Quantum Toys for Quantum Computing: Persistent Currents Controlled by the Spin Josephson Effect

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Quantum devices and computers will need operational units in different architectural configurations for their functioning. The unit should be a simple "quantum toy," an easy to handle superposition state. Here such a novel unit of quantum mechanical flux state (or persistent current) in a conducting ring with three ferromagnetic quantum dots is presented. The state is labeled by the two directions of the persistent current, which is driven by the spin chirality of the dots, and is controlled by the spin (the spin Josephson effect). It is demonstrated that by the use of two connected rings, one can carry out unitary transformations on the input flux state by controlling one spin in one of the rings, enabling us to prepare superposition states. The flux is shown to be a quantum operation gate, and may be useful in quantum computing.

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The realization of quantum mechanical two-level systems and controlling the superposition of the states in experiment is a fundamental but also an interesting subject. Such systems have been intensively studied recently, since controlling them is a starting point of the realization of quantum computers [1]. Such two-level systems, called qubits, have been implemented, for instance, in ion traps [2], nuclear spins [3], and in Josephson junctions [4]. In the case of flux in the Josephson junction, the two-level states are states with persistent currents in a superconducting loop with different directions. The current is induced by a magnetic flux through the ring, and the quantum superposition of the two current states was observed recently by a fine-tuning of the flux [4].

In this Letter, we present a novel quantum mechanical flux state, which is controlled by controlling the spin in a quantum dot. The flux here is due to a persistent current in a conducting ring, but of different origin than the Josephson qubit; namely, the current is induced by spin chirality. By putting three (or more) quantum dots which carry quantum spin, we show that the wave function of the flux is controlled by that of the spins. The realization of the superposition state of flux is thus realized simply by creating a superposition state on one of the spins. We also demonstrate that this system can be used to create entangled states of two or more spins. This "quantum toy" also works as a quantum logic gate, which may be useful in quantum computers. We also discuss the more sophisticated case of two rings coupled, where we can carry out unitary transformations on the current state.

The existence of the spontaneous current in a small ring in contact with three or more ferromagnets when the three magnetization vectors form a finite solid angle was pointed out recently in Ref. [5]. The effect is due to the breaking of the time-reversal symmetry in the orbital motion as a consequence of noncommutativity of the spin algebra, and it is essential that the electron wave function is coherent over the ring. The current was shown to be proportional to the noncoplanarity (spin chirality) of the three magnetizations, $(\boldsymbol{S}_1 \times \boldsymbol{S}_2) \cdot \boldsymbol{S}_3$, where magnetizations are represented by classical vectors $\boldsymbol{S}_1$, $\boldsymbol{S}_2$, and $\boldsymbol{S}_3$.

Here we consider the case where the magnetization is a quantum spin of $S = 1/2$, which is carried by ferromagnetic dots on the ring (see Fig. 1), in which case the same reasoning as in Ref. [5] applies. In coupling quantum spins in dots to conduction electron, one might worry about the Kondo effect, which screens the spin [6]. This does not, however, occur in the present system of small ring, since the screening is suppressed by the discreteness of the energy levels in the ring, $\Delta E \simeq v_F/L$ (~ 2 K in semiconductors if $L = 1\ \mu$m), where v_F and L are the Fermi velocity and the length of the ring. The spins in the dots can then be regarded as qubits. Note that the decoherence time of the electron spin is known to be much larger in general in nanostructures than that for the charge due to the smallness of the spin-orbit coupling [7]. We treat perturbatively the coupling between the conduction electron and the spins in the dots. The equilibrium current at x is calculated from $J(x) = (e\hbar/2m)\mathrm{Im}\mathrm{Tr}[(\nabla_x - \nabla_{x'}) \times G(x, x', \tau = -0)|_{x=x'}]$, where $G(x, x', \tau) \equiv -\langle Tc(x, \tau) \times c^\dagger(x', 0)\rangle$ is the thermal Green

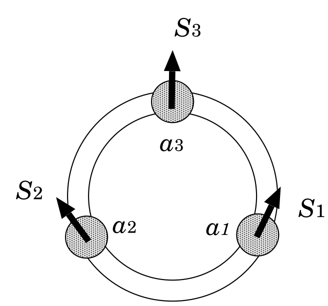

FIG. 1. The system of chirality-driven persistent current with three ferromagnetic dots.

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function, and the trace is over the spin indices. The interaction with the spins in the dots can be expressed by the potential $V(x) = -\Delta \boldsymbol{S}(x) \cdot \boldsymbol{\sigma}$, where $\boldsymbol{S}(x) \equiv \sum_i \hat{\boldsymbol{S}}_i \delta(x - a_i)$, a_i being the position of ferromagnetic dots ($i = 1, 2, 3$), and Δ representing the effective coupling between the electron and quantum spin, $\hat{\boldsymbol{S}}$. The Green function is determined by the Dyson equation, $G = g + gVG$, where the free Green function is denoted by g. By noting that the free Green function is symmetric under spatial reflection, $g(x, x') = g(x', x)$, and by summing over a path contributing to the current and its time-reversed path, the contribution to the current $J(x)$ at nth order in V is shown to be proportional to

$$\sum_{x_i} \mathrm{Tr}[V(x_1)V(x_2)\cdots V(x_n) - V(x_n)\cdots V(x_2)V(x_1)]\nabla_x g(x, x_1)g(x_1, x_2)g(x_2, x_3)\cdots g(x_n, x). \tag{1}$$

The second term in the square bracket corresponds to the contribution from the time-reversed path. Since $\mathrm{Tr}[V(x_1)V(x_2) - V(x_2)V(x_1)] = \Delta^2 S^\mu(x_1)S^\nu(x_2) \times \mathrm{Tr}[\sigma^\mu \sigma^\nu] = 0$, we immediately see that the leading contribution is from the third order with $x_i \in F_i$, which reads

$$\hat{J}(x) = \frac{e\hbar}{m}\int \frac{d\omega}{2\pi} f(\omega)\nabla_x \mathrm{Im}[g_{x1}g_{12}g_{23}g_{3x'}]|_{x'=x} \times 4\Delta^3(\hat{\boldsymbol{S}}_1 \times \hat{\boldsymbol{S}}_2)\cdot \hat{\boldsymbol{S}}, \tag{2}$$

where we have used $\mathrm{Tr}[\sigma_i\sigma_j\sigma_k] = 2i\epsilon_{ijk}$, $f(\omega)$ is the Fermi distribution function and $g_{ij} = g^r(a_i - a_j, \omega)$ ($i, j = x, 1, 2, 3$) is the retarded free Green function. In the case of the one-dimensional ring, the result is

$$\hat{J} = J_0 \hat{C}_3, \tag{3}$$

where $\hat{C}_3 \equiv (\hat{\boldsymbol{S}}_1 \times \hat{\boldsymbol{S}}_2)\cdot \hat{\boldsymbol{S}}_3$ and $J_0 = -2e(v_F/L) \times \cos(k_F L)(\Delta/\epsilon_F)^3$ [8]. The state of the system is thus specified by a combination of states of the spin qubits $\hat{\boldsymbol{S}}_i$ and a current qubit $\hat{J}$. The current takes a value according to the ''volume'' of the three spins, $(\hat{\boldsymbol{S}}_1 \times \hat{\boldsymbol{S}}_2)\cdot \hat{\boldsymbol{S}}_3$. The magnitude J_0 of the present persistent current is different from the conventional one due to a magnetic flux through the ring [9,10] by a factor of $(\Delta/\epsilon_F)^3$. The appearance of the current is due to the symmetry breaking of the charge [U(1)] sector, as in the case of the current in Josephson junction. But note that here the U(1) symmetry breaking was due to the noncommutativity of spin [SU(2)] sector (“spin Josephson effect”).

Classically, spin chirality $C_3 \equiv (\boldsymbol{S}_1 \times \boldsymbol{S}_2)\cdot \boldsymbol{S}_3$ (with $\boldsymbol{S}_i$'s as classical vectors) vanishes if any of the $\boldsymbol{S}_i$'s are parallel to each other, and is thus read as an XOR operation. To be explicit, we choose $\boldsymbol{S}_3 \parallel z$, and then $C_3 = \frac{1}{2}(S_1^x S_2^y - S_1^y S_2^x)$. If we label the state $\boldsymbol{S}_i = \frac{1}{2}\hat{x}$ as 0 and $\boldsymbol{S}_i = \frac{1}{2}\hat{y}$ as 1, the result of C_3 is written as $C_3(00) = C_3(11) = 0$, $C_3(01) = -C_3(10) = \frac{1}{8}$ [states are labeled by $(\boldsymbol{S}_1\boldsymbol{S}_2)$], and hence $|C_3|$ is classical XOR. We can also label $\boldsymbol{S}_1 = \frac{1}{2}\hat{x}$ as 0 and $-\frac{1}{2}\hat{x}$ as 1 for $\boldsymbol{S}_1$, and $\boldsymbol{S}_2 = \frac{1}{2}\hat{y}$ as 0 and $-\frac{1}{2}\hat{y}$ as 1 for $\boldsymbol{S}_2$, fixing the direction of $\boldsymbol{S}_1$ and $\boldsymbol{S}_2$ in the x and y direction, respectively. We then have $C_3(00) = C_3(11) = \frac{1}{8}$ and $C_3(01) = C_3(10) = -\frac{1}{8}$ and this is another XOR if we read the sign of C_3 as 0 and 1.

Let us see how the quantum operation works. To remove an irrelevant degeneracy due to rotational symmetry, we fix $\boldsymbol{S}_3$ in the z direction. Then the quantum operator $\hat{C}_3$ reduces to $\hat{C}_2 \equiv \frac{1}{2}(\hat{\boldsymbol{S}}_1 \times \hat{\boldsymbol{S}}_2)_z = \frac{i}{4}(\hat{S}_1^+\hat{S}_2^- - \hat{S}_1^-\hat{S}_2^+)$. The eigenvalues λ and eigenstates (represented by $|S_1^z S_2^z\rangle$) of $\hat{C}_2$ are obtained as $\lambda = 0$ for $|+\,+\rangle \equiv |0_+\rangle$ and $|-\,-\rangle \equiv |0_-\rangle$, $\lambda = \frac{1}{4}$ for $(1/\sqrt{2})(|+\,-\rangle + e^{-(\pi/2)i}|-\,+\rangle) \equiv |R\rangle$, and $\lambda = -\frac{1}{4}$ for $(1/\sqrt{2})(|+\,-\rangle + e^{(\pi/2)i}|-\,+\rangle) \equiv |L\rangle$. Note that the current states $|R\rangle$ and $|L\rangle$ correspond to the entangled states as a result of “square-root swap” operation [11]. As is expected from the classical picture of the current appearing when the three spins points in x, y, and z directions, it is useful to describe the spin state by use of different quantization axis for $\boldsymbol{S}_1$ and $\boldsymbol{S}_2$. We choose the axis of $\boldsymbol{S}_1$ as in the x direction, and that of $\boldsymbol{S}_2$ in the y direction. For instance, $|0\rangle = |x\rangle$ and $|1\rangle = |-x\rangle$ for $\boldsymbol{S}_1$ is written as $|\pm x\rangle = (1/\sqrt{2})(|+\rangle \pm |-\rangle)$. Then states of the two spins are expressed in terms of eigenstates of $\hat{C}_2$ as

$$\begin{aligned} |\pm x, \pm y\rangle &= \frac{1}{2}(|0_+\rangle + i|0_-\rangle) \pm \frac{i}{\sqrt{2}}|R\rangle, \\ |\mp x, \pm y\rangle &= \frac{1}{2}(|0_+\rangle - i|0_-\rangle) \pm \frac{i}{\sqrt{2}}|L\rangle. \end{aligned} \tag{4}$$

By taking the expectation values, we see that the classical XOR gate mentioned above is reproduced by taking the expectation value, $\langle \hat{C}_2\rangle$.

In order to implement quantum operations, we need to kill the unwanted state without current, $|0_\pm\rangle$. These states carry finite total $S_z(\equiv S_z^1 + S_z^2)$, $S_z = \pm 1$, and thus are deleted by use of projection into $S_z = 0$ subspace, which we write as P_0. (Note that $|R\rangle$ and $|L\rangle$ are eigenstates of $S_z = 0$.) After the projection, the mapping (4) reduces to

$$P_0|\pm x, \pm y\rangle = \pm\frac{i}{\sqrt{2}}|R\rangle, \qquad P_0|\mp x, \pm y\rangle = \pm\frac{i}{\sqrt{2}}|L\rangle, \tag{5}$$

and we have a direct correspondence between the quantum spin states and two states of the current. The operation here is a modified quantum XOR gate [neglecting the coefficient of $(1/\sqrt{2})$]:

$$\begin{array}{ccc} |\boldsymbol{S}_1, \boldsymbol{S}_2\rangle & & C_2 \\ |00\rangle & \to & |R\rangle \\ |01\rangle & \to & e^{i\pi}|L\rangle \\ |10\rangle & \to & |R\rangle \\ |11\rangle & \to & e^{i\pi}|L\rangle. \end{array} \tag{6}$$

The extra factor of $e^{i\pi}$ can be removed by a single spin operation if one wants. We can easily check that this operation correctly maps the superposition state of the spin into the corresponding superposition state of the current.

The operation is obviously extended to the case of more qubits. For instance, 4-bit operation is carried out by putting five S_i's on a ring, with S_5 fixed in z direction. The current in this case is found (by a similar calculation) to be proportional to the five-spin-chirality, $\hat{C}_{12345}$, obtained as

$$\begin{aligned}\hat{C}_{12345} = {} & [(\hat{\boldsymbol{S}}_1 \times \hat{\boldsymbol{S}}_2) \cdot \hat{\boldsymbol{S}}_3](\hat{\boldsymbol{S}}_4 \cdot \hat{\boldsymbol{S}}_5) + [(\hat{\boldsymbol{S}}_3 \times \hat{\boldsymbol{S}}_4) \cdot \hat{\boldsymbol{S}}_5](\hat{\boldsymbol{S}}_1 \cdot \hat{\boldsymbol{S}}_2) \\ & - [(\hat{\boldsymbol{S}}_2 \times \hat{\boldsymbol{S}}_4) \cdot \hat{\boldsymbol{S}}_5](\hat{\boldsymbol{S}}_1 \cdot \hat{\boldsymbol{S}}_3) \\ & + [(\hat{\boldsymbol{S}}_1 \times \hat{\boldsymbol{S}}_4) \cdot \hat{\boldsymbol{S}}_5](\hat{\boldsymbol{S}}_2 \cdot \hat{\boldsymbol{S}}_3). \end{aligned} \tag{7}$$

We can show that this $\hat{C}_{12345}$ works as XOR and AND operation combined in rather a complex way.

In the gate proposed here, the single qubit operation is achieved by applying a different magnetic field on each qubit, and for this purpose, magnetic scanning-probe tips might be useful [7]. The magnetic field to point the quantum mechanical spin in the desired direction can be a pulse as in the case of pulsed NMR [3]. For successive operation, one needs somehow to translate the quantum information carried by the current into the spin direction, to be used as inputs of the next step calculation, and this may be carried out by combining two rings (see below). The present gate has a great advantage if we just want the result of a single operation [but on $2n$ qubits ($n \geq 1$)].

As is seen from the above consideration, our systems can be used as a preparation tool of an entangled state of two or more spins. For instance, in the case of three spins $\boldsymbol{S}_i$ ($i = 0, 1, 2$), with $\boldsymbol{S}_0 \parallel z$, we can create an entangled state of $|\boldsymbol{S}_1\boldsymbol{S}_2\rangle = (1/\sqrt{2})|+-\rangle \mp i|-+\rangle$ by projecting the current state into $|R\rangle$ or $|L\rangle$, respectively. The current state is implemented by putting magnetic flux through the ring (i.e., by inducing conventional persistent current) [12]. By carrying out unitary transformations for the spins in the above states, we can obtain various superposition states. Entangled state of three spins is also straightforward. We combined two rings as in Fig. 2(a), with one spin $\boldsymbol{S}_2$ in common. Thus the current states for the first ring, J_1, are described as $|R\rangle_1 = |+-\rangle_{12} - i|-+\rangle_{12}$ and $|L\rangle_1 = |+-\rangle_{12} + i|-+\rangle_{12}$, where $|+-\rangle_{12}$ denotes the state of $\boldsymbol{S}_1$ and $\boldsymbol{S}_2$. Let us point $\boldsymbol{S}_4$ on the second ring in an arbitrary direction described by the polar coordinates (θ, ϕ). Then the current state of the second ring is written in terms of $\boldsymbol{S}_2$ and $\boldsymbol{S}_3$ as

$$\begin{aligned}|R\rangle_2 = {} & \frac{1}{2}[\sin\theta(e^{-i\phi}|++\rangle - e^{i\phi}|--\rangle) \\ & - (\cos\theta + i)|+-\rangle - (\cos\theta - i)|-+\rangle]_{23}, \\ |L\rangle_2 = {} & \frac{1}{2}[\sin\theta(e^{-i\phi}|++\rangle - e^{i\phi}|--\rangle) \\ & - (\cos\theta - i)|+-\rangle - (\cos\theta + i)|-+\rangle]_{23}. \end{aligned} \tag{8}$$

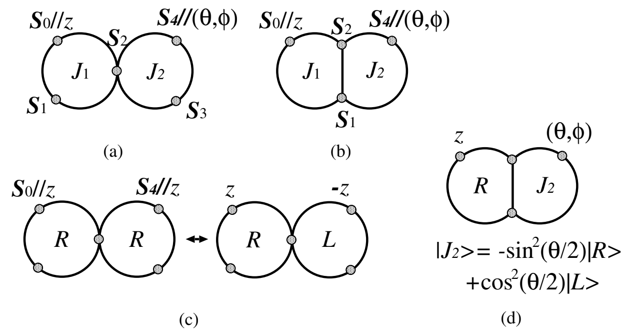

FIG. 2. Two rings coupled (a) with one spin in common and (b) with two spins in common. The current state J_2 in the second ring is a result of a unitary transformation of J_1 specified by (θ, ϕ). (c),(d): An example of operation on the flux by controlling $\boldsymbol{S}_4$. In (d), a superposition state of current in the second ring is created from the R state in the first ring.

Thus if we prepare by the use of the magnetic field the state $|R\rangle$ for both of the rings, i.e., $|R_1R_2\rangle$, the realized spin state on the two rings is

$$\begin{aligned}|R_1R_2\rangle = {} & \frac{1}{2}[-\sin\theta e^{-i\phi}(|+--\rangle + i|-++\rangle) \\ & - (\cos\theta - i)|+-+\rangle \\ & + i(\cos\theta + i)|-+-\rangle]_{123}, \end{aligned} \tag{9}$$

and hence the entanglement of the three spins can be controlled by (θ, ϕ). We notice that for $\theta = 0$, $|R_1R_2\rangle_{\theta=0} = -(e^{-i\pi/4}/\sqrt{2})(|+-+\rangle + |-+-\rangle)_{123}$ and for $\theta = \pi$, $|R_1R_2\rangle_{\theta=\pi} = (e^{i\pi/4}/\sqrt{2})(|+-+\rangle - |-+-\rangle)_{123}$ and this is equal to $-|R_1L_2\rangle_{\theta=0}$. *This means that if we start from the state $|R_1R_2\rangle$ with $\boldsymbol{S}_4 \parallel z$ and flip $\boldsymbol{S}_4$ to be $\boldsymbol{S}_4 \parallel -z$, we obtain a state $|R_1L_2\rangle$; the current in the second ring is reversed. Thus the total flux created by the current is 2 in the initial state, but is switched off to be zero by reversing $\boldsymbol{S}_4$; i.e., by reversing spin we can vanish the flux even if current exist in each ring.* [See Fig. 2(c).]

An alternative way to couple two rings is to share two spins [Fig. 2(b)]. In this case, the currents J_1 and J_2 are both determined by $\boldsymbol{S}_1$ and $\boldsymbol{S}_2$, but the state can again be controllable by $\boldsymbol{S}_4$. In fact, pointing $\boldsymbol{S}_4 \parallel (\theta, \phi)$, the current states of the first ring are translated into the current states of the second ring as (after projection P_0)

$$\begin{aligned}|R\rangle_1 &= \frac{e^{i\pi/4}}{\sqrt{2}}\left[-\sin^2\frac{\theta}{2}|R\rangle_2 + \cos^2\frac{\theta}{2}|L\rangle_2\right], \\ |L\rangle_1 &= \frac{e^{-i\pi/4}}{\sqrt{2}}\left[\cos^2\frac{\theta}{2}|R\rangle_2 - \sin^2\frac{\theta}{2}|L\rangle_2\right]. \end{aligned} \tag{10}$$

Thus one can create from a current in ring one any superposition of $|R\rangle$ and $|L\rangle$ on the second ring. [See Fig. 2(d).]

The readout of the target bit is carried out by measuring the flux arising from the persistent current. Such measurement on a single ring has been successfully carried out in the case of conventional persistent current in a ring of gold [13] and GaAs-AlGaAs [14]. Let us give an estimate of the present effect. We consider as an example a ring of GaAs-AlGaAs as in Ref. [14], where $v_F \simeq 2.6 \times 10^5$ m/s, $\epsilon_F \simeq 1.3 \times 10^{-2}$ eV. For a ring with a diameter of 2 μm, we have $J \simeq 14 \times (\Delta/\epsilon_F)^3$ nA. The coupling Δ depends on the distance of the conducting layer in the semiconductor, but for the case where it is close to the interface with the ferromagnet, Δ/ϵ_F would be close to the value in the ferromagnet; $\Delta/\epsilon_F \simeq 0.2$ (i.e., effective coupling $\Delta \sim 2.6$ meV). So the current would be 0.1 nA. The flux due to this current is not large but may be detected with the present lock-in technique.

We propose here an alternative detection of the flux state by the use of a Hall-like effect in the four-terminal setup. In the presence of flux (or persistent current), the four-terminal conductance through a ring is expected to be asymmetric with respect to the flux, and a finite difference of the conductance arises when the voltage and current leads are reversed [15]. The difference (which may be regarded as a "Hall conductance," G_H) is expected in our system to be $G_H \simeq (e^2/h) \times (\Delta/\epsilon_F)^3 C_3[\sim e^2/h \times O(10^{-2})]$ for the above estimate and if $C_3 \sim O(1)$. This is of the order of typical atomic size contacts of semiconductors, and would be measurable. The electric measurement, being very sensitive, detection of very small spin chirality C_3 would be possible, as well as the system with smaller coupling Δ.

Much larger current would be obtained if we use a superconducting ring of p-wave order parameter, such as Sr_2RuO_4 [16], since the arising persistent current becomes macroscopic. Some semiconducting materials [such as GaAs)] are known to switch to be ferromagnetic when magnetic impurities are doped; (Ga,Mn)As [17]. Such host materials would show a high polarizability when in contact with ferromagnets, and thus would be suitable for the experimental realization of the present effect, because the coupling Δ will increase and thus the value of the current.

We have shown that the current qubit in a small ring can be controlled by use of spin chirality by attaching three ferromagnetic quantum dots. The physics behind this is the "spin Josephson effect," which is a SU(2) analog of Josephson effect in superconductors. The spin chirality is equivalent to the quantum mechanical Berry phase carried by the spin. This Berry phase is a "fictitious magnetic flux," which does not affect the phenomena in the macroscopic world. In nanoscales, in contrast, it can be used to operate logic gates just in the same way as "real" magnetic flux can. This is a novel architecture of quantum logic gates. The quantum current states are described as entangled states of two or more spins. By use of coupling of two or more rings, unitary transformations can be carried out on the current states and superposition states can be prepared. Experimental demonstration of this quantum toy would be interesting, because this can be used as a unit for quantum computing. Implementation by the use of rings of semiconductors or p-wave superconductors would be, in particular, interesting.

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