Perturbative QCD Analysis of the Nucleon's Pauli Form Factor $F_2(Q^2)$

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We perform a perturbative QCD analysis of the nucleon's Pauli form factor $F_2(Q^2)$ in the asymptotically large Q^2 limit. We find that the leading contribution to $F_2(Q^2)$ has a $1/Q^6$ power behavior, consistent with the well-known result in the literature. Its coefficient depends on the leading- and subleading-twist light-cone wave functions of the nucleon, the latter describing the quarks with one unit of orbital angular momentum. We also derive at the logarithmic accuracy the asymptotic scaling $F_2(Q^2)/F_1(Q^2) \sim (\log^2 Q^2/\Lambda^2)/Q^2$ which describes recent Jefferson Lab data well.

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The electromagnetic form factors are fundamental observables of the nucleon containing important information about its internal nonperturbative structure. Since the first measurement in the mid 1950s, experimental studies of these observables have become an active frontier in nuclear and particle physics. More recently, with the development of novel experimental techniques, the magnetic form factor of the proton and electric and magnetic form factors of the neutron have been measured with unprecedented precision at Jefferson Lab and other facilities around the world [1]. In particular, the Pauli form factor of the proton F_2 extracted from the recoil polarizations at Jefferson Lab reveals a significant difference from previous data and theoretical expectations [2,3]. The data show that the ratio of Dirac to Pauli form factor $Q^2F_2(Q^2)/F_1(Q^2)$ continues climbing at the largest Q^2's measured and seems to scale as $\sqrt{Q^2}$. The result has sparked myriad speculations on its implication about the underlying microscopic structure of the proton [4–8].

A simple dimensional counting rule was devised in Refs. [9,10] to determine the dominant power contribution to hadronic form factors at large momentum transfer. For the hadron helicity-conserving form factor $F_1(Q^2)$, it predicts a dominant scaling behavior $1/Q^4$. The power counting can be justified by QCD factorization theorems which separate short-distance quark-gluon interactions from soft hadron wave functions [11–18]; see Fig. 1. Recently, a more sophisticated formalism has been developed to include higher-order perturbative QCD (PQCD) resummation, or Sudakov form factor, which treats contributions from the small-x partons more appropriately [18–22]. The procedure extends the applicability of leading-order PQCD predictions to moderate Q^2, but does not change the dominant power-law behavior significantly [17,22,23].

The literature on PQCD studies of $F_2(Q^2)$, however, is much meager. Since $F_2(Q^2)$ is related to the hadron helicity-flip amplitude, its power behavior is suppressed compared to $F_1(Q^2) \sim 1/Q^4$, and a natural expectation is then $1/Q^6$. This is confirmed by calculations where the quark masses are the mechanism for the helicity flip [9,24]. Since the up and down current quark masses are negligible, the dominant mechanism for the spin flip in QCD comes from the quark orbital angular momentum and the polarization of an extra gluon which must be included to maintain gauge invariance [25]. A generalized power counting including the parton orbital angular momentum validates the expected scaling law for $F_2(Q^2)$ [26]. An actual PQCD calculation for $F_2(Q^2)$ requires the extension of the usual formalism to include the quark orbital angular momentum; a technology has not yet been systematically developed in the literature. The recent development in classification of the light-cone wave functions [27,28] provides the necessary ingredient to perform these calculations.

In this paper we report on a first PQCD calculation of $F_2(Q^2)$ at asymptotically large momentum transfer Q^2. Modulo logarithms, we confirm that the leading contribution to $F_2(Q^2)$ goes like $1/Q^6$. The coefficient of the leading-power term depends on leading-order (twist-three) and next-to-leading order (twist-four) light-cone wave functions. The latter is the probability amplitude for one of the quarks to carry one unit of orbital angular momentum. We compute the leading-order perturbative kernel and estimate the coefficient using models for the light-cone wave functions. We comment on the role of the Sudakov form factor in regulating possible end-point singularities in the phase space integrals. We also derive with the logarithmic accuracy the asymptotic scaling $F_2(Q^2)/F_1(Q^2) \sim (\log^2 Q^2/\Lambda^2)/Q^2$ which we find to describe the data unexpectedly well.

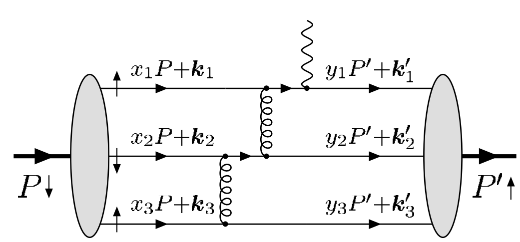

FIG. 1. A leading QCD diagram contributing to the nucleon's form factors.

To start with, we choose a coordinate system for the process $P + \gamma^* \to P'$ such that P and P' form conjugate light-cone vectors. We let the light-cone components P^+ and P'^- be large in the asymptotic limit, $Q^2 = 2P^+P'^-$. The $F_2(Q^2)$ form factor can be extracted from the helicity-flip matrix element

$$\langle P'_\uparrow | J^\mu | P_\downarrow \rangle = F_2(Q^2) \bar{u}_\uparrow(P') \frac{i\sigma^{\mu\alpha} q_\alpha}{2M} u_\downarrow(P), \tag{1}$$

where J^μ is the quark electromagnetic current and $u(P)$ is an on-shell spinor of the nucleon with momentum P.

A perturbative analysis of the above matrix element is done by computing leading-order Feynman diagrams with two gluon exchanges (for a typical graph see Fig. 1) in which the initial (final) nucleon contains three quarks with longitudinal momenta x_iP (y_iP') and transverse momenta $\boldsymbol{k}_i$ ($\boldsymbol{k}'_i$) of order $\Lambda_{\rm QCD}$. Two hard-gluon exchanges ensure that the three quarks in the final-state propagate collinearly after the injection of a large momentum transfer q^μ. Since up and down quarks are light, quark helicity is approximately conserved during the hard scattering. Therefore, to produce a nucleon-spin flip, the quark orbital angular momentum in the initial and final states must differ by one unit. The leading contribution to $F_2(Q^2)$ comes from the configurations in which the quarks in the initial or the final state carry a zero unit, and those in the other state carry one unit, of orbital angular momentum. For definiteness, let us assume that the final-state quarks are symmetric in azimuth.

To isolate the leading contribution, we expand the hard part of the diagram in $\boldsymbol{k}_i^2/Q^2$ in the limit of large Q^2 and $x_i \neq 0$ (a procedure dubbed the collinear expansion). We will comment on the $x_i \to 0$ case later. Since the final-state nucleon has no orbital angular momentum, we throw away all the subleading terms in $\boldsymbol{k}'_i$. On the other hand, the quarks in the initial state nucleon have one unit of orbital angular momentum, and hence the hard part must have linear terms in the quark transverse momenta. Therefore, the leading hard part we are interested in has a structure $\boldsymbol{k}_i T(x_1, x_2, x_3, y_1, y_2, y_3; Q^2)$.

Let us consider the simplification of the nucleon wave functions after the collinear expansion. For definiteness, we consider the proton form factor. The light-cone wave function for the final state is

$$|P_\uparrow\rangle_{1/2} = \frac{1}{12}\int \frac{[dx][d^2\boldsymbol{k}]}{\sqrt{x_1x_2x_3}} \psi_1(\kappa_1, \kappa_2, \kappa_3) \varepsilon^{abc} u^\dagger_{a\uparrow}(\kappa_1) \times \{u^\dagger_{b\downarrow}(\kappa_2) d^\dagger_{c\uparrow}(\kappa_3) - d^\dagger_{b\downarrow}(\kappa_2) u^\dagger_{c\uparrow}(\kappa_3)\}|0\rangle, \tag{2}$$

where the argument κ_i is a shorthand notation for $(x_i, \boldsymbol{k}_i)$, and the integration measures for the quark momenta are

$$[dx] \equiv dx_1 dx_2 dx_3 \delta(x_1 + x_2 + x_3 - 1),$$

$$[d^2\boldsymbol{k}] \equiv d^2\boldsymbol{k}_1 d^2\boldsymbol{k}_2 d^2\boldsymbol{k}_3 \delta^{(2)}(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3).$$

If the hard part has no dependence on $\boldsymbol{k}'_i$, we can ignore the transverse momentum dependence in the quark creation operators and define the twist-three amplitude

$$\Phi_3(x_1, x_2, x_3) = 2\int [d^2\boldsymbol{k}] \psi_1(\kappa_1, \kappa_2, \kappa_3).$$

Then the final nucleon state can be simplified to

$$|P_\uparrow\rangle_{1/2} = \frac{1}{24}\int \frac{[dx]}{\sqrt{x_1x_2x_3}} \Phi_3(x_1, x_2, x_3) \varepsilon^{abc} u^\dagger_{a\uparrow}(x_1) \times \{u^\dagger_{b\downarrow}(x_2) d^\dagger_{c\uparrow}(x_3) - d^\dagger_{b\downarrow}(x_2) u^\dagger_{c\uparrow}(x_3)\}|0\rangle. \tag{3}$$

Since the quarks in the initial nucleon state must have a total helicity 1/2, only the following wave function component is relevant [27,28]:

$$|P_\downarrow\rangle_{1/2} = \frac{1}{12}\int \frac{[dx][d^2\boldsymbol{k}]}{\sqrt{x_1x_2x_3}} \{\bar{k}_{1\perp}\psi_3 + \bar{k}_{2\perp}\psi_4\} \times (\kappa_1, \kappa_2, \kappa_3) \varepsilon^{abc} u^\dagger_{a\uparrow}(\kappa_2) \times \{u^\dagger_{b\downarrow}(\kappa_1) d^\dagger_{c\uparrow}(\kappa_3) - d^\dagger_{b\downarrow}(\kappa_1) u^\dagger_{c\uparrow}(\kappa_3)\}|0\rangle, \tag{4}$$

where we use the notation $\bar{k}_\perp \equiv k^x - ik^y$.

The spin-isospin structure of this wave function is exactly the same as that in Eq. (3). After the collinear expansion, the above wave function will be convoluted with a transverse momenta of quarks. We define the twist-four amplitudes via

$$\Phi_4(x_2, x_1, x_3) = 2\int \frac{[d^2\boldsymbol{k}]}{Mx_3} \boldsymbol{k}_3 \cdot \{\boldsymbol{k}_1\psi_3 + \boldsymbol{k}_2\psi_4\}(\kappa_1, \kappa_2, \kappa_3),$$

$$\Psi_4(x_1, x_2, x_3) = 2\int \frac{[d^2\boldsymbol{k}]}{Mx_2} \boldsymbol{k}_2 \cdot \{\boldsymbol{k}_1\psi_3 + \boldsymbol{k}_2\psi_4\}(\kappa_1, \kappa_2, \kappa_3).$$

Apart from a gluon-potential dependent term, the above light-cone amplitudes are the same as those defined in Ref. [29]. In the following, we include the former so that the result is gauge invariant. To the order that we are working, the gluon-potential terms arise from perturbative diagrams with explicit gluons attached to the nucleon blobs. These contributions are generally considered as dynamically suppressed [30].

We introduce effective nucleon wave functions for the initial state after integrating over the transverse momenta weighted with a momentum factor from the hard part,

$$|P_\downarrow\rangle_{1/2}[\boldsymbol{k}_1] = \frac{M}{24}\int \frac{[dx]}{\sqrt{x_1x_2x_3}} x_1 \Psi_4(x_2, x_1, x_3) \epsilon^{abc} u^\dagger_{a\uparrow}(x_1) \times \{u^\dagger_{b\downarrow}(x_2) d^\dagger_{c\uparrow}(x_3) - d^\dagger_{b\downarrow}(x_2) u^\dagger_{c\uparrow}(x_3)\}|0\rangle, \tag{5}$$

$$|P_\downarrow\rangle_{1/2}[\boldsymbol{k}_3] = \frac{M}{24}\int \frac{[dx]}{\sqrt{x_1x_2x_3}} x_3 \Phi_4(x_1, x_2, x_3) \epsilon^{abc} u^\dagger_{a\uparrow}(x_1) \times \{u^\dagger_{b\downarrow}(x_2) d^\dagger_{c\uparrow}(x_3) - d^\dagger_{b\downarrow}(x_2) u^\dagger_{c\uparrow}(x_3)\}|0\rangle, \tag{6}$$

where the argument on the left-hand side indicates the momentum being averaged.

Using the twist-three and -four amplitudes, we can write the $F_2(Q^2)$ form factor in the following factorized form:

$$F_2(Q^2) = \frac{1}{32}\int [dx][dy]\{x_3\Phi_4(x_1,x_2,x_3)T_\Phi(\{x\},\{y\}) + x_1\Psi_4(x_2,x_1,x_3)T_\Psi(\{x\},\{y\})\} \times \Phi_3(y_1,y_2,y_3), \quad (7)$$

where $\{x\} = (x_1, x_2, x_3)$. Let us see how to extract the hard part, T, in Fig. 1.

We use the nomenclature and strategy of Ref. [11] by calling the top quark line 1, the middle 2, etc., without committing them to a specific flavor. Given the spin-isospin structure of the initial and final-state nucleon wave functions, we assume that the first and third quarks have spin up and the second one spin down. Call the amplitudes T_i when the electromagnetic current acts on particle i. Clearly because of symmetry, T_3 can be obtained from T_1 by a suitable exchange of variables (see below). Using the SU(6) wave function and the quark charge weighting, we find the flavor structure of the hard part for the proton,

$$T^p(\{x\},\{y\}) = \frac{2e_u}{3}T_1 + \frac{e_u+e_d}{3}(T_2+T_3) + \frac{e_u}{3}(T_1' + T_3') + \frac{e_d}{3}T_2', \quad (8)$$

where we have omitted the argument of T_i on the right-hand side, and T_i' has y_1 and y_3 interchanged. For the neutron, $u \leftrightarrow d$.

To find the expression for the hard part, we must calculate perturbative diagrams displayed in Fig. 2. A straightforward evaluation yields

$$T_{1\Psi}(\{x\},\{y\}) = \frac{32M^2C_B^2}{Q^6}(4\pi\alpha_s)^2 x_3(G_{11} - G_{12}), \quad (9)$$

$$T_{1\Phi}(\{x\},\{y\}) = \frac{32M^2C_B^2}{Q^6}(4\pi\alpha_s)^2[-x_1G_{11} - \bar{x}_1G_{12}],$$

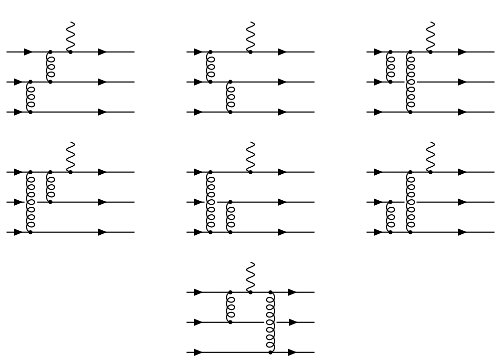

FIG. 2. Perturbative diagrams contributing to the hard part of F_2. Mirror symmetric graphs have been added.

$$T_{2\Psi}(\{x\},\{y\}) = \frac{32M^2C_B^2}{Q^6}(4\pi\alpha_s)^2 \times [x_3(G_{22} - \tilde{G}_{21} - \tilde{G}_{22}) - \bar{x}_3G_{21}],$$

and $T_{2\Phi} = T_{2\Psi}(1 \leftrightarrow 3)$, $T_{3\Psi} = T_{1\Phi}(1 \leftrightarrow 3)$, and $T_{3\Phi} = T_{1\Psi}(1 \leftrightarrow 3)$, where $\bar{x} \equiv 1 - x$. $C_B = 2/3$ is a color factor and the contribution from the twist-four wave functions of the final-state nucleon has also been included. The functions G_{ij} are defined as

$$\begin{aligned} G_{11} &= \frac{1}{x_1x_2x_3^2y_2y_3^2\bar{y}_3}, \\ G_{12} &= \frac{1}{x_3^2\bar{x}_1^2y_3\bar{y}_1^2} + \frac{1}{x_2x_3\bar{x}_1^2y_2\bar{y}_1^2} + \frac{1}{\bar{x}_1^2x_3^2\bar{y}_1y_3^2} - \frac{1}{x_2x_3^2\bar{x}_1y_2y_3\bar{y}_3}, \\ G_{21} &= \frac{1}{x_1^2x_3\bar{x}_3y_1^2y_3\bar{y}_1}, \\ G_{22} &= \frac{1}{x_1x_3^2\bar{x}_2y_3^2\bar{y}_2} + \frac{1}{x_1^2x_3^2y_1y_3\bar{y}_1}, \end{aligned} \quad (10)$$

and $\tilde{G}_{21} = G_{21}(1 \leftrightarrow 3)$, $\tilde{G}_{22} = G_{22}(1 \leftrightarrow 3)$.

To determine the normalization for $F_2(Q^2)$, we need to know the light-cone distribution amplitudes Φ_3, Φ_4, and Ψ_4, which can only be obtained by solving QCD nonperturbatively. However, the scale evolution of these amplitudes selects at the asymptotically large Q^2 the leading component with a fixed small-x_i behavior. For example, the asymptotic form of Φ_3 is $x_1x_2x_3$ [11], whereas that of Φ_4 is x_1x_2. In Ref. [29] a set of phenomenological amplitudes satisfying these asymptotic constraints has been proposed on a basis of conformal expansion.

With the above wave functions, the integrals over momentum fractions x_i and y_i have logarithmic singularities, indicating that the factorization breaks down when one of the quarks in the wave function becomes soft [19,20]. It has been suggested that the higher-order PQCD resummation, or the Sudakov form factor, suppresses the contribution at small x and provides an effective cutoff for the integrals at $x \sim \Lambda^2/Q^2$, where Λ is a soft scale related to the size of the nucleon [21–23,31,32]. The outcome is that the x_i integrations contribute an extra Q^2-dependent factor $\log^2 Q^2/\Lambda^2$, compared to $F_1(Q^2)$. Physically, the end-point divergencies indicate that quarks with different rapidity contribute equally to the hard scattering. Since the contribution from quarks with very large rapidity (small x) is suppressed by the Sudakov form factor, this Q^2 dependence reflects simply the kinematic broadening of the quark (and gluon) rapidity range with increasing nucleon momentum.

For an estimate, we use asymptotic wave functions [29]:

$$\Phi_3 = 120x_1x_2x_3f_N, \qquad \Phi_4 = 12x_1x_2(f_N + \lambda_1), \qquad \Psi_4 = 12x_1x_3(f_N - \lambda_1), \quad (11)$$

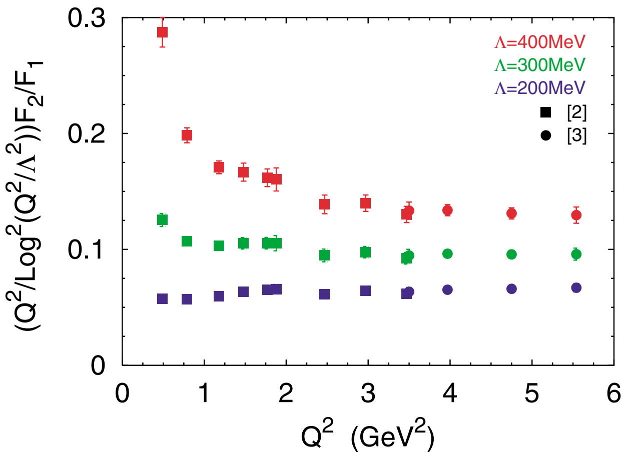

FIG. 3 (color online). JLab data plotted in terms of the leading PQCD scaling. The lower, middle, and upper data points correspond to $\Lambda = 200$, 200, and 400, respectively.

with $f_N = 5.3 \times 10^{-3}\ \mathrm{GeV}^2$ and $\lambda_1 = -2.7 \times 10^{-2}\ \mathrm{GeV}^2$. With a choice of $\Lambda = 0.3$ GeV, $Q^6 F_2^p(Q^2)$ is roughly 0.6 GeV^6 for $Q^2 = (5\text{–}20)\ \mathrm{GeV}^2$, about 1/3 of the Jefferson Lab data at $Q^2 = 5\ \mathrm{GeV}^2$ [3]. Of course, to get a more realistic PQCD prediction in this regime, one must have the quark distribution amplitudes appropriate at this scale. However, from the comparison between the data and PQCD predictions for $F_1(Q^2)$ [8], we believe that asymptotic PQCD is unlikely to be the dominant contribution to $F_2(Q^2)$ at $Q^2 = 3\text{–}5\ \mathrm{GeV}^2$: one must take into account higher-order corrections and higher-twist effects.

Coming back to the scaling behavior of the ratio $F_2(Q^2)/F_1(Q^2)$ for which the Jefferson Lab data have stimulated much discussion in the literature, PQCD predicts the power-law scaling $1/Q^2$. With the new result for $F_2(Q^2)$, we can determine its scaling up to logarithmic accuracy. The strong coupling constant in the ratio simply cancels. The wave function evolution yields a factor of $\alpha_s^{32/(9\beta)}(Q^2)$ for $F_1(Q^2)$ and $\alpha_s^{8/(3\beta)}(Q^2)$ for $F_2(Q^2)$ from the leading nonvanishing contribution, where $\beta = 11 - 2n_f/3$. Thus PQCD predicts that $(Q^2/\log^{2+8/(9\beta)}Q^2/\Lambda^2)[F_2(Q^2)/F_1(Q^2)]$ scales as a constant at large Q^2, $8/(9\beta) \ll 1$. Surprisingly, the Jefferson Lab data plotted this way [ignoring the small $8/(9\beta)$] exhibits little Q^2 variation for a range of choices of Λ as shown in Fig. 3. Since we do not expect the asymptotic predictions for $F_{1,2}(Q^2)$ to work at these Q^2, the observed consistency might be a sign of precocious scaling as a consequence of delicate cancellations in the ratio. A more detailed discussion on this issue along with more thorough phenomenological analyses will be given in a separate publication.

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