalescence factors drop out [see Eq. (3)].

These relations are valid only for rare processes. This is *not* the case at high constituent phase space densities, when most quarks recombine into hadrons, and hence the number of hadrons is *linearly* proportional to that of quarks, $dN_h(p_\perp) \propto dN_q(p_\perp)$.

At lower constituent densities, coalescence processes become relatively rare and therefore the usual coalescence formalism works. On the other hand, most quarks hadronize via fragmentation into hadrons. Nevertheless, depending on how quickly the parton phase space density drops with increasing $p_\perp$, there can be a region of *hadron* transverse momenta that is populated dominantly via coalescence. The reason for this is that hadrons from coalescence have *larger* momenta than the average quark momentum, $dN_h^{\rm coal}(p_\perp) = C_h[dN_q(p_\perp/n)]^n$, $(n = 2, 3)$, whereas hadrons from fragmentation carry only a *fraction* $z < 1$ of the initial quark momentum, $dN_h^{\rm frag}(p_\perp) \sim dN_q(p_\perp/z)$.

At very low parton densities, e.g., at very high transverse momentum, the fragmentation process wins, in accordance with the QCD factorization theorem. For example, a power law parton spectrum $dN_q/p_\perp dp_\perp \sim Ap_\perp^{-\alpha}$ implies $dN_h^{\rm coal}/dN_h^{\rm frag} \sim C_h A^{n-1} p_\perp^{-(n-1)\alpha} \to 0$ at high $p_\perp$.

Therefore, in heavy-ion collisions there can be three qualitatively different phase space regions. At very large transverse momenta particle production is dominated by independent parton fragmentation. At lower transverse momenta coalescence prevails, which region can itself be subdivided into two parts: a very low $p_\perp$(high phase space density) region where Eq. (1) is *not* applicable, and a moderate density (higher $p_\perp$) region, where Eq. (1) is valid. Because the density of produced particles depends on the centrality of the collision, the "boundaries" of these regions depend on centrality. Only detailed quantitative studies [25,26] of the relative contributions of the various hadronization processes, which are beyond the scope of this Letter, could determine where the exact bounds are. Alternatively, the limits can be deduced from comparison with the experimental data.

*Anisotropic flow.*—For brevity we discuss only elliptic flow as the most important and interesting case. However, Eqs. (2), (4), and all conclusions below also apply (i) when azimuthal anisotropies $v_k(p_\perp) \equiv \langle\cos(k\phi)\rangle_{p_\perp}$ of any order are present and (ii) to any anisotropy coefficient $v_k$ instead of $v_2$, even in the former most general case.

In the coalescence region, meson and baryon elliptic flow are given by that of partons via

$$v_{2,M}(p_\perp) \approx 2v_{2,q}\left(\frac{p_\perp}{2}\right), \qquad v_{2,B}(p_\perp) \approx 3v_{2,q}\left(\frac{p_\perp}{3}\right), \qquad (2)$$

as follows from Eq. (1) and $v_2 \ll 1$. For example, if partons have only elliptical anisotropy, i.e., $dN_q/p_\perp dp_\perp d\Phi = (1/2\pi)dN_q/p_\perp dp_\perp[1 + 2v_{2,q}\cos(2\Phi)]$, then

$$v_{2,B}(p_\perp) = \frac{3v_{2,q}(p_\perp/3) + 3v_{2,q}^3(p_\perp/3)}{1 + 6v_{2,q}^2(p_\perp/3)},$$
$$v_{2,M}(p_\perp) = \frac{2v_{2,q}(p_\perp/2)}{1 + 2v_{2,q}^2(p_\perp/2)}. \qquad (3)$$

Figure 1 illustrates the effect of quark coalescence on baryon and meson elliptic flow compared to parton elliptic flow. The latter is shown schematically by the solid line. At small transverse momenta, parton $v_2(p_\perp) \propto p_\perp^2$, as follows from general analyticity considerations. This region before $v_2$ becomes approximately linear in $p_\perp$ could be relatively small (depending on the effective mass of partons). At higher transverse momenta $p_\perp >$ 1–2 GeV, parton elliptic flow saturates as predicted by parton transport [4], and then, possibly already above $p_\perp \gtrsim 4$ GeV, decreases according to perturbative QCD parton energy loss calculations [3]. The curve for baryon (meson) elliptic flow has been obtained by simply rescaling the parton curve by a factor of 3 (2) both vertically and horizontally. As discussed above, for very low and very high $p_\perp$, we boldly use Eq. (2) beyond its region of applicability but doing so does not affect the discussion.

There are three qualitatively different regimes in Fig. 1:

(i) In the small $p_\perp$ region where $v_2(p_\perp)$ increases faster than linearly, $v_{2,B} < v_{2,M} < v_{2,q}$. It is not clear to

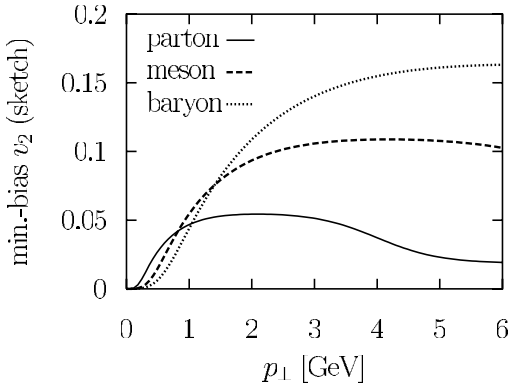


FIG. 1. Qualitative behavior of baryon and meson elliptic flow as a function of $p_\perp$ from quark coalescence.

what extent the coalescence picture is applicable in this region but it is interesting that the data do exhibit such a behavior. This ordering follows naturally from hydrodynamics, where flow decreases with increasing particle mass [10–12]. Similar mass dependence could also arise in a coalescence model because heavier hadrons can be formed by quarks with larger relative momentum (ignored in the current approach).

(ii) In the intermediate $p_\perp$ region where $v_2(p_\perp)$ depends linearly on transverse momentum, $v_{2,B} \approx v_{2,M}$.

(iii) At large $p_\perp$, where parton $v_2(p_\perp)$ increases slower than linearly, baryon flow becomes larger than that of mesons, $v_{2,B} > v_{2,M} > v_{2,q}$, by as much as 50%. Parton collective flow saturation, predicted for $p_\perp >$ 1–2 GeV by parton transport [4], results in saturating meson/baryon flow at $p_\perp >$ 2–4 GeV that is *amplified two/threefold* compared to that of partons. Saturation sets in at 50% higher $p_\perp$ for baryons than for mesons. In addition, any eventual decrease of parton elliptic flow at very high $p_\perp$ would happen at 2 to 3 times larger $p_\perp$ for hadrons.

The high-$p_\perp$ results above strongly differ from those obtained in Ref. [27]. The reason is that, unlike Eq. (2), in Ref. [27] the coalescence of quarks was considered to be independent of their relative momenta and therefore hadron elliptic flow at high $p_\perp$ was similar to that of a high-$p_\perp$ quark.

If not all quarks show the same elliptic flow, further differentiation occurs because in that case

$$v_{2,B=abc}(p_\perp) \approx v_{2,a}(p_\perp/3) + v_{2,b}(p_\perp/3) + v_{2,c}(p_\perp/3),$$
$$v_{2,M=\bar{a}b}(p_\perp) \approx v_{2,\bar{a}}(p_\perp/2) + v_{2,b}(p_\perp/2). \quad (4)$$

For example, strange quarks may have a smaller $v_2(p_\perp)$ than light quarks at high $p_\perp$ because heavy quarks are expected to lose less energy in nuclear medium [28] while at low $p_\perp$ due to the mass dependence of hydrodynamic flow. If $v_2^s < v_2^q$, elliptic flow decreases with increasing relative strangeness content within the baryon and meson bands, i.e., $v_2^p > v_2^\Lambda \approx v_2^\Sigma > v_2^\Xi > v_2^\Omega$ and $v_2^\pi > v_2^K > v_2^\phi$.

*Possible solution to the opacity puzzle.*—While hadronization via 1 parton $\rightarrow 1\pi$ or independent fragmentation approximately preserves elliptic flow at high $2 < p_\perp <$ 6 GeV [4], quark coalescence enhances $v_2$ 2 times for mesons and 3 times for baryons. Hence, the same hadron elliptic flow can be reached from 2–3 times smaller parton $v_2$, which requires smaller parton opacities, i.e., initial parton densities and/or cross sections. The amplification also allows the RHIC $v_2$ data to exceed geometric upper bounds derived based on a nuclear absorption model [29](the data are compatible with those constraints only for idealistic "sharp sphere" nuclear distributions [2]). Those bounds apply to the parton $v_2$ and thus are 2 (3) times higher for mesons/baryons.

To determine the reduction of parton opacity quantitatively, we rely on the results of Ref. [4] that computed gluon elliptic flow as a function of the transport opacity, $\chi \equiv \int dz \sigma_{\rm tr} \rho(z) \approx \sigma dN/d\eta\,/(940\ {\rm mb})$, from elastic parton transport theory for a minijet scenario of Au + Au at $\sqrt{s} = 130A$ GeV at RHIC. Those results can be conveniently parametrized as $v_2(p_\perp, \chi) = v_2^{\rm max}(\chi) \tanh[p_\perp/p_0(\chi)]$, where $v_2^{\rm max}$ is the saturation value of elliptic flow, while $p_0$ is the $p_\perp$ scale above which saturation sets in. For the estimates here we assume that all gluons convert, e.g., via $gg \rightarrow qq$, to quarks of similar $p_\perp$ and hence $v_2^q(p_\perp) = v_2^g(p_\perp)$.

As shown in Fig. 2, the increase of elliptic flow with opacity is weaker than linear, $v_2^{\rm max} \sim \chi^{0.61}$. Therefore, a 2–3 times smaller parton elliptic flow, which is needed to match the charged particle $v_2$ data from RHIC in our coalescence scenario, corresponds to 3–6 times smaller parton opacities $\sigma dN/d\eta(b=0) \sim$ 7000–15 000 mb than those found in Ref. [4]. The lower (upper) value applies if high-$p_\perp$ hadrons are mostly baryons (mesons). Based on preliminary PHENIX data [30] showing $\pi_0/h^\pm \approx 0.5$ between $2 < p_\perp < 9$ GeV, one may expect mesons/baryons $\approx 1$, in which case $\sigma dN/d\eta(b=0) \approx$ 10 000 mb.

In Ref. [4] only collective flow was considered and the parton opacity at RHIC was extracted using elliptic flow data from the reaction plane analysis. Taking into account nonflow effects that contributed up to 15%–20% [31] to the first elliptic flow measurements, parton opacities should be further reduced by 25% to $\sigma dN/d\eta(b=0) \sim$ 5000–10 000 mb. For a typical elastic $gg \rightarrow gg$ cross section of 3 mb, this corresponds to an initial parton density $dN/d\eta(b=0) \sim$ 1500–3000, only 1.5–3 times above the Eskola-Kajantie-Ruuskanen-Tuominen perturbative estimate [20].

The remaining much smaller discrepancy is comparable to theoretical uncertainties. For example, perturbative cross section and parton density estimates may be too low. If most hadrons formed via coalescence, the

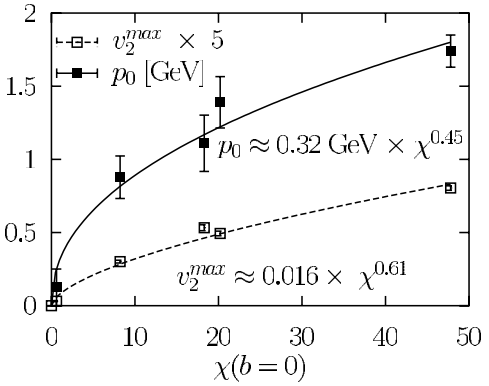


FIG. 2. Fit parameters $v_2^{\rm max}$ and $p_0$ as a function of transport opacity $\chi(b=0)$, for the $v_2(p_\perp)$ results in Ref. [4].

observed hadron multiplicity $dN_h/d\eta \approx 1000$ would imply much higher initial parton densities $dN/d\eta \sim$ 2000–3000. Constituent quark cross sections, $\sigma_{qq} \approx$ 4–5 mb, also point above the $\approx 3$ mb perturbative estimate. One effect that estimate ignores is the enhancement of parton cross sections $\sigma \propto \alpha_s^2/\mu^2$ due to the decrease of the self-consistent Debye screening mass $\mu \sim gT_{\rm eff}(\tau)$ during the expansion. Finally, the contribution of inelastic processes, such as $gg \leftrightarrow ggg$, to the opacity has also been neglected so far. A preliminary study shows [17] that this contribution can be similar to that of elastic processes.

*Summary.*—In this Letter, we studied elliptic flow of hadrons formed from coalescence of quarks with similar momenta. At high $p_\perp > 2$ GeV we found an enhancement of elliptic flow compared to that of partons. With the enhancement, moderate initial parton densities $dN_g/d\eta \sim 1500$–3000 are sufficient to account for the charged particle elliptic flow data from RHIC, providing a possible solution to the RHIC opacity puzzle. At low $p_\perp < 1$ GeV, on the other hand, hadron elliptic flow is suppressed.

Quark coalescence gives a weaker baryon flow than meson flow at low $p_\perp < 0.5$–1 GeV, while the opposite, $v_2^B > v_2^M$, at high $p_\perp > 2$–3 GeV. Assuming all partons have similar elliptic flow, $v_2^B \approx 1.5 v_2^M$ at high $p_\perp$. If, on the other hand, strange quarks show weaker flow than light quarks, elliptic flow at high $p_\perp$ is ordered by relative strangeness content, such that $v_2^p > v_2^\Lambda \approx v_2^\Sigma > v_2^\pi > v_2^K > v_2^\phi$, $v_2^{\Lambda,\Sigma} > v_2^\Xi > v_2^K$, and $v_2^\Xi > v_2^\Omega \approx 3v_2^\phi/2$. These predictions can be readily tested in current and future heavy-ion collision experiments.

We emphasize that the quark coalescence picture and therefore our flow ordering predictions strongly rely on the assumption that quark degrees of freedom are dominant at hadronization. Therefore, experimental support for our predictions may indicate the formation of deconfined nuclear matter in heavy-ion collisions at RHIC energies.

We also note that at very high $p_\perp$ one expects a transition from hadronization via quark coalescence to independent fragmentation. An experimental signature of this may be the reduction of baryon $v_2$ below meson $v_2$.

When this work was in its final stage, two preprints addressing baryon to meson ratio at high $p_\perp$, Refs. [25,26], were submitted to the arXiv.org e-print server. While these studies mainly focus on baryon and meson yields, the underlying physical arguments are very similar to those presented here.

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