Grand Unified Theory Precursors and Nontrivial Fixed Points in Higher-Dimensional Gauge Theories

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(Received 26 February 2003; published 7 August 2003)

Within the context of traditional logarithmic grand unification at $M_{\rm GUT} \approx 10^{16}$ GeV, we show that it is nevertheless possible to observe certain GUT states such as X and Y gauge bosons at lower scales, perhaps even in the TeV range. We refer to such states as "GUT precursors." These states offer an interesting alternative possibility for new physics at the TeV scale, and could be used to directly probe GUT physics even though the scale of gauge coupling unification remains high. Our results also give rise to a Kaluza-Klein realization of nontrivial fixed points in higher-dimensional gauge theories.

DOI: 10.1103/PhysRevLett.91.061601 PACS numbers: 12.10.Kt, 11.10.Hi, 11.10.Kk, 11.25.Wx

One of the most important theoretical challenges in physics is to determine the nature of fundamental theories. Such fundamental theories include theories of grand unification, quantum gravity, and even strings, with each theory carrying its own intrinsic energy scale.

The traditional view of such theories stipulates that their intrinsic energy scales are exceedingly high. In such cases, experimental evidence in favor of such theories is at best indirect. More recently, however, it has been suggested [1–4] that the presence of large extra dimensions might significantly lower the energy scales associated with such theories, perhaps all the way to the TeV range. In such cases, we might hope for direct experimental tests of such theories.

In this Letter, we propose a "hybrid" possibility. Specifically, we consider a higher-dimensional scenario in which the fundamental theories of physics retain their traditional high characteristic energy scales, but in which it is nevertheless possible to obtain *direct,* low-energy evidence of their existence. As we shall see, this will be possible because of the emergence of a nontrivial fixed point which enables a large separation of scales to exist within a single model.

For concreteness, we will consider a scenario in which the unification of gauge couplings retains its traditional logarithmic behavior, with unification occurring near $M_{\rm GUT} \approx 10^{16}$ GeV, as in the minimal supersymmetric standard model (MSSM). However, we shall demonstrate that even within such a scenario, it is possible that certain states associated with the emergence of a grand unified theory (GUT) at this energy scale can actually be extremely light, perhaps even in the TeV range. We shall refer to such states as "GUT precursors." Such precursor states would then provide a direct, experimental window into high-scale, fundamental physics.

We shall work within the context of so-called "orbifold GUT" models [3,5–8], in which the GUT gauge symmetries are broken below the scale of unification by an orbifold compactification defined by $S^1/\mathbb{Z}_2$. If y is the coordinate along the compact extra dimension, states can be either even or odd under $y \to -y$. If standard model fields are even under the orbifold, while GUT fields are odd, then only the zero modes of the standard model fields appear at low energies and the orbifold projection has broken the GUT. However, unlike the Higgs breaking mechanism, where masses of the GUT fields beyond the standard model are parametrically tied to $M_{\rm GUT}$, the masses of the first Kaluza-Klein (KK) modes for these GUT particles are set by the inverse radius of the orbifold. Thus, in cases for which $R^{-1} < M_{\rm GUT}$, we actually begin to observe GUT particles (such as X and Y gauge bosons) *before* we detect actual gauge coupling unification. In other words, these low-lying KK modes of the GUT particles appear as GUT precursors [3], signaling the future emergence of a full gauge coupling unification at an even higher energy scale.

How far below $M_{\rm GUT}$ can the GUT precursors sit? Consider the evolution of the gauge couplings in theories with extra dimensions, which takes the approximate form [3]

$$\alpha_i^{-1}(\Lambda) \approx \alpha_i^{-1}(M_Z) - \frac{b_i}{2\pi}\ln\frac{\Lambda}{M_Z} + \frac{\tilde{b}_i}{2\pi}\ln\Lambda R - \frac{\tilde{b}_i X_\delta}{2\pi\delta}[(\Lambda R)^\delta - 1], \quad (1)$$

where M_Z is our chosen low-energy reference scale; Λ is an arbitrary high-scale cutoff; δ is the number of compactified dimensions; R is their common radius of compactification; and the normalization factor X_δ is the compactification volume with all radii normalized to unity. Likewise, b_i ($\tilde{b}_i$) are the beta-function coefficients of the zero-mode (excited KK) fields.

In cases where the $\tilde{b}_i$ are unequal, we see from Eq. (1) that the power-law evolution of the gauge couplings is different for each gauge coupling. This implies that the relative *differences* between the gauge couplings also evolve with power-law behavior. However, when the $\tilde{b}_i$ are all equal, we see from Eq. (1) that this power-law behavior is universal for all gauge couplings. The relative differences of gauge couplings then evolve purely logarithmically, exactly as in four dimensions.

Despite this fact, it is still important to verify that the individual gauge couplings themselves remain perturbative over the entire energy range from R^{-1} to $\Lambda \equiv M_{\rm GUT}$. If we assume that $\tilde{b}_i \equiv \tilde{b} < 0$ for all i, then the power-law contributions to the gauge couplings push the couplings towards extremely weak values. Indeed, in the limit where $\Lambda R \gg 1$, we find from Eq. (1) that each of the gauge couplings scales in the ultraviolet as

$$\alpha(\Lambda) \approx -\frac{2\pi\delta}{\tilde{b}X_\delta}(\Lambda R)^{-\delta}. \qquad (2)$$

However, even though these couplings are extremely weak, the true loop expansion parameter in such a situation is $\alpha_{\rm eff} \equiv N\alpha$ where $N \equiv X_\delta(\Lambda R)^\delta$ is the number of KK levels that have been crossed. Indeed, $\alpha_{\rm eff}$ describes the effective strength of the gauge interaction, since it characterizes the coupling of each individual KK mode multiplied by the multiplicity of these modes. Thus, for true perturbativity, we must demand $\alpha_{\rm eff} \ll 4\pi$.

Remarkably, this constraint is satisfied no matter how large ΛR becomes. Indeed, we find that $\alpha_{\rm eff} \approx -2\pi\delta/\tilde{b}$ as $\Lambda R \to \infty$, so that the condition for perturbativity becomes $-\delta/(2\tilde{b}) \ll 1$. Thus, if $\tilde{b}$ is sufficiently large and negative, this condition can be satisfied even if $\Lambda R \gg 1$.

As an example, let us consider a scenario in which, as discussed above, the zero-mode fields are those of the MSSM and only the GUT gauge bosons sit in the bulk. For simplicity, we shall take our unified gauge group to be SU(5), and we shall also assume that $\delta = 1$. Since our low-energy theory is $\mathcal{N} = 1$ supersymmetric, the bulk fields necessarily fall into $\mathcal{N} = 2$ supermultiplets. Our bulk fields therefore consist of $\mathcal{N} = 2$ vector multiplets transforming in the adjoint of SU(5), leading to $\tilde{b}_i = \tilde{b} = -10$ for all i. We then find that the effective gauge interaction strength at unification is $\alpha_{\rm eff} \approx 0.63$, which is considerably less than 4π. Note that this remains true even if $\Lambda R \approx 10^{13}$. Thus it is possible for the GUT precursors to appear at the TeV scale even though the (logarithmic) gauge coupling unification does not occur until the usual scale $M_{\rm GUT} \approx 10^{16}$ GeV.

This behavior is illustrated in Fig. 1, where we plot the value of the effective unified coupling $\alpha_{\rm eff}$ at $M_{\rm GUT}$ as a function of R, holding $M_{\rm GUT}$ fixed at its usual four-dimensional value 2×10^{16} GeV. We have taken $\delta = 1$ and $\tilde{b} = -10$, as discussed above. It is clear that the effective coupling remains perturbative for arbitrarily large values of $M_{\rm GUT}R$, saturating at its asymptotic value as early as $M_{\rm GUT}R \approx 100$. Thus, the scale at which our GUT precursors appear can be separated by an arbitrary amount from the scale at which the gauge couplings unify.

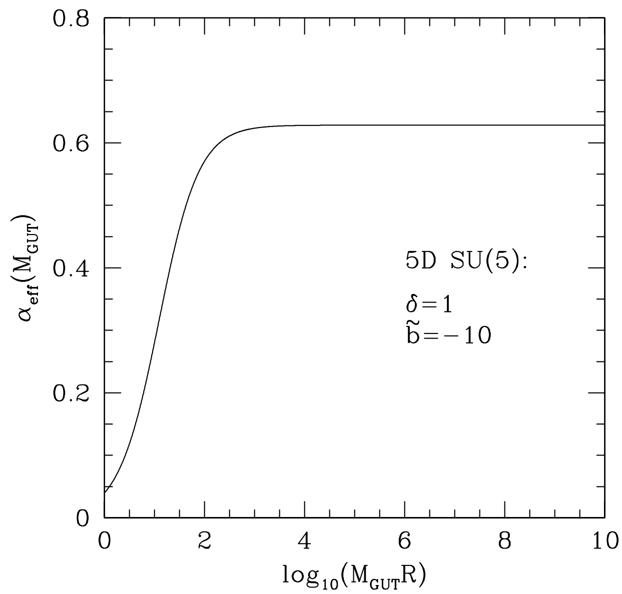

FIG. 1. The effective unified coupling $\alpha_{\rm eff}(M_{\rm GUT})$ as a function of $M_{\rm GUT}R$ for the five-dimensional SU(5) GUT model, as discussed in the text. This coupling remains perturbative for arbitrarily large values of $M_{\rm GUT}R$.

One might worry that two-loop effects might be significant in such a scenario. However, two-loop effects essentially *vanish* in the $\Lambda R \to \infty$ limit, since $\mathcal{N} = 2$ supersymmetry in the bulk ensures that the higher-loop power-law effects are suppressed by a factor of $1/\Lambda R$ relative to the one-loop effects [3,9]. Even when ΛR remains finite, it is straightforward to verify that two- and higher-loop corrections do not substantially alter the logarithmic unification which emerges at one-loop order [3,9].

In this scenario, the asymptotic ultraviolet power-law scaling of gauge couplings towards weak values is exactly compensated by the asymptotic power-law growth of the number of degrees of freedom in the theory in such a way that the product of these two quantities remains a constant. This suggests that the effective strength of the gauge interactions appears to approach a nontrivial fixed point in the ultraviolet. Such behavior for gauge couplings with $\tilde{b} < 0$ was also observed previously in Ref. [10]. In our case it is already apparent from Fig. 1 that as long as $\Lambda R \gtrsim 100$, our theory essentially becomes "scale invariant" in the sense that the ultraviolet physics becomes independent of the low-energy scale R^{-1} at which the GUT precursors appear. We may also rephrase this observation directly in terms of the effective couplings $\alpha_{{\rm eff},i} \equiv N\alpha_i$ where $N \equiv X_\delta(\Lambda R)^\delta$. These effective couplings evolve according to

$$\Lambda\frac{d\alpha^{-1}_{{\rm eff},i}}{d\Lambda} = -\left(\delta\alpha^{-1}_{{\rm eff},i} + \frac{\tilde{b}_i}{2\pi}\right) + \left(\frac{\tilde{b}_i - b_i}{2\pi X_\delta}\right)(\Lambda R)^{-\delta} + \frac{c_i}{2\pi}\,\frac{\alpha_{{\rm eff},i}}{4\pi}(\Lambda R)^{-\delta} + \ldots, \qquad (3)$$

where in the second line we have written the dominant two-loop contributions arising from the bulk and boundary fields running in the loops (with c_i representing a two-loop beta-function coefficient). Thus, even though the individual gauge couplings α_i evolve with power-law behavior, we see from Eq. (3) that for $\Lambda R \gg 1$, the *effective* gauge couplings $\alpha_{\mathrm{eff},i}$ each approach an ultraviolet fixed point at $\alpha_{\mathrm{eff},i} = -2\pi\delta/\tilde{b}_i$. Moreover, if $\tilde{b}_i \equiv \tilde{b}$ for all i, we see that even though the differences of the gauge couplings continue to evolve logarithmically, the fixed-point values of the *effective* gauge couplings all become equal. Thus, in this sense, we see that the effective strengths of the gauge interactions in this theory each flow to a *common* fixed point in the ultraviolet. Note that two- and higher-loop effects merely contribute additional power-law terms in Eq. (3) which again vanish in the $\Lambda R \to \infty$ limit. Such contributions therefore do not alter the ultraviolet fixed-point structure of these theories.

It is natural to interpret these results as indicating the emergence of a nontrivial (interacting) ultraviolet fixed point corresponding to a supersymmetric, higher-dimensional, unified gauge theory. Indeed, such higher-dimensional fixed-point gauge theories are known to exist in uncompactified five and six dimensions [11–13]. Since we expect the ultraviolet (short-distance) limit of our compactified theory to reproduce the physics of an uncompactified higher-dimensional theory, it is tempting to identify the ultraviolet limit of our theory as one of the interacting fixed-point theories discussed in Refs. [12,13].

For example, in the case of SU(N) gauge theory in five dimensions, where matter consists only of n_f "quarks" transforming in the fundamental representation, the necessary and sufficient condition [12] for the existence of an interacting ultraviolet fixed point is $n_f \leq 2N$. This is equivalent to our requirement that $\tilde{b} \leq 0$.

One important by-product of this analysis is that it essentially furnishes us with an alternative, four-dimensional "Kaluza-Klein" realization of these fixed-point theories. In such a realization, the effective higher-dimensional gauge coupling at the fixed point asymptotically emerges in the ultraviolet as the product $\alpha_{\mathrm{eff}} = N\alpha$, and the *dimensionful* gauge coupling in higher dimensions is $\alpha_{4+\delta}(\Lambda) = \Lambda^{-\delta}\alpha_{\mathrm{eff}}(\Lambda)$.

Note that in realistic GUT orbifold models, there can be additional nonuniversal logarithmic contributions to the gauge coupling running. For example, let us consider the case of compactification on an $S_1/(\mathbb{Z}_2 \times \mathbb{Z}_2')$ orbifold with two distinct $\mathbb{Z}_2$ discrete actions [6,7] associated with $y \to -y$ and $y \to \pi R - y$. With this orbifold choice, only the standard model fields have zero modes, but this occurs at the expense of splitting the complete GUT multiplets at each KK level into a subset at even levels, with beta-function coefficients $(\tilde{b}_1, \tilde{b}_2, \tilde{b}_3) = (0, -4, -6)$, and a subset at odd levels, with $(\tilde{b}_1', \tilde{b}_2', \tilde{b}_3') = (-10, -6, -4)$. This results in a staggered, "zigzag" running for the gauge couplings which averages to a universal power law running with an effective radius $R/2$, along with a nonuniversal logarithmic correction. Specifically, the gauge couplings now run according to

$$\alpha_i^{-1}(\Lambda) \approx \alpha_i^{-1}(M_Z) - \frac{b_i}{2\pi}\ln\frac{\Lambda}{M_Z} + \frac{\tilde{b}}{4\pi}\ln\frac{\Lambda R}{2} - \frac{\tilde{b}}{2\pi}\left[\left(\frac{\Lambda R}{2}\right) - 1\right] - \frac{\tilde{b}_i'}{2\pi}Y, \quad (4)$$

where $\tilde{b} \equiv \tilde{b}_i + \tilde{b}_i' = -10$ for all i, where we have neglected certain universal additive constants, and where the nonuniversal logarithm is given by

$$Y \equiv \sum_{n=0}^{(\Lambda R-2)/2} \ln\frac{2n+2}{2n+1} \approx \frac{1}{2}\ln\frac{\pi\Lambda R}{2} \quad (5)$$

with the last approximation holding in the $\Lambda R \gg 1$ limit. Given this running, we then find that the three gauge couplings continue to experience an approximate unification. With $R^{-1} \sim$ TeV, the unification scale is unfortunately quite high ($M_{\mathrm{GUT}} \approx 10^{21}$ GeV), but increasing R^{-1} not only improves the accuracy of the resulting unification but also lowers the unification scale. Asymptotically, with $R^{-1} \approx 10^{15}$ GeV, we obtain an essentially *exact* unification at $M_{\mathrm{GUT}} \approx 10^{17}$ GeV.

The results in this Letter prompt a number of important questions, both phenomenological and theoretical. Among the most important phenomenological questions is the issue of proton decay. Ordinarily, light X and Y gauge boson precursors will mediate rapid proton decay. However, as in all low-scale extensions to the standard model, this problem may be cured through the use of split fermions on the branes [14] or through the introduction of extra discrete symmetries [3,15]. Likewise, other phenomenological issues include doublet/triplet splitting and general issues of flavor physics. Although we have not attempted to make a complete GUT model that accommodates these phenomena, one could imagine doing so following the lines of Refs. [6–8] except that we now have the interesting option of extending the energy scales of such models into the TeV range.

Our results in this paper also raise a number of theoretical issues. Although we have shown that the evolution of the gauge couplings is consistent with perturbativity even when the effective higher-dimensional energy interval is large, one must actually verify that *all* correlation functions in the theory remain finite and under control over this large energy range. By counting KK states and vertex factors in diagrams with arbitrary numbers of loops and external legs, it is straightforward to demonstrate that all diagrams in this theory necessarily scale as $(N\alpha)^k\sqrt{\alpha}\ell$ where k and ℓ are non-negative integers. Thus, in the ultraviolet limit, such diagrams either vanish (if $\ell \neq 0$) or approach a fixed finite value (if $\ell = 0$).

Indeed, even though the number of states in this theory is diverging at higher energies, the individual gauge couplings are falling to zero in a compensatory manner. For example, the four-fermion amplitude for tree-level KK exchange in the $\delta = 1$ case becomes

$$A(s) \sim \alpha(s) \sum_n \frac{1}{s - n^2/R^2} \sim \frac{\alpha(s)}{\sqrt{s}} R \cot(\pi R \sqrt{s}) \rightarrow \frac{\alpha_{\rm eff}}{s} \cot(\pi R \sqrt{s}), \quad (6)$$

where we have taken the limit $sR^2 \gg 1$ and identified $\alpha_{\rm eff} \sim (R\sqrt{s})\alpha(s)$ as $s \rightarrow \infty$. Thus, since $A(s)$ continues to have the asymptotic energy dependence $\sim 1/s$, no unitarity bounds are violated in the ultraviolet.

Another important theoretical issue for our models concerns gravity. Since our large extra dimension is presumably also felt by gravity, KK gravitons will induce Newton's constant to run more quickly. The standard Gauss-law arguments of Ref. [2] then imply that taking $R^{-1} \sim \mathcal{O}(\text{TeV})$ lowers the effective higher-dimensional Planck scale M_* to approximately 10^{14} GeV. Although this is comfortably within all experimental constraints, this value is slightly below $M_{\rm GUT} \approx 10^{16}$ GeV. This indicates that we reach a region of strong gravity *before* our gauge couplings unify.

Within the context of the $S_1/\mathbb{Z}_2$ orbifold, it turns out that there are additional nonuniversal logarithmic contributions which actually lower the unification scale to approximately 10^{13} GeV [16]. On the other hand, for the $S_1/(\mathbb{Z}_2 \times \mathbb{Z}_2')$ orbifold, the unification scale generally exceeds 10^{16} GeV. One could then either restrict the GUT precursor scale to the range $R^{-1} \gtrsim 10^{10}$ GeV or lower the value of $M_{\rm GUT}$ to 10^{14} GeV by introducing further states with appropriate gauge quantum numbers into the theory [17].

Thus, to summarize, we have shown that it is possible for GUT precursor states to appear with masses that are extremely light compared with the scale of gauge coupling unification. This suggests a possible new TeV-scale direction for orbifold GUT models. Indeed, more generally, we have seen that ultraviolet embeddings into fixed-point theories can be used to provide a new method for maintaining or stabilizing a wide separation of energy scales within a single model. Using this technique, hybrid models with coexisting high and low energy scales can therefore be constructed in a variety of contexts. Equally importantly, however, our four-dimensional Kaluza-Klein realization of such fixed points should also provide a new technique for the study of such theories and their properties under various compactifications, both with and without supersymmetry breaking and gauge symmetry breaking. These and other directions await exploration.

K. R. D. was supported in part by the National Science Foundation under Grant No. PHY-0071054, and by the Research Corporation. E. D. was supported in part by the RTN European Program HPRN-CT-2000-00148. We thank K. Agashe, A. Delgado, C. Grojean, K. Intriligator, G. D. Kribs, J. Mourad, Y. Nomura, M. J. Strassler, and especially E. Poppitz for discussions. K. R. D. and T. G. acknowledge the hospitality of the Aspen Center for Physics where this work was initiated, and K. R. D. and E. D. acknowledge the hospitality of the Theoretical Physics Institute at the University of Minnesota where this work was completed.

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