Tokamak Equilibria with Reversed Current Density

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Observations of nearly zero toroidal current in the central region of tokamaks (the "current hole") raises the question of the existence of toroidal equilibria with very low or reversed current in the core. The solutions of the Grad-Shafranov equilibrium equation with hollow toroidal current density profile including negative current density in the plasma center are investigated. Solutions of the corresponding eigenvalue problem provide simple examples of such equilibrium configurations. More realistic equilibria with toroidal current density reversal are computed using a new equilibrium problem formulation and computational algorithm which do not assume nested magnetic surfaces.

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Experiments on JET [1] and JT-60 [2] have demonstrated improved confinement regimes with reversed shear and nearly zero toroidal current density in the central region. Such configurations are the result of high-bootstrap current regimes at high beta which, together with low recirculation power for the current drive, lead to a reduced cost and increased efficiency of a fusion device. The magnetic equilibria with current density reversal are also related to the ac tokamak operation [3], another attractive option. Toroidal axisymmetric equilibria with zero current density in a finite region in the plasma core (current hole) were analytically [4] and numerically [5] investigated. However, a reversed current density in the core, in the sense of a core current density having the opposite sign as the total plasma current, prevents the existence of equilibrium solutions with nested flux surfaces, except for one-dimensional cases (e.g., a circular cylinder). More precisely, a closed magnetic flux surface with identically vanishing poloidal magnetic field can exist only if the current density is zero everywhere inside that surface [4,6].

On the other hand, more general axisymmetric equilibria with reversed current density and poloidal field do exist. These equilibria are characterized by the presence of axisymmetric magnetic islands. Simple examples are readily given by the eigenfunctions of the Grad-Shafranov operator. The eigenvalue problem

$$-R^2\nabla\cdot\left(\frac{\nabla\psi}{R^2}\right) = \lambda\psi \tag{1}$$

with $\psi = 0$ at the boundary defines force-free equilibrium configurations with a toroidal current density $j_\phi = \lambda\psi/R$. An equilibrium solution with nested flux surfaces $\psi =$ const corresponds to the lowest eigenvalue and an eigenfunction without nulls inside the domain. Plenty of other eigenfunctions provide a range of equilibria with a different topology of the magnetic surfaces. In the limit of the infinite aspect ratio and the circular cross section (circular cylinder), the solutions of Eq. (1) are

$$\lambda = z_{m,k}^2, \quad \psi = J_m(\sqrt{\lambda}\rho)e^{im\theta}, \quad m, k = 0, 1, 2, \ldots, \tag{2}$$

where (ρ, θ) are polar coordinates; J_m and $z_{m,k}$ are Bessel functions and their zeros. Toroidicity and cross-section shaping lead to the coupling of poloidal harmonics and splitting of multiple eigenvalues. Solutions of the eigenvalue problem (1) were found on a standard fixed grid by the inverse iteration method using the same difference scheme as in the CAXE code [7]. Taking the value of λ from (2) as an initial guess the method converges to the closest eigenvalue and yields the corresponding eigenfunction. In tokamaks with an finite aspect ratio, 2D-modified $m = 0$, $k > 0$ eigenfunctions were found in the spectrum (together with the 2D-modified $m > 0$ multipole eigenfunctions). The eigenfunctions exhibit poloidal field reversal and magnetic island formation with the X points inside the plasma domain (in contrast to the multipoles with X points at the boundary). Moderate cross-section ellipticity leaves the X points inside the domain (Fig. 1).

The presence of nested flux surfaces in the outer region with positive current density (assuming negative eigenfunction values in the center) suggests the existence of more realistic equilibrium configurations with current density reversal near the plasma center. One such possibility is discussed in [8] in relation to the current hole modeling.

Negative current density at the magnetic axis implies poloidal field reversal as follows from the expression for a circular cylinder: $\rho B_\theta = \int_0^\rho \rho' j_\phi(\rho')d\rho'$. Consequently, the magnetic surface with $B_\theta = 0$ corresponds to zero toroidal current within it and the negative current region lies inside that surface. However, the surface with

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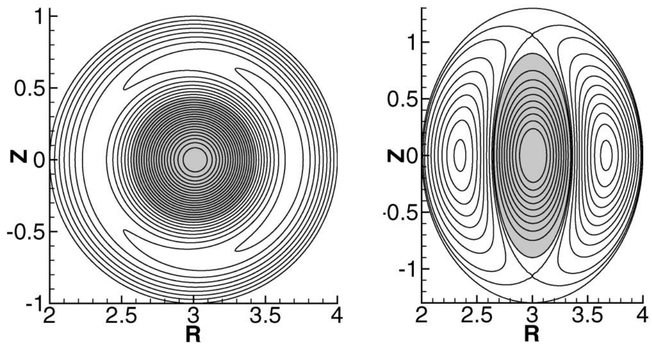

FIG. 1. Level lines of the eigenfunctions ψ from the $m = 0$, $k = 1$ branch for the plasma boundary aspect ratio $A = 3$, circular (elongation $E = 1$) and elliptic $E = 1.3$ cross sections. The shaded regions correspond to $j_\phi < 0$.

identically vanishing poloidal field ($\nabla\psi = 0$) and nonzero current inside exists only in a purely 1D case. Any 2D effect (toroidicity or shaping) leads to the formation of an $n = 0$ magnetic island in place of that surface.

Two flux functions $p(\psi)$ and $f(\psi)$ should be prescribed to define the equilibrium toroidal current density

$$j_\phi = Rp' + \frac{1}{R}ff',$$

where $p' = dp/d\psi$, $f' = df/d\psi$. Only force-free equilibria with zero pressure gradient $p' = 0$ are considered here.

The first difficulty in posing a correct equilibrium problem with current density reversal is the different specification of the flux functions in the negative current density region in the core and the positive current density region outside it. This can be done if the two regions are delimited by some closed magnetic surface. Then the simplest choice is to use constant ff' both inside, $ff' = h_{\mathrm{in}} < 0$, and outside that surface, $ff' = h_{\mathrm{out}} > 0$. In the cylindrical limit a 1D solution with nested flux surfaces exists for any prescribed coordinate $\rho = \rho_d$ of the circular delimiting magnetic surface and for any pair of constants, h_{in} and h_{out}, in the current density definition. In general 2D cases, the position and shape of the delimiting surface should be calculated along with the solution of the equilibrium problem. Our method is to prescribe the diameter (horizontal size) of the delimiting surface $2a_d$ (in units of the plasma minor radius) and the current ratio $I_{\mathrm{in}}/I_{\mathrm{out}}$. In this Letter, we show that this formulation can provide equilibrium solutions for a wide range of parameters.

On the basis of the CAXE code [7] the following numerical procedure has been developed. First, an approximation of the delimiting surface is obtained by tracing the flux surface through some reference node of the computational grid using current values of the function ψ. The corresponding index line of the polar grid is adapted to the delimiting surface and a usual Picard iteration is performed with current densities h_{in}, h_{out} adjusted to preserve the prescribed currents I_{in}, I_{out}. To provide the required shift of the adaptive grid, a feedback procedure connected with the change of the delimiting surface position between the iterations is implemented (in non-up-down symmetric cases not only a horizontal, but also a vertical shift feedback is needed). In contrast to the linear eigenvalue problem, for which all possible solutions can be found, the general equilibrium problem is nonlinear and it is not possible to make strict mathematical statements about the existence and unicity of solutions. The numerical results below are merely examples of possible solutions.

Figure 2 demonstrates that, for a prescribed delimiting surface diameter, its shape and the shape of the islands change according to the value $I_{\mathrm{in}}/I_{\mathrm{out}}$. In Fig. 3 the elongation of the delimiting surface and the ratio of the current densities are shown versus $I_{\mathrm{in}}/I_{\mathrm{out}}$ for a fixed value of a_d. For negative values of $I_{\mathrm{in}}/I_{\mathrm{out}}$ approaching zero the elongation of the delimiting surface decreases (becoming oblate) and the X points approach the delimiting surface. This could possibly explain the limited range of the admissible negative values of $I_{\mathrm{in}}/I_{\mathrm{out}}$: no equilibrium solution was found in the series with fixed delimiting surface diameter for $I_{\mathrm{in}}/I_{\mathrm{out}}/a_d^2 \gtrsim -0.5$. The current density ratio $h_{\mathrm{in}}/h_{\mathrm{out}}$ is nonmonotonic versus $I_{\mathrm{in}}/I_{\mathrm{out}}$ and two different equilibrium solutions exist with the same $h_{\mathrm{in}}/h_{\mathrm{out}}$ and different $I_{\mathrm{in}}/I_{\mathrm{out}}$ values. For the normalized current ratio $I_{\mathrm{in}}/I_{\mathrm{out}}/a_d^2$ all functions shown in

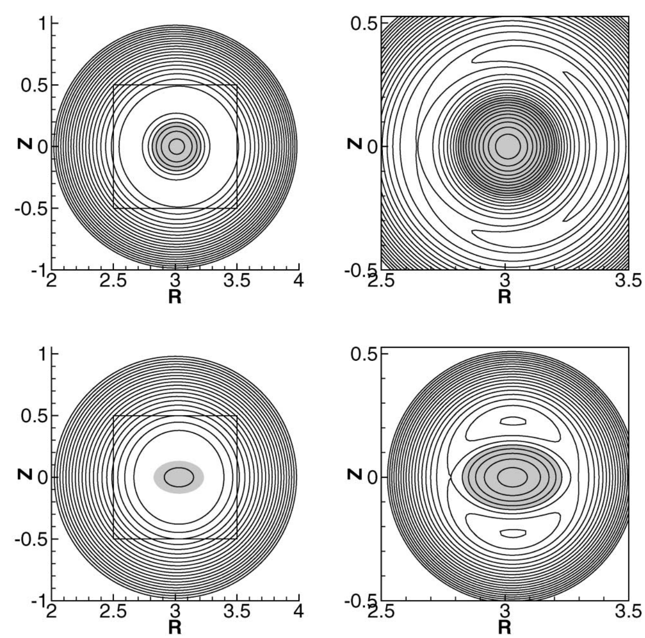

FIG. 2. Magnetic surfaces for two values of $I_{\mathrm{in}}/I_{\mathrm{out}}$: -0.1 (upper row) and -0.03 (lower row). Aspect ratio $A = 3$, circular cross section. The shaded regions correspond to the delimiting surface with $a_d = 0.2$ and $j_\phi < 0$ inside. Expanded central regions are shown on the right.

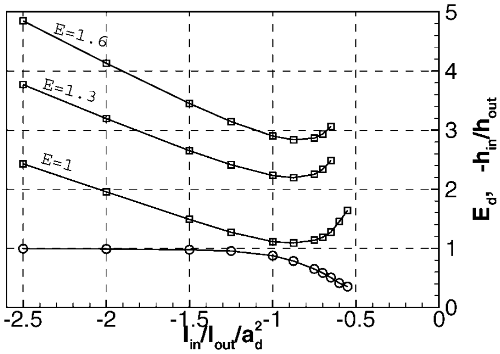

FIG. 3. Delimiting surface elongation E_d (circles) for boundary elongation $E = 1$ and current density ratio h_{in}/h_{out} (squares) versus $I_{in}/I_{out}/a_d^2 < 0$. $A = 3$, $a_d = 0.2$.

Fig. 3 depend weakly on the boundary aspect ratio and the delimiting surface diameter. However, the current density limit significantly varies with the boundary elongation.

Prescribing the horizontal size $2a_d$ and the current ratio $I_{in}/I_{out} < 0$ does not guarantee the uniqueness of the equilibrium solution. Several solutions can exist for sufficiently negative I_{in}/I_{out} (Fig. 4). However, for a given value of $I_{in}/I_{out} < 0$ there is at most only one solution with internal X points and nested flux surfaces near the boundary. All other solutions (multipole type) exhibit X points at the surface and therefore cannot be embedded into an external nested flux surface configuration without breaking the poloidal magnetic field continuity.

The value $I_{in}/I_{out} = -1$ corresponds to the equilibrium with vanishing total plasma current, in which case the normal derivative of ψ at the boundary must change sign (it is zero on average and can be identically zero only in the 1D case). This implies the existence of X points at the plasma boundary, where $d\psi/dn = 0$. For sufficiently negative $I_{in}/I_{out} < -1$ the poloidal field eventually reverses in the whole plasma volume and a configuration with nested flux surfaces is restored (Fig. 5). The kind of solution with nested flux surfaces (corresponding to reverse current density near the edge rather than in the core as long as $|I_{in}| > |I_{out}|$) exists for any delimiting surface diameter. Moreover, for a_d close to 1 there is only one solution branch of the same type as in Fig. 5 in contrast to multiple solutions for small a_d values. The equilibrium sequence from Figs. 4 and 5 leading to the total current

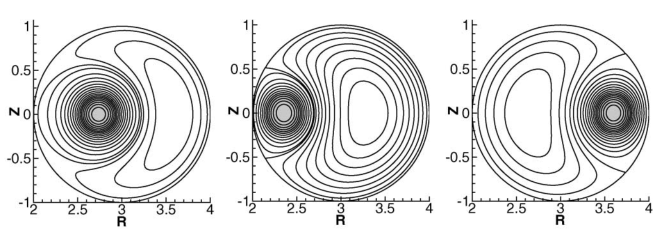

FIG. 4. Several equilibrium solutions with the same values of $a_d = 0.2$ and current ratio $I_{in}/I_{out} = -0.6$. $A = 3$.

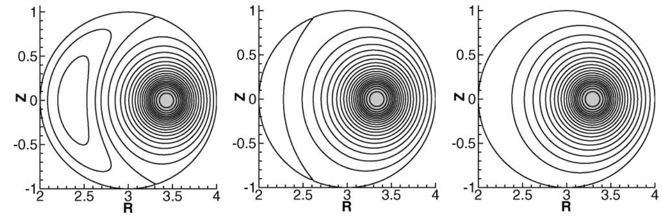

FIG. 5. Equilibrium solutions for low values of current ratio $I_{in}/I_{out} = -1$ (left), $I_{in}/I_{out} = -1.5$ (center), and $I_{in}/I_{out} = -2$ (right). $A = 3$, $a_d = 0.2$.

reversal shows that plasma equilibria can subsist during the ac operation of tokamaks, as was demonstrated experimentally [3].

The same equilibrium formulation can be used with positive values of $I_{in}/I_{out} > 0$ to get nested flux surface configurations with a current hole equilibrium in the limit $I_{in}/I_{out} \to +0$. The delimiting surface elongation depends quite strongly on the current ratio in the range $0.1a_d^2 < I_{in}/I_{out} < a_d^2$, when the current profile becomes hollow (the value $I_{in}/I_{out}/a_d^2$ corresponds to the current density ratio h_{in}/h_{out} for a delimiting surface similar to the plasma boundary) with the elongation limit always higher than that of the plasma boundary (Fig. 6).

In a recent paper [9] a quite detailed reconstruction of a JET equilibrium with extreme shear reversal was reported. The equilibrium formulation described above provides a possibility of finding similar equilibria with different magnetic field structures in the current hole. To model the hollow current density outside the current hole the following parametrization was used:

$$ff' = (1 - f_1)a^{e_1}(1 - a)^{e_2}/c + f_1(1 - a),$$

$e_1 = e_2a_*/(1 - a_*)$, $c = a_*^{e_1}(1 - a_*)^{e_2}$, where the exponent $e_2 = 2$ and the parameter $a_* = 0.1$ are prescribed. The flux surface label a outside the delimiting flux surface with $\psi = \psi_{del}$ is defined as $a = (\psi - \psi_{del})/(\psi_{bou} - \psi_{del})$, where ψ_{bou} is the value at the plasma boundary. A constant $ff' = f_1 = 0.01$ was specified inside the

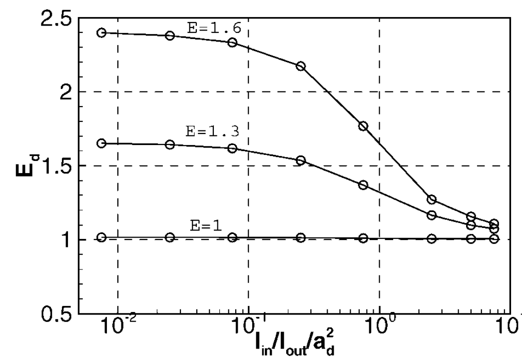

FIG. 6. Delimiting surface (current hole) elongation E_d versus $I_{in}/I_{out}/a_d^2 > 0$. $A = 3$, $a_d = 0.2$, boundary elongations $E = 1$, $E = 1.3$, and $E = 1.6$.

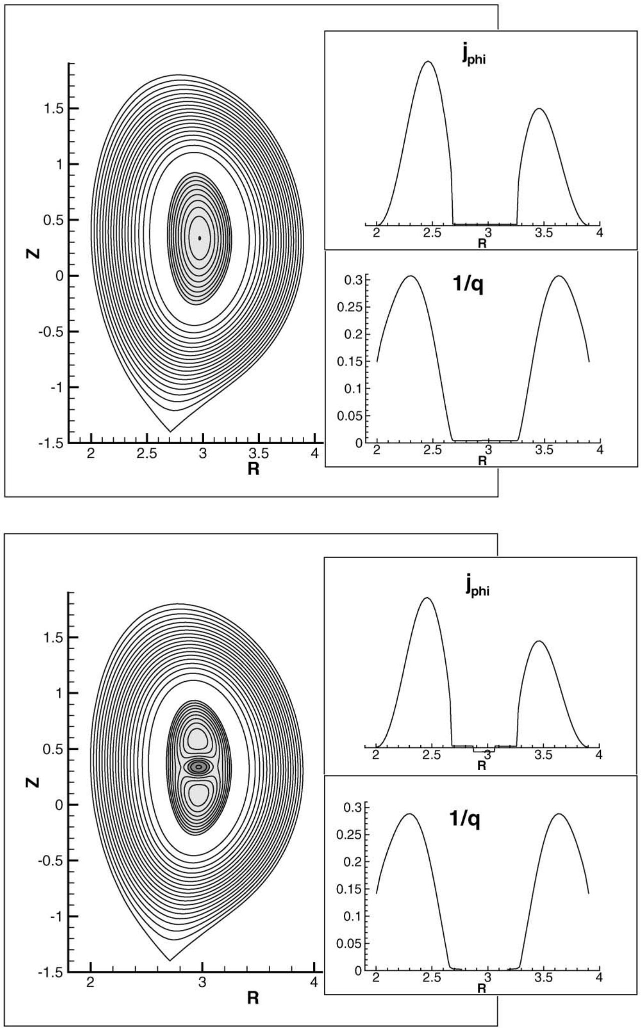

FIG. 7. Magnetic surfaces for the current hole equilibria with JET geometry. The shaded regions correspond to the delimiting surfaces. The level lines of ψ are given for equal steps in ψ inside and outside the current hole (the step inside is 2 orders of magnitude lower). In the insets the toroidal current density and the rotational transform profiles in the plasma equatorial plane are shown.

delimiting surface (“current hole”) with $a_d = 0.3$. This gives the equilibrium with nested flux surfaces shown in Fig. 7 (upper panel) with the current inside the current hole about 0.3% of the total current (the value of the rotational transform in the plasma center is $1/q \sim 1/200$). For the second equilibrium (Fig. 7, lower panel) one more delimiting surface was introduced with $a_d^- = 0.1$ and constant negative current density $h_{\mathrm{in}} = -0.037$ inside it and $h_{\mathrm{out}} = 0.01$ outside. The resulting elongation of the negative current region is $E_d^- = 0.65$ which is close to the limiting value. In both cases the elongation of the current hole $E_d = 2.1$ is higher than the boundary elongation $E = 1.7$. The ideal MHD stability of the equilibrium with nested flux surfaces was calculated using the KINX code [10]. Because of a sufficiently high global shear ($q_{95} = 5.75$, $q_{\mathrm{min}} = 3.25$) the equilibrium is stable against external kink modes.

In summary, tokamak equilibrium solutions with current density and poloidal field reversal have been investigated. The proposed equilibrium problem formulation allowed us to compute a wide range of equilibria with negative current in the plasma core. In plasmas with a finite aspect ratio or elongation, the current reversal leads to the formation of axisymmetric magnetic islands and these have been computed self-consistently. The question of the relevance of the proposed equilibrium solutions to experiments is open. While the approach presented here is applicable to more general plasma profiles and finite β, force-free configurations seem to be a suitable model for the current hole investigation because of the very flat pressure profile inside it. The MHD stability of 2D equilibria with negative central current should be investigated. In particular, it would be a natural extension of current hole stability and evolution studies [5,9,11]. In the context of the possible ac operation of tokamaks, we have shown that a wide range of equilibria can exist during total current reversal, including cases with a large dipole current, in agreement with experiments [3].

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