Accelerating Cosmologies from Spacelike Branes

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We point out that the recently proposed model of a flat four-dimensional universe with accelerated expansion in string or M theory is a special case of time-dependent solutions that the author found under the name of spacelike (S) branes. We also show that similar accelerating models can be obtained from S branes if the internal space is chosen to be hyperbolic or flat spaces.

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In a recent paper [1], Townsend and Wohlfarth proposed a solution of $(4+n)$-dimensional vacuum Einstein equations in string or M theory with compact hyperbolic internal space which exhibits accelerated expansion (hereafter referred to as accelerating solution). This is an interesting cosmological model since astronomical observations show that the universe is not only expanding but also is undergoing accelerated expansion [2]. Recent measurements of the cosmic microwave background seem to further support the accelerated expansion in an inflationary epoch [3]. Related discussions can be found in Refs. [4–6].

On the other hand, an interesting class of time-dependent solutions have been found in the supergravity theories in higher dimensions, the low-energy effective theories of superstring or M theory. These are the spacelike brane solutions (S branes) which were proposed in connection with tachyon condensations and de Sitter/conformal-field-theory correspondence [7–12], but the present interest is concerned with their property as time-dependent solutions. In particular, the analysis in Ref. [12] is quite general to discuss time-dependent solutions, and one may wonder if there is any connection between the S brane and above solutions.

At first sight, it may appear that there is no connection since the above accelerating solution is the one to the vacuum Einstein equations, whereas S branes are a class of solutions with background antisymmetric tensors. It is true for the solutions in Refs. [7–11] since these necessarily involve nonzero field strengths. However, we would like to point out that our solutions in Ref. [12] are sufficiently general to cover the accelerating solution, which is actually a special case of the time-dependent solutions that the present author derived. We show that our solutions [12] reduce to the accelerating solution if we put the field strength to zero and choose constants appropriately. In addition, we show that more general S brane solutions exhibit similar accelerated expansion if we choose the compact internal space to be hyperbolic. It turns out that actually the internal flat space is also allowed for accelerating solution, thus providing a wider class of solutions appropriate for cosmology. Following the usual convention, we use Sq branes for those with $(q+1)$-dimensional Euclidean world volume.

The solution in Ref. [1] is the one for $(4+n)$-dimensional vacuum Einstein equations

$$ds^2 = e^{3nt/(n-1)}K^{-n/(n-1)}ds_E^2 + e^{-6t/(n-1)}K^{2/(n-1)}d\Sigma_n^2, \tag{1}$$

where n is the dimension of the internal hyperbolic space, which is compactified, and

$$ds_E^2 = -S^6dt^2 + S^2d\mathbf{x}^2, \tag{2}$$

describes the four-dimensional spacetime with

$$S(t) = e^{-(n+2)t/2(n-1)}K^{n/2(n-1)},$$
$$K(t) = \frac{\sqrt{3(n+2)/n}}{(n-1)\sinh[\sqrt{3(n+2)/n}|t|]}. \tag{3}$$

If we take the time coordinate η defined by

$$d\eta = S^3(t)dt, \tag{4}$$

the metric (2) describes a flat homogeneous isotropic universe with scale factor S. The condition for expanding the four-dimensional universe is that

$$\frac{dS}{d\eta} > 0. \tag{5}$$

Accelerated expansion is obtained if, in addition,

$$\frac{d^2S}{d\eta^2} > 0. \tag{6}$$

It has been shown that these can be satisfied for $n=7$ and for a certain period of negative t which is the period that our universe is evolving ($t<0$ and $t>0$ are two disjoint possible universes) [1].

We are now going to show that our solutions in Ref. [12] reduce to this if we set the field strength to zero. Our action consists of gravity coupled to a dilaton ϕ and m different n_A-form field strengths in arbitrary dimensions

d. It describes the bosonic part of the $d = 11$ or $d = 10$ supergravities if we choose the parameters suitably.

The solutions are given by

$$ds_d^2 = \prod_A [\cosh \tilde{c}_A (t - t_A)]^{2[(q_A+1)/\Delta_A]} \Bigg[e^{2c_0 t + 2c_0'} \{ -e^{2ng(t)} dt^2 + e^{2g(t)} d\Sigma_{n,\sigma}^2 \} + \sum_{\alpha=1}^{p} \prod_A [\cosh \tilde{c}_A (t - t_A)]^{-2(\gamma_A^{(\alpha)}/\Delta_A)} e^{2\tilde{c}_\alpha t + 2c_\alpha'} dx_\alpha^2 \Bigg], \tag{7}$$

$$E_A = \sqrt{\frac{2(d-2)}{\Delta_A}} \frac{e^{\tilde{c}_A(t-t_A) - \epsilon_A a_A c_\phi'/2 + \sum_{\alpha \in q_A} c_\alpha'}}{\cosh \tilde{c}_A (t - t_A)},$$

$$\tilde{c}_A = \sum_{\alpha \in q_A} c_\alpha - \frac{1}{2} c_\phi \epsilon_A a_A, \tag{8}$$

$$\phi = \sum_A \frac{(d-2)\epsilon_A a_A}{\Delta_A} \ln \cosh \tilde{c}_A (t - t_A) + \tilde{c}_\phi t + c_\phi',$$

where $d = p + n + 1$, A denotes the kinds of q_A branes, the time derivatives of E_A give the value of the field strengths of antisymmetric tensors, a_A is the parameter for the coupling of dilaton and forms, and $\epsilon_A = +1(-1)$ corresponds to electric (magnetic) fields. The coordinates x_α, $(\alpha = 1, \ldots, p)$ parametrize the p-dimensional world-volume directions and the remaining coordinates of the d-dimensional spacetime are the time t and coordinates on compact n-dimensional spherical ($\sigma = +1$), flat ($\sigma = 0$), or hyperbolic ($\sigma = -1$) spaces, whose line elements are $d\Sigma_{n,\sigma}^2$. We have also defined

$$\Delta_A = (q_A + 1)(d - q_A - 3) + \tfrac{1}{2} a_A^2 (d - 2),$$

$$\gamma_A^{(\alpha)} = \begin{cases} d - 2 \\ 0 \end{cases} \quad \text{for} \quad \begin{cases} x_\alpha \text{ belonging to } q_A \text{ brane} \\ \text{otherwise,} \end{cases} \tag{9}$$

and

$$g(t) = \begin{cases} \frac{1}{n-1} \ln \frac{\beta}{\cosh[(n-1)\beta(t-t_1)]} & :\sigma = +1 \\ \pm \beta (t - t_1) & :\sigma = 0 \\ \frac{1}{n-1} \ln \frac{\beta}{\sinh[(n-1)\beta|t-t_1|]} & :\sigma = -1, \end{cases} \tag{10}$$

t_A, t_1 and c are integration constants which satisfy

$$c_0 = \sum_A \frac{q_A + 1}{\Delta_A} \tilde{c}_A - \frac{\sum_{\alpha=1}^{p} c_\alpha}{n - 1}, \qquad c_0' = -\frac{\sum_{\alpha=1}^{p} c_\alpha'}{n-1},$$

$$\tilde{c}_\alpha = c_\alpha - \sum_A \frac{\gamma_A^{(\alpha)} - q_A - 1}{\Delta_A} \tilde{c}_A, \tag{11}$$

$$\tilde{c}_\phi = c_\phi + \sum_A \frac{(d-2)\epsilon_A a_A}{\Delta_A} \tilde{c}_A.$$

These must further obey the condition

$$\frac{1}{n-1} \left(\sum_{\alpha=1}^{p} c_\alpha \right)^2 + \sum_{\alpha=1}^{p} c_\alpha^2 + \frac{1}{2} c_\phi^2 = n(n-1)\beta^2. \tag{12}$$

So the solutions look sufficiently complicated that it may not be easy to find the connection with the accelerating solution (1).

Let us restrict these to a single S brane in $d = 11$ and set the field strength to 0. Remember that the world volume of q branes lies in $(q+1)$-dimensional space and not in time. For 11-dimensional supergravity, we have electric $SM2$ branes ($S2$ branes in 11-dimensional supergravity), magnetic $SM5$ branes, and no dilaton $a_A = 0$, $c_\phi = 0$. Here we note that the relation between $\tilde{c}_A$ and c_α in Eq. (8) is derived under the assumption that we have the independent field strengths E_A. In the absence of these, we can disregard this relation and set $\tilde{c}_A$ to zero. We find that the solution (7) takes the form (1)–(3) with

$$S(t) \equiv e^{-(n+2)(ct+c')/2(n-1) + ng(t)/2}, \tag{13}$$

where we have set $c \equiv c_1 = c_2 = c_3$ and $c' \equiv c_1' = c_2' = c_3'$. It then follows that our solutions reproduce the accelerating one (1) if we further set $p = 3$, $q_A = 2$, $c = 1$, $c' = 0$, $t_1 = 0$, and $\sigma = -1$ [hyperbolic case in (10)] with β determined by Eq. (12).

We note that there is a slight generalization in our solutions that allows constant parameters c and c'. We have also examined the possibility if similar accelerating solutions can be obtained for flat and spherical internal spaces. It turns out that neither the flat nor spherical internal spaces give accelerating cosmologies; the condition for expansion can be satisfied, but both cases give always a decelerating universe.

We now show that our $SM2$ brane also gives four-dimensional models of the accelerating universe. We will find that here the flat internal space also allows this kind of model. We choose $d = 11$, $q_A = 2$, $c \equiv c_1 = c_2 = c_3$, and $c' \equiv c_1' = c_2' = c_3'$. Our solutions (7) then reduce to

$$ds_{11}^2 = [\cosh 3c(t - t_A)]^{-7/6} e^{-7g(t) + 7c'/2} ds_E^2 + [\cosh 3c(t - t_A)]^{1/3} e^{2g(t) - c'} d\Sigma_{7,\sigma}^2, \tag{14}$$

where the four-dimensional part is given by

$$ds_E^2 = -[\cosh 3c(t - t_A)]^{3/2} e^{21g(t) - 9c'/2} dt^2 + [\cosh 3c(t - t_A)]^{1/2} e^{7g(t) - 3c'/2} d\mathbf{x}^2. \tag{15}$$

Comparing this solution with Eqs. (1)–(3), we find that our solutions have precisely the same form with $S(t)$ given by

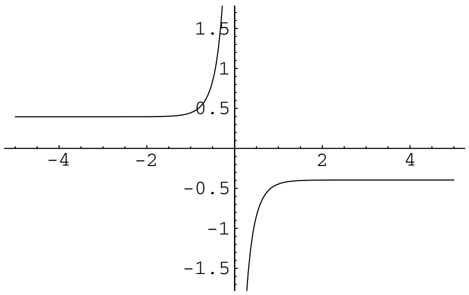

FIG. 1. The lhs of Eq. (17).

$$S(t) = [\cosh 3c(t - t_A)]^{1/4} e^{7g(t)/2 - 3c'/4}. \qquad (16)$$

We then define the time η by Eq. (4) and examine if the conditions for expansion (5) and accelerated expansion (6) are satisfied. For $t_A = t_1 = 0$ and $\sigma = -1$ (hyperbolic space), we find the condition (5) is

$$n_1(t) \equiv \frac{3}{4}\tanh(3ct) - \frac{\sqrt{21}}{4}\coth(3\sqrt{3/7}ct) > 0, \qquad (17)$$

and the condition (6) gives

$$\frac{3}{2\sqrt{2}}\sqrt{\frac{1}{\cosh^2(3ct)} + \frac{1}{\sinh^2(3\sqrt{3/7}ct)}} - n_1(t) > 0. \qquad (18)$$

The left-hand side (lhs) of Eqs. (17) and (18) for $c = 1$ are shown in Figs. 1 and 2, respectively. We see that there is a certain period of negative time that these conditions are satisfied, exactly as the solution (1). The period of the accelerated expansion can be adjusted by changing the constant c. Just as the accelerating solution (1), the universe is decelerating as $t \to -\infty$ ($\eta \to 0$) and $t \to 0$ from $t < 0$ ($\eta \to \infty$) [1]. The singularity at $t = 0$ of the function $S(t)$ is at an infinite proper time future for any event with $t < 0$, and our universe simply separates into two with $t < 0$ and $t > 0$.

If the internal space is chosen to be flat ($\sigma = 0$), the conditions (5) and (6) give

$$n_2(t) \equiv \frac{3}{4}\tanh(3ct) + \frac{\sqrt{21}}{4} > 0, \qquad (19)$$

$$\frac{3}{2\sqrt{2}}\frac{1}{\cosh(3ct)} - n_2(t) > 0, \qquad (20)$$

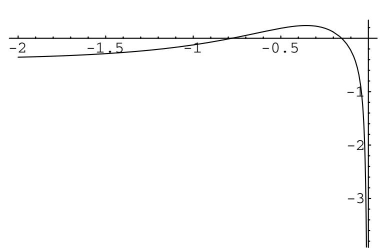

FIG. 2. The lhs of Eq. (18).

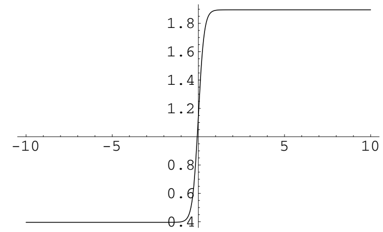

FIG. 3. The lhs of Eq. (19).

where we have chosen the plus sign in Eq. (10) since the minus sign cannot give expanding universe. We find that these conditions can also be satisfied for negative t as shown in Figs. 3 and 4, which are the results again for $c = 1$. Here we note that the universe is decelerating as $t \to -\infty$ ($\eta \to 0$) and $t > 0$. There is no singularity at $t = 0$ and the time η starts from 0 (at $t = -\infty$) to $\eta = \infty$ ($t = \infty$). The accelerated expansion is realized for a certain period before $t = 0$.

On the other hand, if we choose the internal space to be spherical ($\sigma = +1$), we find that the conditions (5) can be satisfied for negative t but (6) cannot be satisfied for any value of the time.

As we have remarked above, the period of the accelerated expansion can be changed by modifying the constant c for hyperbolic and flat internal spaces, but the expansion factor during the accelerated expansion (the ratio of the scale factors at the starting time and ending time) does not change. One typically obtains a factor such as 3, which is too small to explain the horizon or flatness problems as a model of inflation at the early universe. However, it is possible that solutions of a large amount of inflation can be found in this kind of model with suitable modifications. Also, the situation may change if we take into account quintessence field from matters. Another possibility is that the model may be used for explaining the present accelerated expansion of the universe. Details of the analysis on these problems will be reported elsewhere [13].

Though we have not examined other cases in ten-dimensional supergravities, the only other S brane solution that can give four-dimensional universe is the $SD2$ brane, which can be obtained from $SM2$ brane by dimensional reduction and is expected to show similar behavior.

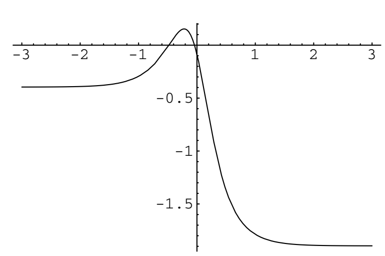

FIG. 4. The lhs of Eq. (20).

However, it would be interesting to further examine other possible solutions.

In summary, we have shown that the accelerating solution (1) is a special case of the solutions in Ref. [12]. We have also shown that the S-brane solutions can give interesting accelerating universe models for the compact internal hyperbolic and flat spaces. Other interesting time-dependent N-brane solutions have been found in Ref. [14]. It would be interesting to examine if this class of solutions can give similar interesting cosmological models and also try to further extend our analysis to other S brane solutions. We hope to discuss these problems elsewhere.

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[1] P. K. Townsend and M. N. R. Wohlfarth, Phys. Rev. Lett. **91**, 061302 (2003).
[2] A. G. Riess *et al.*, Astrophys. J. **560**, 49 (2001).
[3] A. Lewis and S. Bridle, Phys. Rev. D **66**, 103511 (2002).C. L. Bennett *et al.*, astro-ph/0302207.
[4] C. M. Hull and N. P. Warner, Classical Quantum Gravity **5**, 1517 (1988).
[5] P. K. Townsend, J. High Energy Phys. **11** (2001) 042.
[6] G. W. Gibbons and C. M. Hull, hep-th/0111072.
[7] M. Gutperle and A. Strominger, J. High Energy Phys. **04** (2002) 018.
[8] C. M. Chen, D. V. Gal'tsov, and M. Gutperle, Phys. Rev. D **66**, 024043 (2002).
[9] M. Kruczenski, R. C. Myers, and A. W. Peet, J. High Energy Phys. **05** (2002) 039.
[10] N. S. Deger and A. Kaya, J. High Energy Phys. **07** (2002) 038.
[11] V. D. Ivashchuk, Classical Quantum Gravity **20**, 261 (2003).
[12] N. Ohta, Phys. Lett. B **558**, 213 (2003).
[13] N. Ohta, hep-th/0304172; (to be published).
[14] N. Ohta, Phys. Lett. B **559**, 270 (2003).