

## Atomic Bose and Anderson Glasses in Optical Lattices

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An ultracold atomic Bose gas in an optical lattice is shown to provide an ideal system for the controlled analysis of *disordered* Bose lattice gases. This goal may be easily achieved under the current experimental conditions by introducing a pseudorandom potential created by a second additional lattice or, alternatively, by placing a speckle pattern on the main lattice. We show that, for a noncommensurable filling factor, in the strong-interaction limit, a controlled growing of the disorder drives a dynamical transition from superfluid to Bose-glass phase. Similarly, in the weak interaction limit, a dynamical transition from superfluid to Anderson-glass phase may be observed. In both regimes, we show that even very low-intensity disorder-inducing lasers cause large modifications of the superfluid fraction of the system.

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At zero temperature, the bosonic lattice gases may undergo a quantum phase transition [1] from a superfluid (SF) to an insulator [2]. In the absence of disorder there exist Mott insulator (MI) states, characterized by a fixed (integer) number of bosons per lattice site, a gap in the excitation spectrum, and vanishing superfluid fraction and compressibility. In the presence of disorder, an additional insulating phase, so-called Bose-glass phase (BG), may occur [2]. This phase presents no superfluid fraction, a gapless excitation spectrum, and a finite compressibility. The SF-BG transition has been recently experimentally studied in various physical systems [3], and attracts a continuous theoretical interest [4–7]. In particular, the possibility of a direct MI-SF transition in the presence of disorder remains a controversial issue.

The nature of the disorder-induced insulator phases depends on the interplay between hopping, nonlinear interactions, and disorder. In the strong-interaction regime, the cooperation of interactions and disorder leads to the appearance of a BG phase. For weak interactions, the disorder leads to an Anderson-type insulator, or Anderson-glass (AG) [4]. In the latter case, contrary to the BG phase, the interactions tend to delocalize and therefore compete with the disorder.

The detailed analysis of these properties demands an experimentally accessible system in which the disordered Bose lattice gases could be studied in a controlled way. One of the aims of this paper is to show that this goal can be relatively easily accomplished by using cold Bose gases in optical lattices, for which the development of cooling and trapping techniques allows a large degree of control. Recently, Greiner *et al.* [8], following the theoretical suggestion of Jaksch *et al.* [9], have observed the SF to MI quantum phase transition in an optical (non-disordered) lattice loaded with <sup>87</sup>Rb atoms.

In the case of an optical lattice a pseudorandom potential can be dynamically generated by growing on an al-

ready existing (main) optical lattice a second (additional) one with a different wavelength [9–11]. Based on this idea, we study in this Letter the dynamical generation of the BG and AG phases in a many-body system under realistic conditions. In addition, we discuss the alternative generation of a truly random potential by superimposing a speckle pattern. In the first part of this Letter, we consider the regime of strong interactions where the filling factor is noninteger, showing that the adiabatic turn-on of the disorder may lead to a dynamical transition into the BG phase. In the last part of the Letter, we analyze the weak interaction regime, and, in particular, the possibility to achieve a dynamical transition into the AG phase. For both regimes of interactions, the SF fraction [12] is calculated. We show that under realistic conditions even very low-intensity disorder-inducing lasers lead to a dramatic reduction of the SF fraction, indicating the appearance of BG or AG phases.

We consider an ultracold Bose gas in a 2D optical lattice. We analyze the case of <sup>23</sup>Na, but our results also apply to other species. We assume the atoms as tightly confined in the transversal ( $z$ ) direction by a harmonic trap of frequency  $\omega_z/2\pi = 6$  kHz, so that the wave function in  $z$  remains the Gaussian ground state. No additional harmonic confinement is assumed in the  $xy$  plane.

The optical lattice is formed by the main laser beams, whereas additional lasers are responsible for the introduction of (quenched) pseudodisorder. The optical potential thus has the following form:

$$V(\mathbf{r}) = V_l(\mathbf{r}) + V_r(\mathbf{r}), \quad (1)$$

where  $\mathbf{r} = [x, y]$ , the main lattice is  $V_l(\mathbf{r}) = V_0[\cos^2(kx) + \cos^2(ky)]$ , and the secondary lattice is  $V_r(\mathbf{r}) = V_1[\cos^2(\mathbf{k}_1\mathbf{r}) + \cos^2(\mathbf{k}_2\mathbf{r})]$ . The main (additional) beam's intensity and/or detuning controls the value of  $V_0$  ( $V_1$ ). In the following we assume  $k_1 = k_2 \neq k$ , and the directions  $\mathbf{k}_1 \propto [-0.5, 1]$  and  $\mathbf{k}_2 \propto [-1, 0.5]$ .

The pseudorandomness of the potential is determined by  $q = k_1/k$ , i.e., the ratio between the wavelengths of the main ( $\lambda = 2\pi/k$ ) and the additional lattices. For commonly used Nd:YAG and Ti:sapphire lasers  $q = 1064/795 = 1.338$ . We note that a pseudorandom lattice can also be achieved by splitting off part of the main laser beams and creating the additional incommensurate lattice by interfering these light beams at an angle.

We assume that the energies involved in the system are much smaller than the energy separation between the first and the second band of the lattice, and consequently we can reduce our analysis to the first band. In that case, the physics of the atomic lattice Bose gas is governed by the Bose-Hubbard Hamiltonian [9]:

$$H = -\sum_{\langle i,j \rangle} J_{ij} a_i^\dagger a_j + \frac{U_0}{2} \sum_i n_i(n_i - 1) + \sum_i W_i n_i. \quad (2)$$

$J_{ij} = J_0 + \delta J_{ij}$  are the tunneling (hopping) coefficients between nearest neighbors. They differ slightly from  $J_0$  [9] by a correction of the form  $\delta J_{ij} = -\int d^3 r w^*(\mathbf{r} - \mathbf{r}_i) V_r(\mathbf{r}) w(\mathbf{r} - \mathbf{r}_j)$ , where  $w(\mathbf{r} - \mathbf{r}_j)$  are the Wannier functions for the lowest energy band. Clearly, the contributions of the additional beams to the tunneling, vanish on ‘‘average,’’  $\langle \delta J_{ij} \rangle \approx 0$ . In the following we consider  $V_0 = 25E_R$  and  $V_1 \approx 0.05E_R$ , where  $E_R$  is the photon recoil energy. In this case we have checked numerically that  $|\delta J_{ij}/J_0| < 0.1\%$ , hence the major contribution to  $J_{ij}$  comes from  $J_0$ , and the model reduces to the ordinary Bose-Hubbard model with constant tunneling [4–7]. In the Hamiltonian (2),  $U_0 \propto a$  is the coupling constant [9] for the interparticle interactions ( $a$  is the scattering length), and

$$W_i = \int d^3 r w^*(\mathbf{r} - \mathbf{r}_i) V_r(\mathbf{r}) w(\mathbf{r} - \mathbf{r}_i) \quad (3)$$

are the pseudorandom on-site energies, which, may introduce significant effects even for very small  $V_1/V_0$ .

The disorder-induced phases are characterized by a vanishing SF fraction, which is determined studying the system sensitivity to changes of boundary conditions. To this aim, we employ the *boost* method [12], resulting in substitution in (2):  $J_{ij} \rightarrow J_{ij} e^{i\varphi_{ij}}$ . The angles  $\varphi_{ij}$  are defined as follows: if  $i = (x_i, y_i)$  and  $j = (x_j, y_j)$ , then for  $y_i = y_j$ ,  $\varphi_{ij} = \text{sgn}(x_i - x_j)\varphi/M$ , and otherwise  $\varphi_{ij} = 0$ , where  $M$  is the lattice size in the  $x$  direction. Physically, this choice of  $\varphi_{ij}$  corresponds to a constant current per lattice, proportional to  $\varphi$ , in the positive  $x$  direction. The SF fraction is then obtained from the corresponding ground state energy  $E(\varphi)$  as

$$\rho_s = \frac{M^2 E(\varphi) - E(0)}{N J_0 \varphi^2}, \quad (4)$$

where  $N$  is the number (mean) of atoms. A similar expression can be easily derived for a 1D case. In the following, we denote by  $Y$  the ratio of  $N$  to the number

of lattice sites. The ground state is obtained from the minimization of  $\langle \Psi | H - \mu N | \Psi \rangle$ , where  $\mu$  is the chemical potential. We employ the Gutzwiller ansatz  $|\Psi\rangle = \prod_i \sum_n f_n^{(i)} |n\rangle_i$ , where  $f_n^{(i)}$  are the amplitudes of having  $n$  atoms at an  $i$ th lattice site. We have observed that it is numerically easier to calculate the ground state using static methods for a nondisordered and nontwisted case ( $V_1 = 0, \varphi = 0$ ), and then adiabatically evolve such a state, first to  $\varphi \neq 0$ , and then for a constant  $\varphi$  to  $V_1 \neq 0$  (first evolving  $V_1$  and then  $\varphi$  should give the same result). The evolution is performed by means of the dynamical Gutzwiller approach [13], in which we solve the equations

$$i\dot{f}_n^{(i)} = \left[ \frac{U_0}{2} n(n-1) + nW_i \right] f_n^{(i)} + \Phi_i^* \sqrt{n+1} f_{n+1}^{(i)} + \Phi_i \sqrt{n} f_{n-1}^{(i)}, \quad (5)$$

where  $\Phi_i = -\sum_{\langle i,j \rangle} J_{ij} e^{i\varphi_{ij}} \langle \Psi | a_j | \Psi \rangle$ .

Another useful quantity characterizing the state of a Bose lattice gas is the condensate fraction, defined as the highest eigenvalue of the one particle density matrix,  $\rho_{ij} = \langle \Psi | a_i^\dagger a_j | \Psi \rangle$ , divided by the number of particles [14]. This quantity is important for experiments, since it determines the phase coherence, and thus the contrast in interference measurements [8].

In order to study the dynamical transition into the BG phase, it is convenient to prepare the system in the SF phase in the presence of dominating interactions and weak tunneling, when  $Y$  is noninteger. Therefore, we consider an initial system deeply in the SF regime, with almost 100% of SF fraction and  $Y = 0.75$ . Then, the intensity of the main laser is adiabatically increased in 20 ms, obtaining a very large  $U_0/J_0 \approx 70$ . Both the condensate and the SF fraction decrease during this process down to approximately 30%, being nonzero only due to noninteger value of  $Y$  (see inset of Fig. 1). As a next step, the disorder is turned on adiabatically in about 0.5 s, by switching on the additional laser beams. The condensate and also the SF fraction decrease dynamically during this process (Fig. 1). Ultimately, the condensate fraction does not tend to 0, but to a very small value of about 2%, due to the finite size of the systems and the approximate character of the Gutzwiller approach [15,16]. In contrast, the SF fraction tends to zero faster. Note that the superfluidity is rapidly lost, although at any time the additional lattice is very much weaker than the main one. This fact could at first glance seem surprising, but it is due to the small values of  $J_0$  and  $U_0$  ( $10^{-3}E_R$  and  $7 \times 10^{-2}E_R$ , respectively). Thus even for  $V_1$  being of the order of a few percent of  $E_R$  ( $V_1/V_0 \ll 1$ ), the value of  $|W_i|/U_0 \sim 1$ . Together with the low value of  $J_0/U_0$ , this explains why the system enters the BG phase for such a weak additional lattice.

We stress at this point that our calculations do not include an additional inhomogeneous trapping in the  $xy$

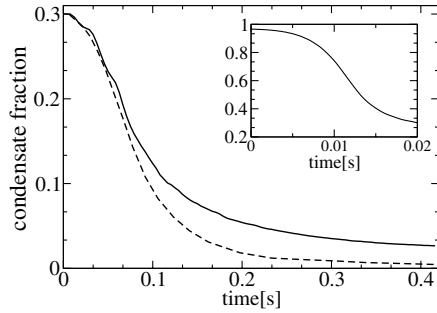


FIG. 1. Condensate fraction (solid line) during the dynamical SF to BG transition induced by a superimposed additional optical lattice, for  $^{27}\text{Na}$  atoms placed on the main  $40 \times 40$  lattice with  $\lambda = 1064$  nm,  $q = 1.338$ ,  $Y = 0.75$ ,  $V_0/E_R = 25$ , and  $V_1(t)/E_R = 0.059t/T$ , where  $T = 0.42$  s is the total time of evolution. The inset shows the first preparation step (see text), for which  $V_0(t) = V_{\text{SF}} + (V_{\text{MI}} - V_{\text{SF}})t/T$  [ $V_1(t) = 0$ ], where  $V_{\text{SF}}/E_R = 7$ ,  $V_{\text{MI}}/E_R = 25$ , and  $T = 0.02$  s. The dashed line presents the SF fraction.

plane, which, for the low  $J_0$  we consider, may result in the formation of MI domains [17] if the filling factor at the trap center is larger than 1. For sufficiently shallow traps, the central BG region should dominate the physics for a finite disorder. For filling factors lower than 1, the systems is expected to be fully in the BG phase.

It is interesting to compare results obtained with the quasidisordered perturbation induced by the additional lattice, and those achieved using a purely random optical potential coming from a speckle pattern. We generated speckle pattern in a way described in [18] and checked that it gives correctly both the autocorrelation function and the probability distribution. The speckle pattern induces a potential  $V_s(\vec{r})$ , which is characterized by its mean value  $V_2 = \langle V_s(\vec{r}) \rangle$ , and by the average speckle size  $\Gamma$ . As above, the disorder-induced corrections to the tunneling are minor, and the most significant contribution of the speckle potential appears in the form of  $W_i$  coefficients (3), defined in this case by means of the  $V_s(\vec{r})$  potential. Not surprisingly, values of  $V_2$  comparable to those previously considered for  $V_1$  lead to a transition from the SF to Bose-glass phase. We have investigated this issue for  $\Gamma = 0.34, 1.37, 2.75\lambda$ . The obtained results are similar to those resulting from quasirandom perturbation generated by additional beams (see Fig. 2).

In the final part of this Letter, we discuss the non-interacting regime, where the adiabatic turn-on of the disorder can lead to a dynamic transition into the AG phase. The weakly interacting regime, and even the non-interacting one, could be achieved by reducing the  $s$ -wave scattering length by means of Feshbach resonances [19]. For simplicity we consider a 1D optical lattice case where the perturbation is either generated by two noncommensurate standing waves,  $V_r(x) = V_1[\cos^2(k_1x) + \cos^2(k_2x)]$  with  $k_1 \neq k_2$ , or by a laser speckle potential  $V_s(x)$  characterized by a mean value  $V_2$ . The Hamiltonian (2) reduces then to the form

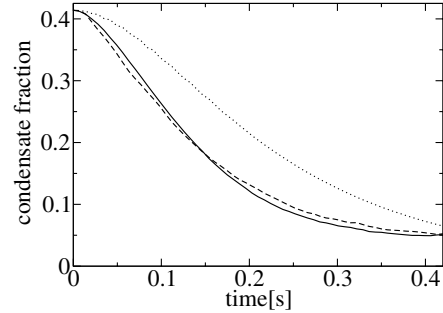


FIG. 2. Condensate fraction during the dynamical SF to BG transition induced by a superimposed speckle pattern, for  $^{23}\text{Na}$  atoms placed on the main  $40 \times 40$  lattice with  $\lambda = 1064$  nm,  $Y = 1.5$ ,  $V_0/E_R = 25$ , and  $V_2/E_R = 0.048t/T$ , with  $T = 0.42$  s. The solid (dashed) line refers to speckles of average size  $\Gamma = 0.34\lambda$  ( $\Gamma = 1.37\lambda$ ). For comparison, the dotted line shows the same transition in the quasidisordered case (parameters as in Fig. 1, but  $Y = 1.5$ ). We expect that, similarly as in Fig. 1, the decrease of the SF fraction is faster than that of the condensate fraction.

$$H = -\sum_{\langle i,j \rangle} J_{ij} a_i^\dagger a_j + \sum_i W_i n_i, \quad (6)$$

which becomes the famous Anderson's hopping model [20], if the  $W_i$  coefficients are *randomly* distributed. Similarly, as above  $J_{ij}$  are almost unchanged by the presence of disorder. The single-particle character of the noninteracting problem greatly simplifies the calculations and allows for an exact treatment. In the absence of interactions, 100% of atoms condense at zero temperature, each in a single-particle state  $|\Psi\rangle = \sum_n c_n a_n^\dagger |\text{vac}\rangle$ . We have analyzed the dynamical evolution of the amplitudes  $c_n$  during the turn-on of the disorder as well as their ground state distributions; see Fig. 3.

Because of the finite size of the system, which implies discreteness of the excitation spectrum, the application of the boost method in the absence of disorder results in the 100% SF fraction. When the typical energy of disorder becomes comparable to the energy separation between the energy levels in the absence of disorder, the SF fraction is expected to vanish (see Fig. 3). We clearly observe a transition from a delocalized SF state to an Anderson-localized one, where the occupation probabilities of neighboring sites decrease *exponentially* with the distance from the localization centers [20]. From the upper plot in Fig. 3 we conclude that in order to drive the system into the AG phase smaller intensities are needed for the speckle pattern than for the superimposed incommensurable lattice, since as expected the speckle pattern results in a "more disordered" distribution of the  $W_i$  coefficients (see middle plot in Fig. 3). It is especially worth noting that in the case of a superimposed incommensurable lattice the periodicity of the lattice reflects itself in periodically *Anderson-localized* domains, but the localization phenomena is still present, as shown in Fig. 3. We observed the same quantitative phenomena

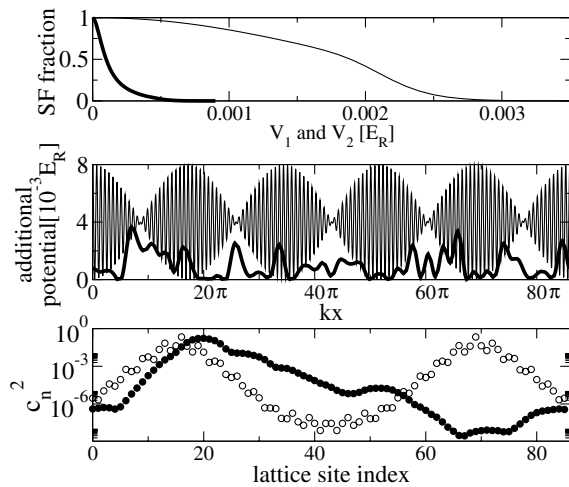


FIG. 3. Transition from the SF to AG phase for  $^{23}\text{Na}$  atoms placed on a 1D main lattice of 86 sites, with  $\lambda = 1064$  nm and  $V_0/E_R = 16$ . Upper plot: superfluid fraction versus amplitude of the disordered potential. The thick curve indicates the speckle-induced transition ( $\Gamma = 1.34\lambda$ ), whereas the normal line refers to the transition generated by a superimposed lattice with  $k_1/k = 1.338$ ,  $k_2/k = 1.396$  (see text). Middle plot: dependence of  $V_r(x)$  (normal line) and  $V_s(x)$  (thick line) for  $V_1/E_R = 0.004$  and  $V_2/E_R = 0.001$ . Lower plot: occupation numbers of the ground state of the Hamiltonian (6) in the presence of speckles (solid circles) and additional beams (open circles), for  $V_1/E_R = 0.004$  and  $V_2/E_R = 0.001$ .

also for ratios  $k_1/k$  and  $k_2/k$  different from the ones listed in the caption of Fig. 3. We have performed also time-dependent simulations based on Eq. (6) to determine the time scale for the adiabatic transition into the AG phase. For a main lattice wavelength  $\lambda = 1064$  nm the adiabatic evolution lasts a few seconds. This may be shortened using higher frequency lattice beams since evolution time scales as  $\lambda^2$ .

The experimental observation of the SF to BG (or to AG) transition should be relatively easy to accomplish in a setup such as that of Ref. [8]. The BG and AG phases can be detected by observing the interference pattern after removing the lattice for different intensities of the superimposed laser beams. The insulator character of the BG and AG phases will be revealed by the disappearance of the interference fringes. The phases can be additionally characterized by measuring their gapless excitation spectrum, in an experiment similar to that of Ref. [8].

In summary, we have proposed an experimentally feasible method of creating a system whose physics is governed by the disordered Bose-Hubbard model. We have shown how the onset of small perturbation of the lattice potential may result in a dynamical transition from a superfluid regime into Bose-glass or Anderson-localized phases. This transition occurs within experimentally feasible time scales and can be easily controlled, allowing for a detailed analysis of disorder-induced transitions. Our proposal stimulates in this sense new interesting

experimental possibilities including studies of Anderson localization in 2D systems and the investigation of the SF to MI transition in the presence of disorder.

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