

Relaxation and the Zeno Effect in Qubit Measurements

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We consider a qubit interacting with its environment and continuously monitored by a detector represented by a point contact. Bloch-type equations describing the entire system of the qubit, the environment, and the detector are derived. Using these equations we evaluate the detector current and its noise spectrum in terms of the decoherence and relaxation rates of the qubit. Simple expressions are obtained that show how these quantities can be accurately measured. We demonstrate that due to interaction with the environment, the measurement can never localize a qubit even for infinite decoherence rate.

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An account of decoherence and relaxation in quantum evolution of a two-level system (qubit), interacting with an environment and a measurement device, has become a problem of crucial importance in quantum computing. Numerous publications have appeared on this subject dealing with interactions either with a measurement device (detector) [1–3] or with the environment (a thermal bath) [4,5]. Generally, the simultaneous influence of an environment and a detector on a qubit is very important for understanding qubit measurements because the environment and the detector act on the qubit in different ways. For instance, the environment at zero temperature relaxes the qubit to its ground state. As a result the qubit finally appears in a pure state, even though it was initially in a statistical mixture. On the other hand, the measurement device puts the qubit in a statistical mixture, even if it was initially in a pure state.

One of the most striking measurement effects in which the role of relaxation has not been investigated is the so-called Zeno paradox [6]. It consists of total freezing of a qubit in the limit of continuous measurement. Usually, it is associated with the projection postulate in the theory of quantum measurements. Indeed, it follows from the Schrödinger equation that the probability of a quantum transition from an initially occupied state of a qubit is $P(\Delta t) = a(\Delta t)^2$, where a is a factor which depends on the system [6]. If we assume that Δt is the measurement time which determines the time scale on which the system is projected into the initial state, then after N successive measurements the probability of finding the qubit in its initial state, at time $t = N\Delta t$, is $P(t) = [1 - a(\Delta t)^2]^{t/(\Delta t)}$. Thus $P(t) \rightarrow 1$ for $\Delta t \rightarrow 0$, $N \rightarrow \infty$, and $t = \text{const}$. Including the environment into the Schrödinger equation for the entire system one would expect from the above arguments that the relaxation processes could affect only the coefficient a but cannot destroy the qubit localization in the limit of $\Delta t \rightarrow 0$.

This conclusion, however, is not correct. We demonstrate in this Letter that any weak relaxation delocalizes

the qubit even in the limit of continuous measurement. It is shown by using new Bloch-type quantum rate equations for the description of a qubit interacting with a detector and its environment. These rate equations are derived from the microscopic Schrödinger equation for the entire system. Using these equations, we determine the qubit behavior without any phenomenological parameters. In addition, we are able to relate the qubit behavior to the detector outcome and therefore establish how decoherence and relaxation rates of the qubit can be accurately measured.

With respect to the contradiction of our results to the Zeno paradox arguments, it is possible that the simple derivation described above is not valid when the environment is taken into account. One reason is that the expansion of $P(t) = at^2 + \dots$ near $t = 0$ is not valid because of the discontinuity of $P'(t)$ at $t = 0$. The latter arises due to the transition to the continuous and sufficiently flat spectrum of the environment [7,8]. We present our result for an environment at zero temperature, where the interplay between decoherence and relaxation is most pronounced. (The detector, however, is far from equilibrium.)

Let us consider the generic example of electrostatic qubit measurements. The qubit is an electron in a double-dot system, whereas the detector is a point contact placed near one of the dots (Fig. 1). When the electron occupies the first well close to the point contact, Fig. 1(a), the current is smaller than in Fig. 1(b) due to the electron electrostatic repulsion. Thus, the electron is continuously monitored by the tunneling current. The entire system can be described by the tunneling Hamiltonian $H = H_0 + H_{\text{PC}} + H_{\text{int}}$, where

$$H_0 = E_1 a_1^\dagger a_1 + E_2 a_2^\dagger a_2 - \Omega_0 (a_1^\dagger a_2 + a_2^\dagger a_1) \quad (1)$$

is the qubit Hamiltonian and H_{PC} , H_{int} describe the point contact detector and its interaction with the qubit:

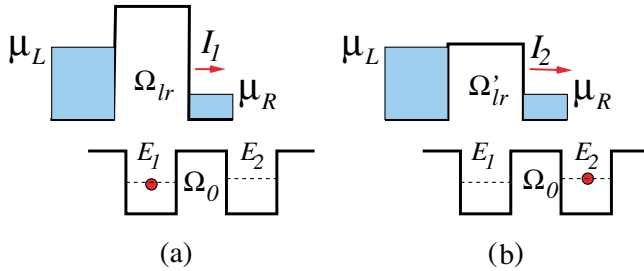


FIG. 1 (color online). A point-contact detector monitoring the electron position in the double dot. $\mu_{L,R}$ denote the chemical potentials in the left and right reservoirs.

$$H_{\text{PC}} = \sum_l E_l a_l^\dagger a_l + \sum_r E_r a_r^\dagger a_r + \sum_{l,r} (\Omega_{lr} a_r^\dagger a_l + \text{H.c.}),$$

$$H_{\text{int}} = \sum_{l,r} \delta\Omega_{lr} a_2^\dagger a_2 (a_r^\dagger a_l + \text{H.c.}).$$

Here $a_l^\dagger(a_l)$ and $a_r^\dagger(a_r)$ are the creation (annihilation) operators in the left and right reservoirs, and Ω_{lr} is the hopping amplitude between the states l and r of the reservoirs. For simplicity we consider electrons as spinless fermions and assume each of the reservoirs is at zero temperature. The interaction term H_{int} generates a change in the hopping amplitude, $\delta\Omega_{lr} = \Omega'_{lr} - \Omega_{lr}$. We assume that the hopping amplitude is weakly dependent on the states l, r , so that it can be replaced by its average value, $\Omega_{lr} \approx \bar{\Omega}$ and $\delta\Omega_{lr} \approx \delta\bar{\Omega}$. Thus the detector current is $I_1 = e2\pi\bar{\Omega}^2\rho_L\rho_R V$ when the electron occupies the first dot, and $I_2 = e2\pi(\bar{\Omega} + \delta\bar{\Omega})^2\rho_L\rho_R V$ when the electron occupies the second dot [1]. Here $\rho_{L,R}$ are the density of states in the reservoirs and $V = \mu_L - \mu_R$ is the bias voltage.

It was shown in [1] that for the case of a large bias voltage V , one can reduce the Schrödinger equation for the entire system to Bloch-type rate equations describing the reduced density matrix of the electron, $\sigma_{ij}(t)$. The diagonal terms of this density matrix, $\sigma_{11}(t)$ and $\sigma_{22}(t)$, are the probabilities of finding the electron in the first dot or in the second dot, respectively. The off-diagonal matrix elements (“coherences”) $\sigma_{12}(t)$, $\sigma_{21}(t)$ describe linear superpositions of these states. One finds [1]

$$\dot{\sigma}_{11} = -i\Omega_0(\sigma_{12} - \sigma_{21}),$$

$$\dot{\sigma}_{12} = i\epsilon\sigma_{12} - i\Omega_0(2\sigma_{11} - 1) - (\Gamma_d/2)\sigma_{12},$$

where $\sigma_{22}(t) = 1 - \sigma_{11}(t)$ and $\sigma_{21} = \sigma_{12}^*(t)$. Here $E_{1,2} = \mp\epsilon/2$ and $\Gamma_d = (\sqrt{I_1/e} - \sqrt{I_2/e})^2 V/2\pi$ is the decoherence rate due to interaction with the detector. Solving Eqs. (3a) and (3b), one obtains that the decoherence term in Eq. (3b) leads to a vanishing of coherences in the limit $t \rightarrow \infty$. Finally the electron density matrix becomes a completely random mixture for *any* initial condition: $\sigma_{ij}(t \rightarrow \infty) = (1/2)\delta_{ij}$.

Now we introduce the environment, represented by a boson bath at zero temperature and interacting with the

electron. First, consider the case in which the electron is not coupled to the detector, $H_{\text{int}} = 0$. Then, for any initial conditions, the electron relaxes to its lowest energy state (E_+) through boson emission. This final state can be found by disregarding any possible renormalization of the electron states due to interaction with the boson field. We thus diagonalize the Hamiltonian H_0 , Eq. (1) by using the following “rotation”: $a_{+,-} = \pm \cos(\theta/2)a_{1,2} + \sin(\theta/2)a_{2,1}$, with $\tan\theta = 2\Omega_0/\epsilon$. Then $H_0 = (\tilde{\epsilon}/2) \times (a_-^\dagger a_- - a_+^\dagger a_+)$, where $\tilde{\epsilon} = (\epsilon^2 + 4\Omega_0^2)^{1/2}$.

In order to account for the above relaxation effects in the qubit evolution, we replace the Hamiltonian H_0 by the following Hamiltonian:

$$H'_0 = H_0 + \sum_\alpha [E_\alpha b_\alpha^\dagger b_\alpha + c_\alpha (a_+^\dagger a_- b_\alpha^\dagger + a_-^\dagger a_+ b_\alpha)],$$

where $b_\alpha^\dagger(b_\alpha)$ are creation (annihilation) operators of a boson with the energy E_α . This Hamiltonian is essentially equivalent to the Lee model and has been investigated in numerous works [9]. In the weak coupling limit this model leads to the same results as the usual spin-boson model, although it includes an additional, direct coupling between the qubit states $a_{1,2}^\dagger|0\rangle$.

If there is no interaction with the detector, $H_{\text{int}} = 0$, and c_α is weakly dependent on E_α , one can trace the boson variables in the Schrödinger equation, by reducing it to the rate equations for the electron density matrix in the basis states, $a_\pm^\dagger|0\rangle$ [9],

$$\dot{\sigma}_{--}(t) = -\Gamma_r \sigma_{--}(t),$$

$$\dot{\sigma}_{+-}(t) = i\tilde{\epsilon}\sigma_{+-}(t) - (\Gamma_r/2)\sigma_{+-}(t),$$

with $\sigma_{++}(t) = 1 - \sigma_{--}(t)$ and $\sigma_{-+}(t) = \sigma_{+-}^*(t)$. Here $\Gamma_r = 2\pi c_\alpha^2 \rho_\alpha$ is the relaxation rate and ρ_α is the density of boson states. Equations (5a) and (5b) obviously reproduce an exponential decay (relaxation) of the electron from the upper level E_- to the ground level E_+ : $\sigma_{--}(t) = \exp(-\Gamma_r t)$. Note that the off-diagonal density-matrix element $\sigma_{+-}(t)$ vanishes in the limit $t \rightarrow \infty$, similar to $\sigma_{12}(t)$ in Eq. (3b). Yet, the disappearance of off-diagonal density-matrix elements does not necessarily imply dephasing. Indeed, in the case of relaxation, Eqs. (5a) and (5b), the diagonal term $\sigma_{--}(t)$ vanishes as well for $t \rightarrow \infty$. As a result, the qubit finally appears in a pure state (the ground state), in contrast to Eqs. (3a) and (3b) leading to the statistical mixture. Now let us include the interaction with the detector. Then the Hamiltonian of the entire system becomes $H' = H'_0 + H_{\text{PC}} + H_{\text{int}}$. It is useful to return to the original qubit basis $a_{1,2}^\dagger|0\rangle$, in which H_{int} , Eq. (2), has a simple form. The corresponding rate equations for the qubit density matrix $\sigma_{ij}(t)$ are obtained by tracing the detector and boson degrees of freedom. These equations can be written directly by using the method of Refs. [1,10]. We obtain

$$\dot{\sigma}_{11} = -i\Omega_0(\sigma_{12} - \sigma_{21}) - \Gamma_r(\kappa\epsilon/2\tilde{\epsilon})(\sigma_{12} + \sigma_{21}) - (\Gamma_r/4)[1 + (\epsilon/\tilde{\epsilon})^2](2\sigma_{11} - 1) + \Gamma_r(\epsilon/2\tilde{\epsilon}), \quad (6a)$$

$$\dot{\sigma}_{12} = i\epsilon\sigma_{12} - [i\Omega_0 + \Gamma_r(\kappa\epsilon/2\tilde{\epsilon})](2\sigma_{11} - 1) + \Gamma_r[\kappa - (1/2)\sigma_{12} - \kappa^2(\sigma_{12} + \sigma_{21})] - (\Gamma_d/2)\sigma_{12}, \quad (6b)$$

where $\kappa = \Omega_0/\tilde{\epsilon}$. (Similar equations were obtained by Korotkov [11] in the weak coupling limit by using phenomenological arguments.)

Solving Eqs. (6a) and (6b) for $\epsilon = 0$, one obtains the qubit density matrix for the stationary state ($\dot{\sigma} = 0$)

$$\bar{\sigma} = \sigma(t \rightarrow \infty) = \begin{pmatrix} 1/2 & y/(1+2y) \\ y/(1+2y) & 1/2 \end{pmatrix}, \quad (7)$$

where $y = \Gamma_r/\Gamma_d$. This describes a heated qubit with an effective temperature $T_{\text{eff}} = 2\Omega_0/\ln(1+4y)$. This heating is caused by the measurement process [12].

Now we investigate how the relaxation affects the time dependence of the qubit density matrix. Consider again the symmetric case, $\epsilon = 0$. Let us evaluate the probability of finding the electron in the first dot, $\sigma_{11}(t)$. By solving Eqs. (6) for the initial conditions $\sigma_{11}(0) = 1$, $\sigma_{12}(0) = 0$, we find

$$\sigma_{11}(t) = \frac{1}{2} + \frac{e^{-\Gamma_r t/2}}{4}(C_1 e^{-e-t} + C_2 e^{-e+t}), \quad (8)$$

where $e_{\pm} = \frac{1}{4}(\Gamma_d \pm \Omega)$, $\Omega = \sqrt{\Gamma_d^2 - 64\Omega_0^2}$, and $C_{1,2} = 1 \pm (\Gamma_d/\Omega)$. Thus, for the case of weak decoherence, $\Gamma_d \ll 8\Omega_0$, the electron displays damped oscillations between the dots with the Rabi frequency, $\sqrt{(2\Omega_0)^2 - (\Gamma_d/4)^2}$.

For strong coupling to the detector, $\Gamma_d \gg 8\Omega_0$, the situation is different. If $\Gamma_r = 0$, the electron would stay in the same dot for a long time (“quantum Zeno” effect). The dwell time τ_Z obtained from Eq. (8) is $\Gamma_d/8\Omega_0^2$. Thus the increase of Γ_d leads to a freezing of the electron, which is totally localized in the limit of $\Gamma_d \rightarrow \infty$. This result is consistent with the Zeno paradox, based on the

projection postulate. Indeed, the “measurement time” Δt is inversely proportional to Γ_d . (See, for instance, [3,13].) Also one can observe from Eq. (8) that for small Δt , and $\Gamma_r = 0$ $P(\Delta t) = [1 - \sigma_{11}(t)] \propto (\Delta t)^2$. However, as follows from Eq. (8), the interaction with the environment essentially destroys the Zeno effect. We find that the Zeno time in this case is $\tau_Z^{-1} = (\Gamma_r/2) + (8\Omega_0^2/\Gamma_d)$. Therefore the continuous measurement cannot localize the electron for a long time, even if $\Gamma_d \rightarrow \infty$. The corresponding dwell time is restricted by the relaxation rate Γ_r^{-1} . The disappearance of the Zeno paradox can be understood as follows. It is crucial to observe that the relaxations due to the environment and the detector are factorizable, as seen in Eq. (8). As a result of this factorizability, an effective linear in the Δt term, generated by purely exponential decay for small Δt , appears in the expansion of $P(\Delta t)$. This leads to the elimination of the Zeno effect [7,8].

Now we relate the qubit behavior to the corresponding observable quantities. For this, we have to include the detector states in the rate equations (6). We thus introduce the (reduced) density matrix $\sigma_{ij}^{(nn')}(t)$, where index n denotes the number of electrons that have arrived in the right reservoir by the time, t [8]. This density matrix is related to the previous one by $\sigma_{ij}(t) = \sum_n \sigma_{ij}^{(n)}(t)$, where $\sigma_{ij}^{(n)} \equiv \sigma_{ij}^{(nn)}$. Starting from the microscopic Schrödinger equation for the entire system and using the same method as in Refs. [1,8,10] we can demonstrate that in the limit of high bias voltage V of the detector, Fig. 1, the off-diagonal density-matrix elements $\sigma_{ij}^{(nn')}$ are decoupled from the diagonal elements $\sigma_{ij}^{(n)}$ in the equation of motion [8]. As a result we arrive at the following Bloch-type rate equations:

$$\dot{\sigma}_{11}^{(n)} = -i\Omega_0(\sigma_{12}^{(n)} - \sigma_{21}^{(n)}) - \Gamma_r(\kappa\epsilon/2\tilde{\epsilon})(\sigma_{12}^{(n)} + \sigma_{21}^{(n)}) - \frac{\Gamma_r}{4}(\beta_-^2 \sigma_{11}^{(n)} - \beta_+^2 \sigma_{22}^{(n)}) - \frac{I_1}{e}(\sigma_{11}^{(n)} - \sigma_{11}^{(n-1)}), \quad (9a)$$

$$\dot{\sigma}_{22}^{(n)} = i\Omega_0(\sigma_{12}^{(n)} - \sigma_{21}^{(n)}) + \Gamma_r(\kappa\epsilon/2\tilde{\epsilon})(\sigma_{12}^{(n)} + \sigma_{21}^{(n)}) + \frac{\Gamma_r}{4}(\beta_-^2 \sigma_{11}^{(n)} - \beta_+^2 \sigma_{22}^{(n)}) - \frac{I_2}{e}(\sigma_{22}^{(n)} - \sigma_{22}^{(n-1)}), \quad (9b)$$

$$\dot{\sigma}_{12}^{(n)} = i\epsilon\sigma_{12}^{(n)} - i\Omega_0(\sigma_{11}^{(n)} - \sigma_{22}^{(n)}) + \Gamma_r[\kappa(\beta_-^l \sigma_{11}^{(n)} + \beta_+^l \sigma_{22}^{(n)}) - (1/2)\sigma_{12}^{(n)} - \kappa^2(\sigma_{12}^{(n)} + \sigma_{21}^{(n)})] - \frac{I_1 + I_2}{2e}\sigma_{12}^{(n)} + \frac{\sqrt{I_1 I_2}}{e}\sigma_{12}^{(n-1)}, \quad (9c)$$

where $\beta_{\pm} = 1 \pm (\epsilon/\tilde{\epsilon})$ and $\beta'_{\pm} = 1 \pm (\epsilon/2\tilde{\epsilon})$. Tracing Eqs. (9) over n and using $\sigma_{11}(t) + \sigma_{22}(t) = 1$ we obtain Eqs. (6).

Equations (9) allow us to evaluate the average detector current and its shot-noise power spectrum. The (ensemble) average current is given by

$$I(t) = e \sum_n n \dot{P}_n(t) = (I_1 - I_2)\sigma_{11}(t) + I_2, \quad (10)$$

where $P_n(t) = \sigma_{11}^{(n)}(t) + \sigma_{22}^{(n)}(t)$ is the probability of finding n electrons in the collector by time t . As expected, the average current is directly related to the occupation of the

first dot. The shot-noise power spectrum can be calculated via the McDonald formula [14,15]

$$S(\omega) = \frac{e^2 \omega}{\pi} \int_0^{\infty} dt \sin(\omega t) \frac{d}{dt} N_R^2(t), \quad (11)$$

where $N_R^2(t) = \sum_n n^2 P_n(t)$.

Now we investigate how the relaxation and decoherence rates can be extracted from $I(t)$ and $S(\omega)$. Consider first the stationary detector current $\bar{I} = I(t \rightarrow \infty)$ for the symmetric case ($\epsilon = 0$). It follows from Eqs. (7) and (10) that $\bar{I} = (I_1 + I_2)/2$, so that it is independent

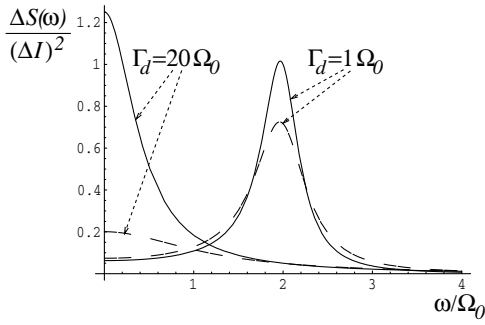


FIG. 2. The excess noise power spectrum of the detector current for different decoherence and relaxation rates. The solid curves correspond to $\Gamma_r = 0$, and the dashed curves to $\Gamma_r = 0.1\Omega_0$.

of $y = \Gamma_r/\Gamma_d$. Therefore this ratio cannot be extracted from \bar{I} for $\epsilon = 0$. However, for a nonsymmetric qubit, $\epsilon \gg \Omega_0$, the detector current becomes sensitive to y . Indeed, $\bar{I} = (I_1 + I_2)/2$ for $\Gamma_r = 0$ and $\Gamma_d \neq 0$, but $\bar{I} \approx I_1$ for $\Gamma_d = 0$ and $\Gamma_r \neq 0$ (since the relaxation puts the system into the lowest energy state, $E_+ \sim E_1$ for $\epsilon \gg \Omega_0$). Using Eqs. (6) and (10) we obtain for $\epsilon \gg \Omega_0$

$$\bar{I} = \Delta I \frac{y + (\Omega_0/\epsilon)^2}{y + 2(\Omega_0/\epsilon)^2} + I_2, \quad (12)$$

where $\Delta I = I_1 - I_2$.

Since in the symmetric case $\epsilon = 0$, the relaxation does not affect the stationary current \bar{I} . It instead affects its transient properties, which are reflected in the shot-noise spectrum of the detector current, $S(\omega)$, given by Eq. (11). One can write $S(\omega) = S_0 + \Delta S(\omega)$, where the first term $S_0 = e(I_1 + I_2)$ is the Schottky noise and the second term is the excess noise generated by the qubit dynamics. Generally the analytic expression for $\Delta S(\omega)$ is rather lengthy. We therefore present it only for $\epsilon = 0$ and in two limits: $\Gamma_d, \Gamma_r \ll \Omega_0$ and $\Gamma_d \gg \Omega_0$. In the first case the excess noise can be very well approximated by a Lorentzian

$$\Delta S(\omega) = \frac{(\Delta I)^2(\Gamma_d + 2\Gamma_r)}{(\Gamma_d + 2\Gamma_r)^2 + 16(\omega - 2\Omega_0)^2}. \quad (13)$$

This corresponds to the result of Korotkov, obtained by a Bayesian approach (“continuous” wave function collapse) in the weak coupling limit [11].

The second case, $\Gamma_d \gg \Omega_0$, corresponds to the Zeno effect regime. We find

$$\Delta S(\omega) = \frac{(\Delta I)^2(16\Gamma_d\Omega_0^2 + \Gamma_r\Gamma_T^2)}{4[\Gamma_d^2\omega^2 + 4(\omega^2 - 4\Omega_0^2)^2] + \Gamma_r\Gamma_d\Gamma_T^2}, \quad (14)$$

where $\Gamma_T = \Gamma_d + \Gamma_r$.

The excess noise, $\Delta S(\omega)$, for different values of Γ_d, Γ_r and $\epsilon = 0$ are shown in Fig. 2. As expected, for $\Gamma_d \lesssim \Omega_0$, the Rabi oscillations generate a peak in the noise spectrum at $\omega = 2\Omega_0$. The relaxation modifies this peak according to Eq. (13). In the case of large decoherence rate $\Gamma_d \gg \Omega_0$ and $\Gamma_r = 0$, the qubit is in the regime of the

Zeno effect. This leads to a telegraph noise [11], resulting in a peak at $\omega = 0$. This peak, however, is strongly diminished in the presence of relaxation, even for a small Γ_r , as given by Eq. (14).

This strong dependence provides a new way to measure the relaxation rate of a quantum system in experiments. As seen in Fig. 2, one can measure the relaxation rate of a qubit via the noise spectrum of the detector at zero frequency. Specifically, the relaxation rate can be lower by 2 or more orders of magnitude compared with the dephasing rate and still change the noise spectrum significantly. With such sensitivity, one may more accurately measure the relaxation rate because one can increase the output signal by increasing the coupling between the qubit and the detector.

In summary, we found that a qubit interacting with its environment and with a detector can be described by a set of modified Bloch-type equations in which the decoherence and relaxation process are clearly distinguished. The most interesting result of our analysis is that there is no Zeno paradox when the relaxation due to the environment is taken into account. In addition, we obtained simple analytical expressions for the detector current and its noise spectrum. Using these findings, we proposed a new and possibly more accurate way to measure the qubit decoherence and relaxation rates.

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