Is There a Ferromagnet to Ferromagnetic *s*-Wave Superconductor Phase Transition?

Recently, Karchev *et al.* [1] have found the first mean-field theory with coexisting ferromagnetism and *s*-wave superconductivity (sf state) in ferromagnetic metals. They claimed that the sf state is favorable in energy as compared to the normal ferromagnetic (nf) state around their solution $JM = 2\Delta + 0^+$, where J, M, and Δ are the Heisenberg exchange, the spontaneous magnetization, and the BCS gap, respectively. In this Comment, we point out that this conclusion is not firmly established due to the following negligence: They have compared the energy of the sf state to that of a state without both ferromagnetism and superconductivity. We point out that their sf state is possibly *unstable* against the *true* nf state with nonzero M as solved from their model.

First of all, let us point out that there is no pure ferromagnetic solution in the parameter limit $r = Jm^* p_F/4\pi^2 \gg 1$ of Ref. [1], where $p_F = \sqrt{2m^*\mu}$ is the Fermi wave vector when the free electron band $\epsilon_{\mathbf{p}} = p^2/2m^* - \mu$ is used. The spontaneous magnetization M is obtained by numerically solving

$$M = \frac{1}{12\pi^2}[(p_F^2 + m^*JM)^{3/2} - (p_F^2 - m^*JM)^{3/2}], \quad (1)$$

i.e., Eq. (9) of Ref. [1] at $\Delta = 0$. This equation contains all three quantities in the "large" parameter r. We do our calculations for all possible values of J, m^*, and p_F. We found that there is a solution of $M \neq 0$ only when $r < 1.06$. This condition in parameter space is well known in research of itinerant ferromagnetism using the free electron dispersion $p^2/2m^*$. Naturally, there is no phase transition from a normal ferromagnetic state to coexisting ferromagnetic and *s*-wave superconducting states within their parameter limit since there is no ferromagnetic normal state at all.

We do not agree with the author of Ref. [1] to present the energy difference between the sf state and the nf state by the BCS condensation energy only. The condensation energy is actually the energy difference between sf and free electron states. This is just the lowest energy argument presented in Ref. [1] via Eqs. (19) and (20). The special mean-field solution of $JM = 2\Delta + 0^+$ is adopted when they derive Eq. (20) from Eq. (19); i.e., M is vanishingly small when $\Delta \to 0$. In other words, they compared the energy of their solution with the energy of the free Fermi sphere. Of course, the state with long range order will have lower energy. The notation $\Omega_{\mathrm{sf}} - \Omega_{\mathrm{nf}}$ in Eqs. (19) and (20) of Ref. [1] is misleading. It should be replaced by $\Omega_{\mathrm{sf}} - \Omega_n$ (where Ω_n stands for the energy of free electron gas), which is well known in standard textbooks. Reference [1] fails to derive the true $\Omega_{\mathrm{sf}} - \Omega_{\mathrm{nf}}$, which is the most important point in their paper. The authors of Ref. [1] should have claimed that they had found coexisting ferromagnetism and *s*-wave superconductivity in free electron metals instead of ferromagnetic metals as experimentally discovered [2].

The spontaneous magnetization M is changed drastically from the normal ferromagnetic state to the ferromagnetic superconducting state in the parameter region where $M \neq 0$ and $\Delta = 0$ solution exists. This may contribute an important portion of positive energy difference besides the negative BCS condensation energy in the model proposed by Ref. [1]. Our numerical calculation shows that their sf state is possibly *unstable* against the *true* nf state with nonzero M as solved from their model. So it is still not clear whether the sf state can have lower energy than the *true* ferromagnetic normal state.

In conclusion, we point out that there is no phase transition from ferromagnetic metals to coexisting ferromagnetic and *s*-wave superconducting states as claimed in [1]. The reason is simply that there is *no ferromagnetic* normal state solution in the parameter region where their solution works. Thus, Ref. [1] is irrelevant to recent experimental discovery [2]. The theoretical coexistence of ferromagnetism and *s*-wave superconductivity in ferromagnetic metals is still questionable.

Note added.—After submission of this Comment, Jackiewicz and two of the authors of Ref. [1] presented a numerical study [3] of the model proposed in [1]. In this preprint, they make clear they are calculating $\Omega_{\mathrm{sf}} - \Omega_n$. They do find that the coexisting state has the lowest energy in a vanishingly small parameter region for $1.001 < J < 1.005$ at $g = 2$. But their results confirm our point that the sf state is *unstable* against the *true* nf state in most of the parameter space. Especially, it certainly cannot rebut our point here that the ferromagnet to ferromagnetic *s*-wave superconductor phase transition at $r = Jm^* p_F/4\pi^2 \gg 1$ of [1] is void.

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