

Generating a Superposition of Spin States in an Atomic Ensemble

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A method for generating a mesoscopic superposition state of the collective spin variable of a gas of atoms is proposed. The state consists of a superposition of the atomic spins pointing in two slightly different directions. It is obtained by using off resonant light to carry out quantum nondemolition measurements of the spins. The relevant experimental conditions, which require very dense atomic samples, can be realized with presently available techniques. Long-lived atomic superposition states may become useful as an off-line resource for quantum computing with otherwise linear operations.

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The generation of superpositions of macroscopically different quantum “cat” states has been attracting fundamental interest since they illustrate the superposition principle in an extreme setting [1]. Recently, such states have also been proposed as an off-line resource for a quantum computer, with which the computer can work using only linear operations [2].

Quantum superpositions have been demonstrated using both a few microwave photons in a cavity [3] and currents in superconducting interference devices [4]. There are proposals for realizing such states in different physical systems; see, for instance, [5] for those which involve Bose-Einstein condensates (BEC). Methods for generating superpositions of two coherent states $|\alpha\rangle + |-\alpha\rangle$ of the electromagnetic (e.m.) field have been suggested, but they require a perfect quantum nondemolition (QND) measurement of the photon number [6], which is impractical at present.

In this paper, we propose a method for realizing a superposition state of the collective spin variables of a gas of alkali atoms. Specifically the state that would be realized is a coherent superposition of the atomic spins pointing in two slightly different directions. Our method is another step within the growing field devoted to developing a quantum interface between light and atomic ensembles. Strong coupling between collective degrees of freedom of multiatom ensembles and multiphoton pulses of light can be achieved in free space and has been recently used to demonstrate squeezing of the collective spin [7,8] and entanglement between two atomic samples [9,10]. This should be contrasted with the strong coupling of individual photons to individual atoms, which requires high finesse cavities [3,11].

The method we propose here advances these techniques beyond the continuous variables approximation. Indeed in our proposal a strong atom-photon coupling has to be realized with a small number of atoms. The relevant ex-

perimental conditions, which require very dense atomic samples, seem to be realizable using presently available techniques. The transfer of the atomic ensemble spin state onto the state of light is also in principle possible [12,13], and therefore superpositions of coherent states of light could also be realized using our method.

The collective spin variables for the ground state of an atomic ensemble $\hat{\mathbf{J}} = \sum_i \hat{\mathbf{J}}^{(i)}$, where $\hat{\mathbf{J}}^{(i)}$ is the total angular momentum of the i th atom in the ground state, obey the commutation relations $[\hat{J}_x, \hat{J}_y] = i\hat{J}_z$. For simplicity we assume that each atom is a spin 1/2 system. We start with the coherent spin state with maximal total angular momentum (i.e., all spins parallel), whereupon $\hat{J}_x^2 + \hat{J}_y^2 + \hat{J}_z^2 = (N_a/2)[(N_a/2) + 1]$ where N_a is the number of atoms. We will suppose that the evolution of the collective spin is limited to unitary transformations without dissipation. Under these conditions the total length of the spin vector is conserved, i.e., it stays on the surface of the same Bloch sphere. Throughout the paper the spin vector stays polarized close to the $+z$ direction, but not necessarily exactly in this direction. We can then define the rescaled variables $\hat{x}_A = \sqrt{(2/N_a)}\hat{J}_x$, $\hat{p}_A = \sqrt{(2/N_a)}\hat{J}_y$, $\hat{n}_A = (N_a/2) - \hat{J}_z$, which satisfy

$$[\hat{x}_A, \hat{p}_A] = i + O(\hat{n}_A/N_a), \quad (1)$$

$$\frac{\hat{x}_A^2}{2} + \frac{\hat{p}_A^2}{2} = \hat{n}_A + \frac{1}{2} + O(\hat{n}_A^2/N_a). \quad (2)$$

Thus \hat{x}_A and \hat{p}_A are analogous to the position and momentum operators, or to the quadrature operators of a single mode of the e.m. field, and \hat{n}_A is analogous to the occupation number of a harmonic oscillator, or to the number of photons in a mode of the e.m. field. The new feature of the present model, as compared to the earlier work, is that it goes beyond the harmonic oscillator

approximation since the deviation of \hat{J}_z from $N_a/2$ is taken into account to first order.

The qualitative outline of the procedure for the cat state generation goes as follows; see Fig. 1. As shown in [8,9,12] a component of the collective spin operator can be measured in a QND way. A measurement of \hat{J}_x produces a squeezed state of the atomic spin with uncertainties $\Delta x_A^2 = 2\Delta J^2/N_a = 1/(2\xi^2)$ and $\Delta p_A^2 = \xi^2/2$, where ξ denotes the degree of squeezing. Such a state has a ‘‘banana’’ shape localized around the pole of the Bloch sphere. The critical next step is to carry out a QND measurement of \hat{J}_z , i.e., of the number operator \hat{n}_A . Of central importance is that we require only ‘‘weak’’ QND measurements of the atomic population, i.e., measurements of \hat{n}_A , which do not need to have the single atom accuracy. This measurement can be visualized as intersecting the banana with a plane parallel to the equatorial plane. The distance from the plane to the pole depends on the result of the QND measurement of \hat{n}_A . The ‘‘width’’ of the plane depends on the precision ΔJ with which \hat{n}_A is measured. The resulting state lies at the intersection of the plane and the initial squeezed state. It is a superposition state.

Let us now describe in detail the QND measurements of the atomic spin. The starting point is the effective interaction between the off resonant light and the atomic ground state spin [8,10,12]. Atomic levels and interactions with light are shown in Fig. 1(a). Let us suppose that initially the photons are in the coherent state $|\psi_P\rangle = \int dx_P \exp[-x_P^2/2]|x_P\rangle$ (hereafter we omit for simplicity normalization constants) and that the atoms are polarized in the z direction in the coherent spin state $|0\rangle_{n_A} = \int dx_A \exp[-x_A^2/2]|x_A\rangle$ [the level scheme on top in Fig. 1(a)]. For light propagating along the x axis the effective interaction Hamiltonian is $H_{\text{eff}} = \kappa \sum_i |\uparrow_x\rangle_i \langle \uparrow_x| \hat{a}_L^\dagger \hat{a}_L + |\downarrow_x\rangle_i \langle \downarrow_x| \hat{a}_R^\dagger \hat{a}_R$, where \hat{a}_L, \hat{a}_R are the annihilation operators for left and right circular polarized light [the interaction is pictorially shown in the middle level scheme in Fig. 1(a)]. We introduce the Stokes operators $\hat{S}_x = \hat{a}_L^\dagger \hat{a}_L - \hat{a}_R^\dagger \hat{a}_R$, $\hat{S}_y = \hat{a}_L^\dagger \hat{a}_R + \hat{a}_R^\dagger \hat{a}_L$, $\hat{S}_z = -i\hat{a}_L^\dagger \hat{a}_R + i\hat{a}_R^\dagger \hat{a}_L$ and assume that $\hat{S}_z \simeq N_p$ is close to its maximal value (where N_p is the number of photons in the beam). Then we can define the rescaled position-momentum-like operators $\hat{S}_x = \hat{x}_P \sqrt{N_p}/2$, $\hat{S}_y = \hat{p}_P/\sqrt{N_p}/2$ which obey $[\hat{x}_P, \hat{p}_P] = i$. With this notation the unitary evolution can be written (up to an unimportant overall phase) as

$$U = \exp(-ia\hat{J}_x \hat{S}_x) = \exp(-i\alpha \hat{x}_A \hat{x}_P), \quad (3)$$

where $\alpha = a\sqrt{N_a N_p}/2$.

In the first step a spin squeezed state is obtained as follows [8,10,12]. After the interaction with light the state is, in the p_P basis, $U|\psi_P\rangle|0\rangle_{n_A} = \int dx_A dp_P \exp[-(\alpha x_A - p_P)^2/2] \exp[-x_A^2/2]|x_A\rangle|p_P\rangle$. Upon measuring \hat{p}_P a particular outcome p_P is obtained. The state of the atoms conditioned on the measurement outcome p_P is $\psi_A(p_P) =$

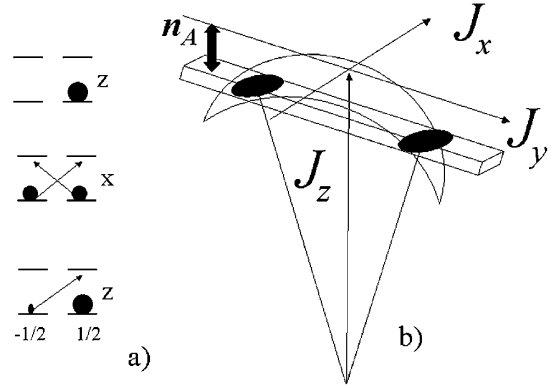


FIG. 1. Generation of the superposition spin state. (a) Atomic levels and interaction with light. Both ground and excited states are spin 1/2 states. The top diagram shows the initial coherent spin state in z representation. The middle diagram shows the same state in x representation along with the virtual transitions used for the generation of the spin squeezed state. The bottom diagram shows the spin squeezed state in z representation along with the interaction used for the cat state generation. (b) The spin squeezed state generated by a QND measurement using the first pulse of light is represented by the ‘‘banana’’ shape. The following QND measurement of the number operator \hat{n}_A corresponds to intersecting the banana with a plane of a finite width. Thus the final state has two components lying at positive and negative values of J_y , i.e., it is a superposition state.

$\int dx_A \exp[-(\alpha x_A - p_P)^2/2] \exp[-x_A^2/2]|x_A\rangle$. This is a squeezed state. By a rotation one can center the state around $p_P = 0$ whereupon one has

$$\begin{aligned} \psi_A^{\text{squeezed}} &= \int dx_A \exp[-(\alpha^2 + 1)x_A^2/2]|x_A\rangle \\ &= \sum_{n_A} c(n_A)|n_A\rangle, \end{aligned} \quad (4)$$

where $c(n_A) = 0$ if n_A is odd, and equals $c(n_A) = [(\xi^2 - 1)/2(\xi^2 + 1)]^{n_A/2} [\sqrt{n_A!}/(n_A/2)!]$ when n_A is even, with $\xi^2 = \alpha^2 + 1$.

The next step is the generation of the cat state from the spin squeezed state by carrying out a QND measurement of \hat{J}_z . Towards this end we send off-resonant right circular polarized light in a coherent state with the mean number of photons N_p along the z axis through the sample [the bottom level scheme in Fig. 1(a)]. The temporal evolution of the light and spins is then given by $U = \exp(-ia\sum_i |\downarrow_i\rangle_i \langle \downarrow_i| \hat{a}_R^\dagger \hat{a}_R)$. We define a translated creation operator by $\hat{a}_R = \sqrt{N_p} + \hat{a}'_R$ and the translated quadrature operators $\hat{x}_R = (\hat{a}'_R + \hat{a}'_R^\dagger)/\sqrt{2}$, $\hat{p}_R = -i(\hat{a}'_R - \hat{a}'_R^\dagger)/\sqrt{2}$ so that the unitary evolution takes the form $U = \exp[-ia\hat{n}_A(N_p + \sqrt{2N_p}\hat{x}_R)]$, where we have dropped a term proportional to $\hat{a}'_R^\dagger \hat{a}'_R$. The term proportional to N_p does not modify the state of the light, but it rotates the atomic spin. This term can be taken into account at later stages of the protocol and is omitted from now on. The part of the unitary transformation relevant for the QND measurement is therefore $U = \exp[-i\beta\hat{n}_A\hat{x}_R]$, where $\beta = a\sqrt{2N_p}$.

Initially light is in the coherent state $\psi_R = \int dx_R \exp[-x_R^2/2]|x_R\rangle$ and the spin squeezed state of atoms is given by Eq. (4). After the interaction of light and atoms, one measures the p_R quadrature and finds an outcome p_R . The state of the atoms conditional on this outcome is

$$\psi_A(p_R) = \sum_{n_A} c(n_A) \exp[-(\beta n_A - p_R)^2/2]|n_A\rangle. \quad (5)$$

For suitable values of α , β , and p_R , this is a cat state. In order to show this one should express the state in the x_A and p_A basis. This can be done using the matrix elements $\langle p_A | n_A \rangle$ (expressible in terms of Hermite polynomials). A numerical calculation of $\psi_A(p_R)$ in the p_A basis is given in Fig. 2 (solid line). To facilitate the analysis of the obtained state we derive approximate analytical expressions for $\psi_A(p_R)$ in the x_A and p_A basis. First note that using the Stirling expansion, the coefficients $c(n_A)$ can be approximated by $c(n_A) \approx [(\xi^2 - 1)/(\xi^2 + 1)]^{n_A/2}$ when n_A is even. Inserting this into Eq. (5) yields

$$\psi_A(p_R) \approx \sum_{n_A \text{ even}} \exp\{-\beta^2[n_A - \mu(p_R)]^2/2\}|n_A\rangle, \quad (6)$$

where

$$\mu(p_R) = \frac{p_R}{\beta} + \frac{1}{2\beta^2} \ln\left(\frac{\xi^2 - 1}{\xi^2 + 1}\right) \approx \frac{p_R}{\beta}. \quad (7)$$

The distribution of number states is thus a Gaussian with mean $\mu(p_R)$ and variance $1/\beta$. We now use the fact that the initial state is strongly squeezed in x_A ; see Eq. (4). Thus we can use the approximation $n_A \approx p_A^2/2$ in Eq. (6). This implies that the wave function in p_A representation is centered around the two values $\pm\sqrt{2\mu(p_R)}$ with the variance $1/[\sqrt{2\mu(p_R)}\beta]$. Hence the wave function is

$$\psi_A(p_R) \approx \int dp_A \{ \exp[-(p_A - \sqrt{2\mu})^2\beta^2\mu] + \exp[-(p_A + \sqrt{2\mu})^2\beta^2\mu] \} |p_A\rangle, \quad (8)$$

$$\approx \int dx_A \exp\left[-\frac{x_A^2}{4\beta^2\mu}\right] \cos[x_A\sqrt{2\mu}] |x_A\rangle, \quad (9)$$

where the second equality is obtained from the first by taking the Fourier transform, and the notations are simplified as $\mu = \mu(p_R)$. Note that the matrix elements $\langle p_A | n_A \rangle$ with n_A even are even functions of p_A which explains the + sign between the two terms in Eq. (8). As seen in Fig. 2, Eq. (8) (dotted line) is a good approximation to the exact wave function.

To verify that the cat state has indeed been obtained, measurements of spin components \hat{x}_A and \hat{p}_A should be carried out. The results of the \hat{p}_A measurement should be distributed according to the two broad peaks described by Eq. (8). The condition for observation of two distinct peaks is that the width of each of them should be less

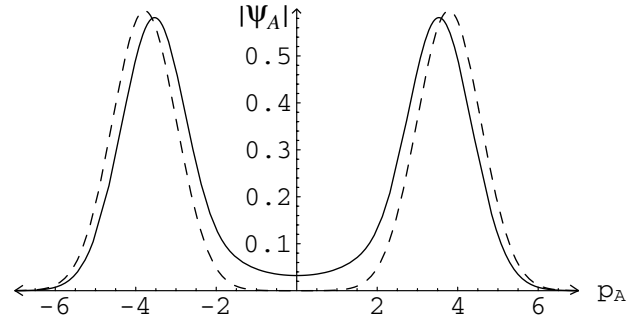


FIG. 2. The solid line shows the amplitude of the state $\psi_A(p_R)$, Eq. (5), in the p_A basis for parameters $\xi^2 = 20$, $p_R/\beta = 7$, and $\beta = 1/3$. The dotted line shows the approximation Eq. (8) for the same values of the parameters.

than the distance between them. The \hat{x}_A measurement should exhibit an interference pattern due to the cosine in Eq. (9). This interference pattern is the proof that the two peaks found in the \hat{p}_A measurement are coherent, and therefore that a quantum superposition between atomic spins pointing in two different directions has been created. The condition for observation of the interference is that the period of the cosine should be less than the width of the Gaussian in (9). In fact both conditions lead to the same inequality, $\mu \geq 1/\beta$. This condition means that by the QND measurement of the population one has removed the components with small n_A , i.e., the components near the origin $p_A = 0$, from the squeezed state leaving a state with two peaks.

There is a second condition which is that the average population μ is limited by the degree of spin squeezing obtained in the first step of our protocol: $\mu \leq \xi^2$. This condition has two origins. The first is that the probability of finding values of μ which exceed ξ^2 is exponentially small. The second is that when this condition is violated it becomes impossible to measure the interference pattern in the x_A basis, and thereby to prove that the state so obtained is indeed a coherent superposition.

Putting these two conditions together we obtain the condition $\beta\xi^2 > 1$ which relates the precision $1/\xi$ with which one can measure the \hat{x}_A quadrature to the precision $1/\beta$ with which one can measure the number operator \hat{n}_A . Using the expressions for β and ξ in terms of a , we can reexpress this condition as

$$\xi^2 \geq N_a^{1/3}. \quad (10)$$

Thus a superposition state can be produced if a high degree of squeezing can be realized with a small number of atoms. In what precedes we have assumed that the quantum states are pure and that there is no noise present. Below we consider decoherence of the atomic state caused by the nonideal QND measurements and describe a possible experimental realization of the proposal.

The off-resonant probe light used for the QND measurements will cause (weak) real excitations of atoms

leading to an incoherent spontaneous process [10]. The significance of this process can be assessed in terms of accessible experimental parameters. A first parameter is the resonant optical depth of the atomic sample, $\kappa_0 = \sigma N_a/A$ where σ is the cross section. The forward scattering amplitude of light is proportional to $\frac{\gamma}{2}/(i\Delta + \gamma/2)$ where γ is the linewidth and Δ is the detuning with respect to resonance. Hence the optical depth at detuning $\Delta \gg \gamma$ is $\kappa_\Delta = \kappa_0 \gamma^2/4\Delta^2$ and the phase shift (or polarization rotation) of light traversing the sample is $\theta_\Delta = \kappa_0 \gamma/2\Delta$. The parameter a defined in Eq. (3) is the phase shift or polarization rotation per atom, hence equal to $a = \theta_\Delta/N_a$. The degree of squeezing that can be obtained is $\xi = a\sqrt{N_a N_p}/2$. Decoherence caused by the probe light can be described by the degree of optical depumping, $\eta = \kappa_\Delta N_p/N_a$, which can be interpreted as the probability that an atom makes an incoherent transition. All these parameters can be put together in the equation $\xi^2 = \kappa_0 \eta/4$. The degree of optical depumping must obey the condition $\eta \leq 1/\xi^2$, otherwise the coherence of the squeezed state or the superposition state will be destroyed. This condition follows from the fact that about ξ^2 of the spins are pointing in the $-z$ direction and the other spins are pointing in the $+z$ direction. If one of the atoms pointing in the $-z$ direction decoheres, then the coherence of the superposition state, or of the parent squeezed state, is seriously affected. Thus the figure of merit for the generation of squeezing and entanglement is the resonant optical depth of the sample which limits the degree of spin squeezing to $\xi^2 \leq \frac{1}{2}\sqrt{\kappa_0}$. Using these conditions and Eq. (10), one finds that the condition for realizing a superposition state can be written as

$$\kappa_0 \geq 4N_a^{2/3}; \quad (11)$$

that is, a large optical depth must be achieved with a small number of atoms.

The condition Eq. (11) can be achieved in an alkali metal BEC. Atomic density of the order of 10^{15} cm^{-3} for 4×10^5 atoms has been reported in [14] for a cigar shaped condensate with the optical density along the long axis on the order of 10^4 . This is marginally close to the condition Eq. (11). The degree of squeezing required for the cat state generation under the above conditions is $\xi^2 \approx 50$. An additional improvement in the efficiency of the spin measurement of a small number of atoms can be achieved by placing the atomic sample in a low-finesse cavity with the mirror transmission coefficients $T = 1 - R$. If the single pass absorption probability and the single pass phase rotation angle θ are much smaller than T , then the phase shift/polarization rotation of light exiting the cavity is $\theta^c = 2\theta/T$, and the degree of optical depumping becomes $\eta^c = 2\eta/T$. Thus the effect of the cavity can be summarized by introducing an effective cross section $\sigma^c = 2\sigma/T$. The condition Eq. (11) therefore becomes $2\kappa_0/T > 4N_a^{2/3}$. For a BEC

sample with the density of 10^{15} cm^{-3} placed in a low-finesse cavity with $T = 5\%$, this condition is met with 10^3 atoms, which according to Eq. (10) requires a feasible degree of spin squeezing $\xi^2 \approx 10$ for the generation of the superposition state. Note that the lifetime of the coherent superposition state is close to the lifetime of the parent spin squeezed state, $\tau_c \xi^{-2}$ [15], where τ_c is the coherence time of the atomic ground state, which for a BEC is at least 0.1 sec [16].

In summary we have proposed a method for generating a coherent superposition of states in which the collective spin of an atomic ensemble points in two slightly different directions. It appears to be realizable using present technology by combining the methods used for QND measurements of collective atomic variables with a BEC in a low-finesse cavity. This mesoscopic superposition state uses the ground state atomic spins which means that it is relatively long lived. It can therefore in principle be used as an off-line nondeterministic resource for quantum logic as described in [2].

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