Light Intensification towards the Schwinger Limit

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A method to generate ultrahigh intense electromagnetic fields is suggested, based on the laser pulse compression, carrier frequency upshift, and focusing by a counterpropagating breaking plasma wave, relativistic flying parabolic mirror. This method allows us to achieve the quantum electrodynamics critical field (Schwinger limit) with present-day laser systems.

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The invention of the chirped pulse amplification (CPA) method and recent development of laser technology led to a stunning increase of the light intensity in a laser focal spot [1]. Electrons in a laser electromagnetic field become relativistic at intensities $I \sim 10^{18}$ W/cm^2. The ion motion strongly affects the relativistic plasma dynamics starting from $I \gtrsim (m_i/m_e) \times 10^{18}$ W/cm^2 (see Ref. [2] and references therein). Nowadays, lasers produce pulses whose intensity approaches 10^{22} W/cm^2 [1]. With a further increase of intensity, we shall meet novel physical processes such as the radiation reaction dominated regimes, which come into play at $I = 10^{23}$–10^{24} W/cm^2 [3], and then the regime beyond $I = 10^{25}$ W/cm^2, where the quantum electrodynamics (QED) description is needed as the recoil of emitted photon momentum becomes comparable with the electron momentum [4]. Near the intensity 10^{29} W/cm^2, corresponding to the QED critical electric field, light can generate electron-positron pairs from vacuum [5,6]. Even before that limit, the vacuum begins to act nonlinearly such as vacuum polarization. These nonlinear effects have attracted a great deal of attention since [5] they lie outside the scope of perturbation theory and shed light on the nonlinear quantum electrodynamics properties of the vacuum. There are several ways to achieve such an intensity. One way was demonstrated in the experiments [7], where a high-energy bunch of electrons interacts with a counterpropagating intense laser pulse. In the reference frame of electrons, the electric field magnitude of the incident radiation was approximately 25% of the QED critical field.

A technically feasible way is to increase the power of the contemporary laser system by some 7 orders of magnitude through megajoule lasers [8], albeit quite expensive. Another way is to increase the frequency of the laser radiation and then focus it onto a tiny region. In this method, x-ray lasers can be used [9]. To achieve more "moderate" intensities, 10^{24}–10^{25} W/cm^2, another scheme was suggested in Ref. [10], where a quasisoliton wave between two foils is pumped by the external laser field up to an ultrahigh magnitude. Another method is based on the simultaneous laser frequency upshifting and the pulse compression. These two phenomena were demonstrated in a broad variety of configurations, where they were caused, in general, by different mechanisms. In particular, the wave amplification reflected at the moving relativistic electron slab was discussed in Ref. [11] (based on the frequency up-shift of radiation reflected at the relativistic mirror, as predicted by Einstein in Ref. [12]); the backward Thompson scattering at relativistic electron bunch was considered in Ref. [13]; the reflection at the moving ionization fronts has been studied in Ref. [14]; "photon acceleration" schemes with copropagating laser pulses in underdense plasma were examined in Ref. [15]; various schemes of the counterpropagating laser pulses and the use of parametric amplification process were discussed in Ref. [16].

In the present Letter, we consider a plasma wakefield in the wave-breaking regime as a tool for generating a coherent radiation of ultrahigh intensity. Compared to the previously discussed schemes, this regime demonstrates both the robustness and coherence of the transformed laser light.

We examine the following scenario. A short intense laser pulse (the "driver pulse") induces wakefield in a plasma. As it is well known [17], the wakefield phase velocity $v_{\mathrm{ph}} = \beta_{\mathrm{ph}} c$ equals the laser pulse group velocity, which is close to the speed of light in a vacuum when the laser pulse propagates in the underdense plasma. The corresponding Lorentz factor is $\gamma_{\mathrm{ph}} = (1 - \beta_{\mathrm{ph}}^2)^{-1/2} \approx \omega_d/\omega_{pe}$, where ω_d is the driver pulse frequency, and ω_{pe} is the Langmuir frequency. The nonlinearity of strong wakefield causes a nonlinear wave profile, including a steepening of the wave and formation of localized maximums in the electron density—the spikes [18]. This amounts to wave-breaking regime (see Ref. [2] and references therein). Theoretically, the electron density in the spike tends to infinity, but remains integrable [2]. A sufficiently weak counterpropagating laser pulse (the "source pulse") will be partially reflected from the density maximum. The reflection coefficient scales as γ_{ph} and the reflected wave vector-potential scales as $\gamma_{\mathrm{ph}}^{-3}$, as shown below. As we see, the electron density maximum acts as a mirror flying with the relativistic velocity $v_{\mathrm{ph}} \approx c$. The frequency of the reflected radiation is up-shifted by factor $(1+\beta_{\rm ph})/(1-\beta_{\rm ph}) \approx 4\gamma_{\rm ph}^2$, in accordance with the Einstein formula [12]. It is important that the relativistic dependence of the Langmuir frequency on the driver pulse amplitude causes parabolic bending of the constant phase surface of the plasma wave, since the driver pulse has a finite transverse size [19]. As a result, the surface where the electron density is maximal has a shape close to a paraboloid. Because we have a curved mirror, the frequency $\tilde{\omega}_s$ of the reflected radiation depends on the angle:

$$\tilde{\omega}_s = \frac{1+\beta_{\rm ph}}{1-\beta_{\rm ph}\cos\theta}\omega_s, \tag{1}$$

where ω_s is the source pulse frequency, and θ is the angle between the reflected wave vector and the direction of the driver pulse propagation in the laboratory frame. The curved mirror focuses the reflected light. The focal spot size is of the order of the diffraction limited size. In the reference frame of the wakefield it is $\lambda_s' = \lambda_s[(1-\beta_{\rm ph})/(1+\beta_{\rm ph})]^{1/2} \approx \lambda_s/2\gamma_{\rm ph}$, where λ_s is the wavelength of the source pulse. In the laboratory frame the focal spot size is approximately $\lambda_s/4\gamma_{\rm ph}^2$ along the paraboloid axis, and $\approx \lambda_s/2\gamma_{\rm ph}$ in the transverse direction. In the focal spot the resulting intensity gain factor scales with $\gamma_{\rm ph}$ as $\gamma_{\rm ph}^{-3} \times (\tilde{\omega}_s/\omega_s)^2 \times (D_s/\lambda_s')^2 = 64(D_s/\lambda_s)^2\gamma_{\rm ph}^3$, where D_s is the diameter of the efficiently reflected portion of the source pulse beam. This value can be great enough to substantially increase the intensity of the reflected light in the focus, even up to the QED critical electric field.

In order to calculate the reflection coefficient, we consider the interaction of an electromagnetic wave with a maximum of the electron density formed in a breaking Langmuir wave. In the laboratory frame, this interaction can be described by the wave equation,

$$\partial_{tt}A_z - c^2\Delta A_z + \frac{4\pi e^2 n(x - v_{\rm ph}t)}{m_e\gamma_e}A_z = 0, \tag{2}$$

where A_z is the z component of the vector potential, γ_e is the electron Lorentz factor, and $\gamma_e \approx \gamma_{\rm ph}$ near the maximum of the density in the wakewave wave-breaking regime.

According to the continuity equation $\partial_t n_e + {\rm div}(n_e v_e) = 0$, the electron density in the stationary Langmuir wave is given by $n = n_0 v_{\rm ph}/(v_{\rm ph} - v_e)$, where the electron velocity v_e varies from $-v_{\rm ph}$ to $v_{\rm ph}$ (see Ref. [18]), and the electron density varies from the minimal value $= n_0/2$ to infinity (integrable). For the breaking plasma wakewave, in every wave period approximately half of the electrons are located in the spike of the electron density. Therefore we can approximate the electron density by $n(x - v_{\rm ph}t) = [1 + \lambda_p\delta(x - v_{\rm ph}t)]n_0/2$, where λ_p is the wakefield wavelength and $\delta(x)$ is the Dirac delta function. This approximation is valid when the density maximum thickness is sufficiently less than the collisionless skin depth c/ω_{pe} and source pulse wavelength in the wakefield rest frame, i.e., when the wakefield is close to the wavebreaking regime.

In the reference frame comoving with the plasma wakewave, Eq. (2) has the same form. The Lorentz transformation to this frame is given by $t' = (t - v_{\rm ph}x/c^2)\gamma_{\rm ph}$, $x' = (x - v_{\rm ph}t)\gamma_{\rm ph}$, $y' = y$, $z' = z$.

We seek for a solution to Eq. (2) in the form $A_z = \mathcal{A}(x')\exp[i(\omega_s't' + k_x'x' + k_y'y' + k_z'z')]$, where $\omega_s' = (\omega_s + v_{\rm ph}k_x)\gamma_{\rm ph}$, $k_x' = (k_x + v_{\rm ph}\omega/c^2)\gamma_{\rm ph}$, $k_\perp' = k_\perp$ are the frequency and wave vector in the moving frame, and $k_x' > 0$. Using this ansatz, from Eq. (2) in the moving frame we obtain

$$\frac{d^2\mathcal{A}}{dx'^2} + q^2\mathcal{A} = \chi\delta(x')\mathcal{A}, \tag{3}$$

where $q^2 = \omega_s'^2/c^2 - k_\perp'^2 - \omega_{pe}^2/(2c^2\gamma_{\rm ph}) > 0$ and $\chi = \omega_{pe}^2\lambda_p/c^2$. This equation is equivalent to the scattering problem at the delta potential. The solution is $\mathcal{A}(x') = \exp(iqx') + \rho(q)\exp(-iqx')$ for $x' \geq 0$ (incident and reflected wave), and $\mathcal{A}(x') = \tau(q)\exp(iqx')$ for $x' < 0$ (transmitted wave), where $\rho(q) = -\chi/(\chi + 2iq)$ and $\tau(q) = iq/(\chi + 2iq)$. In a nonlinear Langmuir wave, its wavelength depends on the wave amplitude [18], and for the breaking wakewave we have $\lambda_p \approx 4(2\gamma_{\rm ph})^{1/2}c/\omega_{pe}$. In this case $\chi = 4(2\gamma_{\rm ph})^{1/2}\omega_{pe}/c$. Taking $\omega_s' = 4\gamma_{\rm ph}^2\omega_s$ into account, we find that the reflection coefficient, defined as a ratio of the reflected to the incident energy flux, in the comoving frame is $\approx (\omega_d/\omega_s)^2/2\gamma_{\rm ph}^3$. In the laboratory frame it is

$$R \approx 8\gamma_{\rm ph}(\omega_d/\omega_s)^2. \tag{4}$$

The intensity $\tilde{I}_{sf}$ in the focal spot of the source pulse, reflected and focused by the electron density maximum in the laboratory frame, is increased by the factor of the order of

$$\tilde{I}_{sf}/I_s \approx 32(\omega_d/\omega_s)^2(D_s/\lambda_s)^2\gamma_{\rm ph}^3. \tag{5}$$

Theoretically, the actual gain can be even greater, because (i) the estimation (4) corresponds to the one-dimensional case, whereas the density modulation in the 3D breaking wakewave is stronger, and (ii) the reflectance (4) of the 3D paraboloidal mirror is greater at the periphery.

We consider the following example. A one-micron laser pulse (driver) generates wakefield in a plasma with density $n_e = 10^{17}$ cm^{-3}. The corresponding plasma wavelength is $\lambda_p \approx 100$ μm. The Lorentz factor, associated with the phase velocity of the wakefield, is estimated as $\gamma_{\rm ph} \approx \omega_d/\omega_{pe} \approx 100$. The counterpropagating one-micron laser pulse with intensity $I_s = 10^{17}$ W/cm^2 (source) is partially reflected and focused by the wakefield cusp. If the efficiently reflected beam diameter is $D_s = 200$ μm, then, according to Eq. (5), the final intensity in the focal spot is $\tilde{I}_{sf} \approx 1.3 \times 10^{29}$ W/cm^2. The driver pulse intensity should be sufficiently high and its beam diameter should be enough to give such a wide mirror, assuming $I_d = 10^{18}$ W/cm^2 and $D_d = 800$ μm.

Thus, if both the driver and source are one-wavelength pulses, they carry 17 and 0.1 J, respectively. We see that in an optimistic scenario the QED critical electric field may be achieved with the present-day laser technology.

To demonstrate the feasibility of the effect of the light reflection and focusing by the breaking wakewave, we performed three-dimensional particle-in-cell (PIC) simulations using the code REMP (relativistic electromagnetic particle-mesh code) based on scheme [20]. In the simulations, the driver pulse propagates in the direction of the x axis. Its dimensionless amplitude is $a_d = 1.7$ which corresponds to peak intensity 4×10^{18} W/cm^2 $\times$ $(1\ \mu\mathrm{m}/\lambda_d)^2$, where λ_d is the driver wavelength. The driver is linearly polarized along the z axis, it has the Gaussian shape, and its FWHM size is $3\lambda_d \times 6\lambda_d \times 6\lambda_d$. The source pulse propagates in the opposite direction. Its wavelength is 2 times greater than the driver wavelength, $\lambda_s = 2\lambda_d$. The source pulse amplitude is chosen to be small, $a_s = 0.05$, to reduce the distortion of the wakewave. The pulse shape is rectangular in the x direction and Gaussian in the transverse direction; its size is $6\lambda_d \times 6\lambda_d \times 6\lambda_d$. To distinguish the electromagnetic radiation of the driver from the source pulses, we set the source pulse to be linearly polarized in the direction perpendicular to the driver polarization, i.e., along the y axis. The laser pulses propagate in the underdense plasma slab with the electron density $n_e = 0.09 n_{cr}$, which corresponds to the Langmuir frequency $\omega_{pe} = 0.3\omega_d$. The plasma slab is localized at $2\lambda_d < x < 13\lambda_d$ in the simulation box with size $22\lambda_d \times 19.5\lambda_d \times 19.2\lambda_d$. The simulations were carried out on 720 processors of the supercomputer HP Alpha Server SC ES40 at JAERI Kansai. The mesh size is $dx = \lambda_d/100$; the total number of quasiparticles is 10^{10} (ten billion). The boundary conditions are absorbing on the x axis and periodic in the transverse direction, both for the electromagnetic fields and quasiparticles. We emphasize that the simulation grid must be, and in fact was chosen to be, fine enough to resolve the huge frequency up-shift given by Eq. (1), exhausting all the supercomputer resources.

The simulation results are presented in Figs. 1 and 2. Figure 1 shows the plasma wakewave induced by the driver laser pulse as modulations in the electron density. We see the electron density cusps in the form of paraboloids. They move with velocity $v_{\mathrm{ph}} \approx 0.87c$; the corresponding gamma factor is $\gamma_{\mathrm{ph}} \approx 2$. Their transverse size is much larger than the wavelength of the counter-propagating source pulse in the reference frame of the wakefield. As seen from the electron density profile along the axis of the driver pulse propagation, the wakewave dynamics is close to the wave-breaking regime. Each electron density maximum forms a semitransparent parabolic mirror, which reflects a part of the source pulse radiation.

In Fig. 2, we present the electric field components. The driver pulse is seen in the cross section of the z component of the electric field in the $(x, y, z = 0)$ plane. The source

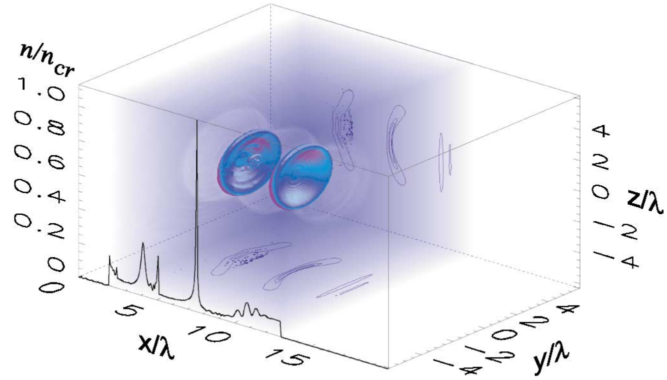

FIG. 1 (color). The electron density in the wake of the driver laser pulse at $t = 14 \times 2\pi/\omega_d$. The $(x, y = -6\lambda, z)$ plane: density profile along the symmetry axis. Blue curves are for density values $n = 0.12$, 0.24, and $0.36 n_{cr}$ on the corresponding perpendicular planes of symmetry; isosurfaces for value $n = 0.15 n_{cr}$; "blue gas" for lower values.

pulse and its reflection are seen in the cross section of the y component of the electric field in the $(x, y = 0, z)$ plane. The part of the source pulse radiation is reflected from the flying paraboloidal mirrors, then it focuses yielding the peak intensity in the focal spot, and finally it defocuses and propagates as a spherical short wave train, whose frequency depends on the wave vector direction, in agreement with Eq. (1). This process is clearly seen in the

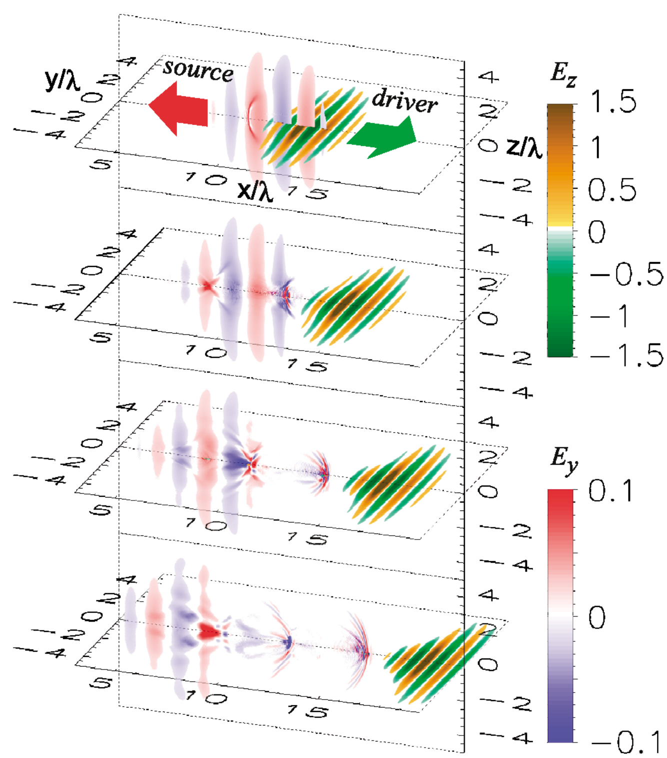

FIG. 2 (color). The cross sections of the electric field components. The $(x, y, z = 0)$ plane: $E_z(x, y, z = 0)$ (green-brown color scale); the plane $(x, y = 0, z)$: $E_y(x, y = 0, z)$ (blue-red color scale) at $t = 16$, 18, 20, and $22 \times 2\pi/\omega_d$ (top-down).

animations produced from the data (see authors' website). The main part of the reflected light power is concentrated within the angle $\sim 1/\gamma_{ph}$; hence, this coherent high-frequency beam resembles a searchlight. The reflected part has the same number of cycles as the source pulse, as expected, since it is Lorentz invariant. The wavelength and duration of the reflected pulse are approximately 14 times less than the wavelength and duration of the source pulse, in agreement with Eq. (1) since $(1 + \beta_{ph})/(1 - \beta_{ph}) \approx 14.4$. The focal spot size of the reflected radiation is much smaller than the wavelength of the source pulse. The electric field in the focal spot is approximately 16 times higher than in the source pulse. Therefore, the intensity increases 256 times in agreement with estimation (5).

We emphasize that the efficient reflection is achievable only when the wakefield is close to the wave-breaking regime and the cusps in the electron density are formed. As we see in the simulations, the reflection and focusing is robust and even distorted (to some extent) wakewave can efficiently reflect and focus the source pulse radiation. We also observe that despite the moderate reflection coefficient, the colossal frequency up-shift and focusing by a sufficiently wide (transversely) wakewave give us a huge increase of the light intensity.

Similar processes may occur in the laser-plasma interaction spontaneously, e.g., when a short laser pulse exciting plasma wakewave is a subject of the stimulated backward Raman scattering or a portion of the pulse is reflected back from the plasma inhomogeneity. Then the backward scattered electromagnetic wave interacts with plasma density modulations in the wakewave moving with relativistic velocity. According to the scenario described above, the electromagnetic radiation, reflected by the wakewave, propagates in the forward direction as a high-frequency strongly collimated (within the angle $\sim 1/\gamma_{ph}$) electromagnetic beam.

We have proposed the scheme of the relativistic plasma wake caustic light intensification, which can be achieved due to the reflection and focusing of light from the maximum of the electron density in the plasma wakewave at close to the wave-breaking regime. The presented results of 3D PIC simulations provide us a proof of the principle of the electromagnetic field intensification during reflection of the laser radiation at the flying paraboloidal relativistic mirrors in the plasma wakewave. With the ideal realization of the described scheme, we can achieve extremely high electric fields (in the laboratory reference frame) approaching the QED critical field with the present-day laser technology. We envision the present example is just one manifestation of what we foresee as the emergence of relativistic engineering.

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