Dynamical Symmetry Breaking as the Origin of the Zero-dc-Resistance State in an ac-Driven System

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Under a strong ac drive the zero-frequency linear response dissipative resistivity $\rho_d(j=0)$ of a homogeneous state is allowed to become negative. We show that such a state is absolutely unstable. The only time-independent state of a system with a $\rho_d(j=0)<0$ is characterized by a current which almost everywhere has a magnitude j_0 fixed by the condition that the nonlinear dissipative resistivity $\rho_d(j_0^2)=0$. As a result, the dissipative component of the dc-electric field vanishes. The total current may be varied by rearranging the current pattern appropriately with the dissipative component of the dc-electric field remaining zero. This result, together with the calculation of Durst *et al.*, indicating the existence of regimes of applied ac microwave field and dc magnetic field where $\rho_d(j=0)<0$, explains the zero-resistance state observed by Mani *et al.* and Zudov *et al.*.

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Recently, two experimental groups [1,2] reported observations of a novel zero-resistance state in two-dimensional electron systems subjected to a dc magnetic field and to strong microwave radiation. When no microwave power is applied, Refs. [1,2] observe a longitudinal resistivity only weakly dependent on the magnetic field, at least for the relatively small fields (filling factor $\nu > 10$) applied in the experiment. However, when a high level of microwave power was applied the resistance developed a strong oscillatory dependence on the applied magnetic field, with an oscillation period controlled only by the ratio of the microwave frequency ω to the cyclotron frequency ω_c. At low T and in certain field ranges, the dissipative resistance was observed to *vanish* within the experimental accuracy.

An important step towards the understanding of these observations was taken in Ref. [3], which presented a calculation of the effect of microwave radiation on the dc linear response conductivity of a two-dimensional electron gas. A crucial result of Ref. [3] (see also Ref. [4] for earlier treatment and Ref. [5] for a detailed analysis) is the existence of the regimes of magnetic field and applied microwave power for which the longitudinal linear response conductivity is negative,

$$\sigma_{xx} < 0. \tag{1}$$

However, in the literature so far a precise connection between a negative linear response conductivity and the experimental observations has not been presented.

In this Letter we show that Eq. (1) by itself suffices to explain the *zero-dc-resistance* state observed in Refs. [1,2], independent of the details of the microscopic mechanism which gives rise to Eq. (1). The essence of our result is that a negative linear response conductance implies that the zero-current state is intrinsically unstable: the system spontaneously develops a nonvanishing local current density, which almost everywhere has a specific magnitude j_0 determined by the condition that the component of the electric field parallel to the local current vanishes. The existence of this instability is shown, under reasonable assumptions, to imply the observed zero-resistance state.

We consider dc transport in a two-dimensional electron gas exposed to a static magnetic field and to an ac electric field. We assume that the local dc-electric field $\boldsymbol{E}$ is related to the local dc-current density $\boldsymbol{j}$ via

$$\boldsymbol{E} = \boldsymbol{j}\rho_d(\boldsymbol{j}^2) + [\boldsymbol{j}\times \boldsymbol{z}]\rho_H, \tag{2}$$

where $\boldsymbol{z}$ is the unit vector normal to the plane of the system. The crucial quantity in Eq. (2) is the longitudinal (dissipative) resistivity $\rho_d(\boldsymbol{j}^2)$ whose dependence on the current determines the physics we consider. The form of $\rho_d(\boldsymbol{j}^2)$ is determined by parameters such as the applied magnetic field $B_{\rm app}$ and the frequency ω and power $\mathcal{P}_{\rm ac}$ of the ac field, which we do not explicitly write. Also, to simplify the discussion we do not consider nonlinear effects in the Hall resistivity ρ_H. This is not crucial for the zero-resistance state; effects of including it in the theory are discussed briefly at the end of the paper.

We assume that $\rho_d(\boldsymbol{j}^2)$ is a real, continuous function of j^2 and that (as found, e.g., in the calculations of Ref. [3]) a range of $B_{\rm app}$, ω, and $\mathcal{P}_{\rm ac}$ exists for which a spatially homogeneous zero-current state is characterized by the negative dissipative resistivity

$$\rho_d(\boldsymbol{j}^2=0) < 0. \tag{3}$$

However, at sufficiently large values of the dc current $\rho_d(j^2)$ must revert to its dark ($\mathcal{P}_{\rm ac}=0$) value because in this limit the microwave radiation will be a small perturbation on the steady state electron distribution function. Continuity implies that there is a value $j = j_0$ at which

$$\rho_d(j_0^2) = 0. \tag{4}$$

We take $\rho_d(j^2)$ to have the form shown in the inset of Fig. 1. The main panel of Fig. 1 shows the current-voltage characteristic following from the assumed form of $\rho_d(j^2)$. Such a dependence was obtained analytically in Ref. [5].

A negative dissipative resistivity is allowed under nonequilibrium conditions, if the system is continuously supplied with energy. In the situation considered here energy conservation requires only that $j^2\rho_d(\boldsymbol{j}^2) + \mathcal{P}_{\rm ac} > 0$. However, a negative resistivity may render the system unstable. Specifically, we now show that in a system described by Eq. (3) with a resistivity curve as shown in the inset in Fig. 1:

(i) A homogeneous, time-independent state characterized by a current j of magnitude *less* than the critical value j_0 defined in Eq. (4) is unstable with respect to inhomogeneous current fluctuations.

(ii) The only possible time-independent state is one in which the current $\boldsymbol{j}$ has magnitude j_0 everywhere except at isolated singular points (vortex cores) or lines (domain walls), implying vanishing dissipative electric field, $\boldsymbol{j} \cdot \boldsymbol{E} = 0$.

An immediate consequence of (ii) is that by adjusting the details of the current pattern, any net dc current less than a threshold value (which we discuss below) can be sustained at vanishing dissipative electric field, so that *any microscopic mechanism of nonequilibrium drive resulting in* $\rho_d(\boldsymbol{j}^2 = 0) < 0$ *leads to the observed* [1,2] *zero dissipative differential resistance*:

$$\frac{dV_x}{dI_{\rm dc}} = 0. \tag{5}$$

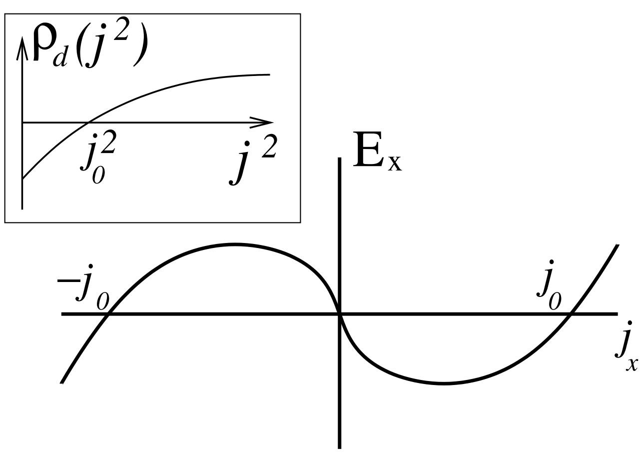

FIG. 1. Assumed dependence of the dissipative (parallel to current) component of the local electric field E_x on the current density j_x. Inset: dependence of the dissipative resistivity on the square of the current.

(Here $I_{\rm dc}$ is a sufficiently weak applied current.) If too large a current is imposed, the current structure will collapse and a nonvanishing resistance will be observed. We emphasize, however, that Eq. (5) is obtained on the assumption that the system is in a steady state. Any current pattern consistent with a boundary condition of a small net current implies the existence of singularities (domain walls or vortices) in the current distribution; finite density of these objects may lead to a small dissipative resistivity.

We pause to discuss the relation of our arguments to previous literature. The instability of systems with absolute negative conductivity is known since the work of Zakharov [6]; for a recent review see [7]. The important new feature of the instability and the domain structure of the present paper is that it occurs at a large Hall angle; as a result the domains for the current coincide with the domains of the electric field directed perpendicularly to the current. We would also like to point out a certain similarity with the model of photoinduced polarization domains proposed by D'yakonov [8] as an explanation of experiments on ruby crystals under intense laser irradiation [9].

We now present our specific arguments. We begin by considering the fluctuations δj about a time-independent homogeneous state of current j_i. Taking the time derivative of Eq. (2) and using the continuity equation,

$$\frac{\partial n}{\partial t} + \boldsymbol{\nabla} \cdot \boldsymbol{j} = 0, \tag{6}$$

and the Poisson equation,

$$\boldsymbol{E} = -\boldsymbol{\nabla}\hat{U}n, \tag{7}$$

we obtain

$$\boldsymbol{\nabla}(\hat{U}\boldsymbol{\nabla} \cdot \boldsymbol{j}) = \frac{\partial}{\partial t}\{\boldsymbol{j}\rho_d(\boldsymbol{j}^2) + [\boldsymbol{j} \times z]\rho_H\}. \tag{8}$$

Here n is the electron charge density and $\hat{U}$ is a nonlocal interaction operator which can be expressed in terms of the Green function of the Laplace equation with appropriate boundary conditions. The crucial point for us is that $\hat{U}$ has non-negative eigenvalues. (We assume that the screening radius is equal to zero and neglect the difference between the electric and electrochemical potentials. This approximation does not alter our main conclusions.)

Writing $\boldsymbol{j}(r, t) = \boldsymbol{j}_i + \delta\boldsymbol{j}(r, t)$, linearizing in $\delta\boldsymbol{j}$, and taking the divergence of both sides of Eq. (8), we find

$$\frac{\partial \boldsymbol{\nabla} \cdot \delta\boldsymbol{j}}{\partial t} = [\boldsymbol{\nabla}(\tilde{\hat{\rho}}_d + \hat{\rho}_H)^{-1}\boldsymbol{\nabla}\hat{U}]\boldsymbol{\nabla} \cdot \delta\boldsymbol{j}, \tag{9}$$

with $\hat{\rho}_H = \rho_H\left[\begin{smallmatrix} 0 & 1 \\ -1 & 0 \end{smallmatrix}\right]$ the usual Hall resistivity tensor,

$$\tilde{\hat{\rho}}_d = \rho_d\boldsymbol{1} + \alpha_j\boldsymbol{j}_i \otimes \boldsymbol{j}_i, \tag{10}$$

and

$$\alpha_j = 2\frac{d\rho_d(\boldsymbol{j}^2)}{d\boldsymbol{j}^2}\bigg|_{j^2=j_i^2}. \quad (11)$$

The Coulomb interaction operator $\hat{U}$ is positive definite, so the stability is determined by the sign of the operator $\boldsymbol{\nabla}(\hat{\rho}_d + \hat{\rho}_H)^{-1}\boldsymbol{\nabla}$ in front of it. Performing a Fourier transform of this operator we see that in order for any solution of Eq. (9) not to grow with time the conditions

$$\rho_d(\boldsymbol{j}^2) \geq 0, \quad (12a)$$

$$\rho_d(\boldsymbol{j}^2) + \alpha_j \boldsymbol{j}^2 \geq 0, \quad (12b)$$

must hold.

We therefore conclude that if at least one of ρ_d or $\rho_d + \alpha_j$ is negative, i.e., if $j_i < j_0$, a homogeneous state of uniform current is unstable. However, we may also show that any state with local current density *larger* than j_0 but net current density *smaller* than j_0 is necessarily time dependent. In this case the condition $\boldsymbol{\nabla} \cdot \boldsymbol{j} = 0$ requires the presence of circulating currents. The integral $\mathcal{J} = \oint_C d\mathbf{l} \cdot \boldsymbol{E}$ along the current flow lines must vanish because $\boldsymbol{\nabla} \times \boldsymbol{E} = 0$. On the other hand, from Eq. (2) and $\boldsymbol{\nabla} \cdot \boldsymbol{j} = 0$, we find $\mathcal{J} = \oint_C d\mathbf{l} \cdot \boldsymbol{j}\rho_d(\boldsymbol{j}^2)$. By construction of the contour $d\mathbf{l}\boldsymbol{j} > 0$. Therefore $\mathcal{J} = 0$ can be satisfied together with the stability condition (12a) only for $\rho_d(\boldsymbol{j}^2) = 0$, i.e., for $j = j_0$.

Finally, we examine the stability of general states with $|\vec{j}(r)| = j_0$. In this case $\rho_d = 0$ but $\alpha > 0$; substitution into Eq, (9) and use of Eq. (6) leads to

$$\left\{\frac{\partial}{\partial t} + (\boldsymbol{\nabla} \cdot [\boldsymbol{j}_0 \times z])([z \times \boldsymbol{j}_0] \cdot \boldsymbol{\nabla})\frac{\alpha \hat{U}}{\rho_H^2}\right\}\delta n = 0, \quad (13)$$

$\boldsymbol{\nabla} \cdot \delta\boldsymbol{j} = -\partial_t \delta n$. Operator $\hat{U}$ is positive definite, while the operator $(\boldsymbol{\nabla} \cdot [\boldsymbol{j}_0 \times z])([z \times \boldsymbol{j}_0] \cdot \boldsymbol{\nabla})$ is Hermitian and is non-negative because it can be presented as $AA^\dagger$. Therefore, the state (4) is not unstable, except possibly at singular points. The investigation of the stability of the current pattern in the vicinity of the singular point requires going beyond the local Eqs. (2) and (13) and will not be done in the present paper.

Moreover, one can see from Eq. (13) that all the perturbations decay in time exponentially with the exception of those for which $[\boldsymbol{j}_0 \times z] \cdot \boldsymbol{\nabla}\hat{U}\delta n = 0$, i.e., with the electric field directed along $\boldsymbol{j}_0$. The physical meaning of these zero modes is all the perturbations of the current which keep $\boldsymbol{j}^2 = \boldsymbol{j}_0^2$ and $\boldsymbol{\nabla} \cdot \boldsymbol{j} = 0$ (most trivial example of such perturbation is a homogeneous rotation of vector $\boldsymbol{j}_0$). These perturbations have zero eigenvalue and, analogously to Goldstone modes, are a straightforward consequence of the symmetry breaking induced by the applied nonlinear drive.

We now consider the physical consequences of our results. We found that a nonequilibrium system which has a negative linear response resistivity is unstable to the formation of a state of nonvanishing local current. Almost everywhere in the sample the current has the magnitude j_0 at which the dissipative resistivity vanishes, but the direction must vary so that the net current is consistent with boundary conditions. The current distribution must contain singular regions, of negligible volume fraction, at which the current takes values different from j_0. The arguments relating to time-independent states given above may be viewed as showing that it is impossible to construct a time-independent singularity structure for distributions involving currents of magnitude greater than j_0, whereas it is possible if in almost all of the sample the current has magnitude j_0. Just as in the theory of superconductivity the detailed nature and structure of the singular regions (domain walls, vortex cores, or other structures) presumably depends both on boundary conditions and on short length scale physics. The question cannot be analyzed within the quasicontinuum/local response function approach used in this paper and is an important topic for future investigation.

For concreteness of further discussion we will consider the obvious choice of singularity shown in Fig. 2, namely, a linear domain wall, separating two domains in which current flows parallel and antiparallel to the domain wall. We believe that a structure involving vortices would lead to essentially identical physics. In the presence of a magnetic field, consideration of the Hall component of the current reveals the existence of a discontinuity in the component of the electric field perpendicular to the boundary. If $\hat{\mathbf{n}}$ is the vector perpendicular to the wall, and $\Delta\boldsymbol{j} = 2j_0$ is the discontinuity in current across the wall (assumed parallel to the wall direction), then the singularity in the electric field is

$$\Delta E = 2\mathbf{n}\rho_H j_0. \quad (14)$$

This discontinuity requires a charge accumulation which, in a two-dimensional situation, is nonlocal (l_0 is a cutoff

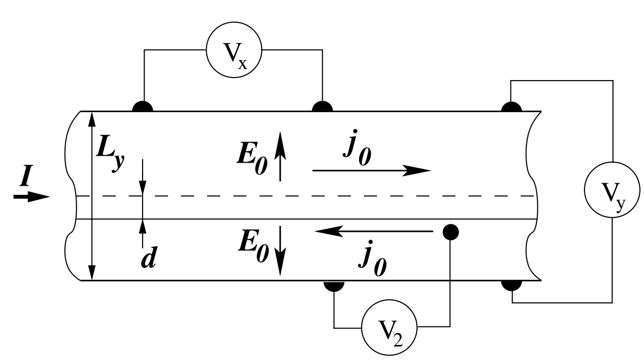

FIG. 2. The simplest possible pattern of the current distribution—domain wall. The net current, I, is accommodated by a shift of the position of the domain wall by the distance d; see text. The electric field in the domain is $E_0 = \rho_H j_0$. The current pattern in the Corbino disc geometry is obtained by connecting the broken edges into a ring.

set by the microscopic structure of the domain)

$$n(\mathbf{r}) \simeq -\rho_H j_0 \ln\left(\frac{|\mathbf{r}\cdot\hat{\mathbf{n}}|}{l_0}\right). \tag{15}$$

This charge distribution may be detectable by local probes. The other possible way to detect dynamical symmetry breaking is to measure spontaneous voltages arising inside the domain, voltage V_2 in Fig. 2.

Figure 2 presents a very natural (albeit probably oversimplified) picture of the experimental situation studied in Refs. [1,2]. In these experiments the current in one direction (say, x) was fixed by current leads to some value I, and the current in the transverse direction was set to zero. The longitudinal (x) and transverse (y) voltages were measured. Referring to Fig. 2, we see that any value of net current I corresponding to a current density much less than j_0 can be obtained simply by adjusting the height of the domain wall: if d is the position of the domain wall relative to the center of the device, then $I = 2dj_0$ with $V_x = 0$. Similarly, the total Hall voltage is the sum of a positive voltage in the upper half of the sample and a negative voltage in the lower half, leading to $V_y = \rho_H[j_0(L_y/2 - d) - j_0(L_y/2 + d)] = -\rho_H I$, which will equal the dark (no microwave) result if ρ_H is not much affected by the ac field. Notice that for the Corbino disc geometry the applied voltage (corresponding to V_y of Fig. 2) can also be accommodated by the shift of the domain wall without the generation of the dissipative current, resulting in the *zero-conductance state* [10].

The equations analyzed in this paper predict threshold behavior in I at low temperature T:V_x is strictly zero for weak applied currents, but if the applied current is large enough that the current density becomes of order j_0 then the domain wall is swept out of the system, and a dissipative state corresponding to current densities greater than j_0 in some parts of the sample will result. Similarly, our equations predict a critical temperature: at very high temperature, the linear response conductivity will be positive even at nonzero (but fixed) microwave power. As T is lowered, σ_{xx} will decrease and at some temperature pass through 0, upon which dissipationless behavior will result. The sharp thresholds in I and T, which are in apparent contradiction with Refs. [1,2], may be artifacts of the simple treatment given here, which assumed a static singularity structure and zero screening radius.

We did not consider the dependence of the Hall conductivity on the applied current. It is easy to see that this dependence does not change the condition (4) for the circulating currents in the state because the Hall coefficient does not cause dissipation. Singular dependence of j_0 on the magnetic field will cause singular features in the magnetic field dependence of the Hall resistivity near the zero-resistance state. The shape and the value of these singularities, however, have nothing to do with the quantized plateaus in the quantum Hall effect.

Finally, we note that we have assumed an isotropic ρ_d. In fact, the presence of an ac drive will lead to a quadrupolar anisotropy (see Ref. [5] for a microscopic derivation) which for the sake of notational clarity we did not write but which can easily be included if desired. This anisotropy will presumably affect the orientations of domain walls, suggesting that it would be interesting to look for differences in threshold behavior for the dc current parallel or perpendicular to the ac current.

To summarize, we have shown that the remarkable zero-resistance state found by Refs. [1,2] may be understood on very general grounds as a consequence of a negative linear response conductivity. Reference [3] has presented a calculation, based on a specific microscopic model, showing that this negative linear response conductivity indeed may occur in the regime of magnetic field and microwave frequency in which the zero-resistance state occurs. Taken together, we believe the present work and Ref. [3] capture the essence of the experimental result.

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