Persistent Currents in Interacting Aharonov-Bohm Interferometers and Their Enhancement by Acoustic Radiation

O. Entin-Wohlman,[1,3] Y. Imry,[2] and A. Aharony[1]

[1]*School of Physics and Astronomy, Raymond and Beverly Sackler Faculty of Exact Sciences, Tel Aviv University, Tel Aviv 69978, Israel*

[2]*Department of Condensed Matter Physics, The Weizmann Institute of Science, Rehovot 76100, Israel*

[3]*Albert Einstein Minerva Center for Theoretical Physics at the Weizmann Institute of Science, Rehovot 76100, Israel*

(Received 7 February 2003; published 22 July 2003)

We consider an Aharonov-Bohm interferometer, connected to two electronic reservoirs, with a quantum dot embedded on one of its arms. We find a general expression for the persistent current at steady state, valid for the case where the electronic system is free of interactions except on the dot. The result is used to derive the modification in the persistent current brought about by coupling the quantum dot to a phonon source. The magnitude of the persistent current is found to be enhanced in an appropriate range of the intensity of the acoustic source.

DOI: 10.1103/PhysRevLett.91.046802 PACS numbers: 73.23.–b, 71.38.–k, 72.15.Gd

It has long been established, both theoretically [1] and experimentally [2], that a magnetic flux which threads an isolated electronic system creates a persistent current at thermal equilibrium. That current, which is equivalent to the thermodynamic orbital magnetic moment of the electrons, arises from the interference of the electronic wave functions and, as long as the electrons are phase coherent, survives the presence of reasonably strong static disorder [3]. When the electrons are coupled to a phonon bath, the naive expectation is that the persistent current will diminish, due to loss of coherence, caused by inelastic processes as well as by renormalization effects due to the "dressing" of the electrons by the phonons. However, it turns out that this is not the whole effect. For example, in the case of strongly localized electrons, the coupling to the acoustic waves gives rise to an additional persistent current [4], brought about by delicate resonant processes [5], which result in a nonmonotonic temperature dependence of the orbital magnetic moment at sufficiently low temperatures. Somewhat related examples are the enhancement of the persistent current in response to an external ac electric noise [6,7] and its peculiar behavior under the effect of a nonequilibrium electron energy distribution [8] or of a periodic oscillating potential [9].

Here we consider the persistent current I_c, circulating in the ring of an Aharonov-Bohm interferometer (ABI), connecting two electronic reservoirs having equal or slightly different chemical potentials, μ_ℓ and μ_r. This small voltage $eV = \mu_\ell - \mu_r$ is not the main source for the deviation from equilibrium in our case. The current is studied also when the system is strongly out of equilibrium due to a coupling to an acoustic source, as in Ref. [10]. In order to retain the coherence of the conduction electrons, our results apply only for size scales small enough to stay coherent at the given temperature. Moreover, the strength of the acoustic source is taken to be such that the additional decoherence due to it is not essential [7]. We take the electronic system to be noninteracting, except at a single "site" on the ABI, dubbed "quantum dot," where the electrons can couple to an external source of sonic (or electromagnetic) waves and/or experience electronic interactions. (For simplicity, we assume that this site has only a single relevant on-site energy level, ϵ_d.) Under these conditions, we obtain a general expression for the persistent current circulating around the Aharonov-Bohm ring [Eq. (2) below]. This expression does not necessitate a near-equilibrium situation. We then use the result to show that a phonon source, interacting with the electrons on the quantum dot, may lead to a considerable enhancement of the persistent current and of the related orbital magnetic moment. This acousto-persistent current exists even when $V = 0$.

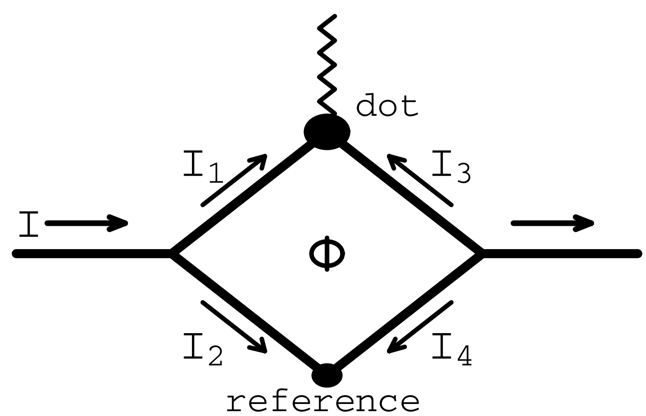

FIG. 1. An ABI, containing a quantum dot on its upper arm and threaded by a magnetic flux Φ. The wavy line symbolizes the phonons radiated by the phonon source into the dot. The lower arm of the ABI contains a "reference" site. The ABI is connected to two electronic reservoirs whose chemical potentials are either equal or have a small difference, allowing a current I to flow from the left to the right.

However, the coupling of the ring to the leads is crucial. Such a relation between the acoustic wave intensity and the orbital magnetic moment opens interesting possibilities for future nanodevices.

Our model system is depicted in Fig. 1. In the steady state, a dc current I is flowing through the ABI. To eliminate spurious circulating currents caused by a geometrical asymmetry of the ring, we calculate the current I_c, *circulating* around the ABI ring under the effect of the Aharonov-Bohm flux Φ, by

$$I_c = \tfrac{1}{2}(I_1 - I_2)|_{\Phi} - \tfrac{1}{2}(I_1 - I_2)|_{-\Phi}. \qquad (1)$$

The persistent current with a coupling to one or more electron reservoirs, but without the additional interactions on the dot, has been studied in Refs. [11–13].

By using the Keldysh technique [14], one can find the partial currents flowing in each arm of the ABI. Moreover, when the electrons are noninteracting, except on the quantum dot, one is able to express all the relevant currents in terms of the *exact* Green function on the dot, G_{dd}, which includes the interactions and the effect of the couplings between the dot and the other parts of the circuit. This Green function has a self-energy which comes from two sources. The first arises solely from the coupling of the dot to the ring and is denoted by $\Sigma_{\rm ext}$. This part can be obtained straightforwardly, as it pertains to noninteracting electrons. $\mathrm{Im}\Sigma_{\rm ext}$ represents the lifetime of the on-site energy level on the dot, turning it into a resonance. The other part of the self-energy, denoted $\Sigma_{\rm int}$, comes from the interactions experienced by electrons residing on the dot [15], e.g., the coupling to the phonon source. This self-energy also depends on the coupling to the other parts of the circuit. A detailed calculation of I_1 and I_2 then yields [16]

$$I_c = \int \frac{d\omega}{i\pi} \frac{f_\ell(\omega) + f_r(\omega)}{2} \left[\frac{\partial \Sigma^A_{\rm ext}}{\partial \Phi} G^A_{dd}(\omega) - cc \right], \qquad (2)$$

where $f_{\ell,r}(\omega) = f(\omega - \mu_{\ell,r})$, with $f(z) \equiv (e^{\beta z} + 1)^{-1}$ being the electron distribution on the left (right) reservoir, having the chemical potential μ_ℓ (μ_r). (It is assumed that the two reservoirs are otherwise identical. We also use a unit electron charge, $e = 1$, and $\hbar = 1$.) The superscripts A (R) refer to the advanced (retarded) Green functions. Since both $\Sigma^A_{\rm ext}$ and G^A_{dd} are even in Φ, due to additive contributions (with equal amplitudes) from clockwise and counterclockwise motions of the electron around the ring (see, e.g., Refs. [12,17,18]), I_c is odd in Φ, as it should be. Another important point to notice is that the frequency integration in Eq. (2) is restricted to the bandwidth of the conduction electrons on the leads. This is because G^A_{dd} attains an imaginary part due only to the coupling of the dot with the band states on the leads, i.e., for ω inside the band.

Several physical remarks on Eq. (2) are called for: (i) The small voltage allowed on the system is not essential; the result also holds when $\mu_\ell = \mu_r$. (ii) Equation (2) averages the flux derivative of the external self-energy over energy, with weights containing the densities of electrons and single-particle states with that energy. However, the flux derivative of the latter does *not* appear. This is reminiscent of the equilibrium case, where the persistent current is given by the flux derivative of the energies, weighed by the populations, without the appearance of the flux derivative of those, as in Eq. (3) below (see, for example, Ref. [4]).

Equation (2) holds under four conditions: (i) The interferometer is *symmetric*, i.e., the left and right couplings of the dot to the interferometer are identical, and so are those of the reference site (see Fig. 1). Otherwise, there appears a term proportional to the asymmetry of the ring, which we omit for clarity. (ii) In order to eliminate the Keldysh Green function $G^<_{dd}$, we have used current conservation, $I_1 + I_3 = 0$, in conjunction with the "wideband" approximation [19], which implies that $\Sigma_{\rm ext}$ is energy independent. (iii) The chemical potential difference $\mu_\ell - \mu_r$ is small, and the system is in the *linear transport regime.* (iv) Last, but not least, is the requirement that the additional dephasing due to the nonequilibrium situation can be neglected.

For noninteracting electrons, our expression for I_c is consistent with the result in Ref. [12]. It generalizes the results of Refs. [11,13] to a steady-state situation. In that noninteracting case, $\Sigma^A_{\rm ext}$ constitutes the entire self-energy of the dot Green function, $G^{0A}_{dd}(\omega) = 1/(\omega - \epsilon_d - \Sigma^A_{\rm ext})$. The persistent current then becomes

$$I^{(0)}_c = \int \frac{2d\omega}{\pi} \frac{f_\ell(\omega) + f_r(\omega)}{2} \frac{\partial \delta^0(\omega)}{\partial \Phi}, \qquad (3)$$

where δ^0 is the phase of the retarded Green function $G^R = (G^A)^*$ for the noninteracting situation,

$$\tan\delta^0(\omega) = -\frac{\mathrm{Im}\Sigma^R_{\rm ext}}{\omega - \epsilon_d - \mathrm{Re}\Sigma^R_{\rm ext}}. \qquad (4)$$

This phase is identical to the transmission phase (the Friedel phase) of the interferometer [15]. However, this simple relation between the transmission phase and the persistent current ceases to hold when the dot self-energy also contains the interaction-induced part, $\Sigma_{\rm int}$, which depends on the flux as well. Thus, interactions eliminate the simple relation between the persistent current and the scattering solution.

Equation (2), which is our first main result, applies to *any* kind of interaction on the dot, including electron-electron interactions. In the rest of this Letter, we apply this general result to study the modification of the persistent current when the dot is coupled to a *phonon source.* Utilizing again the wideband approximation [19,20] and assuming an interaction between the phonons and the electron residing on the dot which is *linear* in the phonon coordinates, we find

$$G^R_{dd}(\omega) = -iK\left[(1-n_d)\int_0^\infty dt e^{i(\omega-\epsilon_d-\Sigma^R_{\text{ext}})t}e^{\Psi(t)} + n_d\int_0^\infty dt e^{i(\omega-\epsilon_d-\Sigma^R_{\text{ext}})t}e^{\Psi(-t)}\right]. \quad (5)$$

Here n_d denotes the electron occupation on the dot [21]. The on-site energy on the dot, ϵ_d, is now renormalized by the polaron shift, $\epsilon_P = \sum_{\mathbf{q}}|\alpha_{\mathbf{q}}|^2/\omega_q$, where $\alpha_{\mathbf{q}}$ is the electron-phonon coupling and ω_q denotes the phonon frequency. Since this renormalization is temperature and flux independent, we omit it in the following. The other phonon variables are contained in K, the Debye-Waller factor, and in $\Psi(t)$. Explicitly,

$$K = \exp\left[-\sum_{\mathbf{q}}\frac{|\alpha_{\mathbf{q}}|^2}{\omega_q^2}(1+2N_q)\right],$$
$$\Psi(t) = \sum_{\mathbf{q}}\frac{|\alpha_{\mathbf{q}}|^2}{\omega_q^2}[N_q e^{i\omega_q t} + (1+N_q)e^{-i\omega_q t}], \quad (6)$$

where N_q is the phonon occupation of the q mode, which is not necessarily the thermal equilibrium one, but may be tuned externally.

Let us now specify to the case of a weak electron-phonon interaction and not too large N_q. Expanding Eq. (5) to lowest order in $|\alpha_{\mathbf{q}}|^2$, one obtains $G^R_{dd}(\omega)$ in terms of the dot Green function of the noninteracting case (i.e., in the absence of the electron-phonon interaction), $G^{0R}_{dd}(\omega)$, at the same frequency ω and at frequencies shifted by the phonon frequencies, $\omega \pm \omega_q$. As a result, the expression for I_c involves the phase $\delta^0(\omega)$ at those frequencies as well. The result is conveniently written in the form

$$I_c = I_c^{(0)} + \Delta I_c, \quad (7)$$

where $I_c^{(0)}$ is the persistent current of the noninteracting interferometer, Eq. (3) above, and ΔI_c is the acousto-persistent current, given, within our approximation, by

$$\Delta I_c = \int \frac{d\omega}{\pi}\frac{f_\ell(\omega)+f_r(\omega)}{2}\times\sum_{\mathbf{q}}\left[A^+_q\frac{\partial}{\partial\Phi}(\delta^0(\omega+\omega_q)+\delta^0(\omega-\omega_q)-2\delta^0(\omega)) + A^-_q\frac{\partial}{\partial\Phi}(\delta^0(\omega+\omega_q)-\delta^0(\omega-\omega_q))\right], \quad (8)$$

where

$$A^+_{\mathbf{q}} = \frac{|\alpha_{\mathbf{q}}|^2}{\omega_q^2}(1+2N_q), \qquad A^-_{\mathbf{q}} = \frac{|\alpha_{\mathbf{q}}|^2}{\omega_q^2}(2n_d-1). \quad (9)$$

The acousto-induced persistent current, ΔI_c, consists of two parts: The first term in Eq. (8) is dominated by the phonon occupations [see Eq. (9)], via $A^+_{\mathbf{q}}$. The second term depends only on the dot's occupation, n_d, and its sign may change according to the relative location of ϵ_d with respect to the Fermi energy. The first term in ΔI_c shows that, by shining a beam of phonons of a specific frequency, the magnitude of the persistent current, and hence of the orbital magnetic moment of the ABI, can be enhanced and controlled experimentally, as long as the temperature of the electronic system and the intensity of the phonon source N_q are low enough to retain coherent motion of the electrons. This intensity is also limited by our weak effective interaction approximation. Similar considerations apply to photons. Both the precise magnitude of these effects and the above bounds depend on the detailed geometry of the dot and on the acoustic (or electromagnetic) mismatch. Such calculations go beyond the scope of the present Letter.

Equation (8) for the acousto-persistent current is our second main result. It is important to appreciate the difference between this result and the corresponding one found earlier [4] for the isolated ring. In the isolated ring, the Holstein process [5] required the emission (absorption) of a specific phonon, with the exact excitation energy of the electron on the ring. In the present case, the coupling to the leads turns the bound state into a resonance, with a width $\Gamma^0_d = -\text{Im}\Sigma^R_{\text{ext}}$ which vanishes when the ring is decoupled from the leads. As a result, there is always some overlap between the tail of the Green function $G^{0R}_{dd}(\omega)$ and the Fermi distribution $f(\omega)$, yielding contributions from Holstein-like processes via phonons with many energies. Indeed, each contribution to ΔI_c contains the phase $\delta^0(\omega)$, which vanishes with Γ^0_d ($\delta^0 \sim \Gamma^0_d/|\epsilon_d|$ far from the resonance). In particular, this results in a nonzero ΔI_c even at zero temperature: In that limit, if $\epsilon_d < \mu_\ell = \mu_r = 0$, then $n_d = 1$. Even with no phonons, $N_q = 0$, the square brackets in Eq. (8) become proportional to $\delta^0(\omega+\omega_q) - \delta^0(\omega)$, reflecting processes which begin by an emission of phonons. None of this remains for the isolated ring, when $\Gamma^0_d = 0$.

To obtain explicit expressions, we now evaluate the frequency integration appearing in Eq. (8). First, since we operate within the linear response regime, the voltage is not essential to our effect and we may safely write in Eq. (8) $f_\ell(\omega) = f_r(\omega) \equiv f(\omega)$ [22]. Furthermore, we take the electronic temperature to be low compared to all other energies, so that $f(\omega) \approx \Theta(-\omega)$. Second, we note that the self-energy Σ^R_{ext} takes a simple form when one invokes the wideband approximation, with a self-energy which is effectively at the middle of the (symmetric) conduction band of the leads [18]. For simplicity, we present results only for the special symmetric ABI (Fig. 1), with the same hopping matrix elements on the left and right branches of the ring and with the hopping from the ring onto each lead equal to that along the lead.

In the absence of the lower arm of the ABI, the self-energy is purely imaginary [20], $\Sigma^R_{\rm ext} = -i\Gamma^0_d$. Adding the lower branch, the self-energy acquires a real part as well, and both the real and imaginary parts depend on the flux. That dependence can be written in terms of T_B and R_B, the transmission and the reflection coefficients of the reference arm alone [18],

$$\begin{aligned} {\rm Re}\Sigma^R_{\rm ext} &= -\Gamma^0_d\sqrt{T_BR_B}\cos^2(\Phi/2), \\ {\rm Im}\Sigma^R_{\rm ext} &= -\Gamma^0_d[1 - T_B\cos^2(\Phi/2)]. \end{aligned} \tag{10}$$

Third, we take the typical phonon frequency to be much smaller than the large bandwidth in the leads. With these approximations the frequency integration in Eq. (8) is easily performed, to yield

$$\Delta I_c = \frac{\Gamma^0_d}{2\pi}\sin\Phi\sum_{\mathbf{q}}\{A^+_{\mathbf{q}}[F(\omega_q) + F(-\omega_q) - 2F(0)] + A^-_{\mathbf{q}}[F(\omega_q) - F(-\omega_q)]\}, \tag{11}$$

where $F(\omega)$ is given in terms of $\delta^0(\omega)$, Eq. (4),

$$F(\omega) = -\sqrt{T_BR_B}\delta^0(\omega) - T_B\ln|\sin\delta^0(\omega)|. \tag{12}$$

Note again that the dependence of the acousto-persistent current on the phonon frequency is fixed by the Friedel phase of the dot at that frequency. To leading order in the strength of the electron-phonon coupling, the magnitude of the first term in ΔI_c is proportional to $A^+_{\mathbf{q}}$ and thus grows linearly with the occupation number of the acoustic modes acting on the dot, N_q. In fact, the acousto-persistent current contains two types of contributions: the part associated with $F(0)$, which simply represents the "trivial" Debye-Waller renormalization of the current, and the novel frequency-dependent parts, which reflect the change in the persistent current due to Holstein-like processes.

In summary, we have derived a general expression for the steady-state current circulating in the Aharonov-Bohm ring, which is also valid when there are electronic interactions on the dot. We used this expression to find the effect of an acoustic (or electromagnetic) source on the persistent current. In particular, we found that, by controlling the intensity of the acoustic wave in a certain frequency range, one may tune the magnitude of the orbital moment. The same calculation can also apply to the photon-induced persistent current.

This project was carried out in a center of excellence supported by the Israel Science Foundation. It was also partially supported by the German-Israeli Foundation (GIF) and by the German Federal Ministry of Education and Research (BMBF), within the framework of the German-Israeli Project Cooperation (DIP). We acknowledge helpful discussions with the late A. G. Aronov and with Y. Levinson.

[1] M. Büttiker, Y. Imry, and R. Landauer, Phys. Lett. **96A**, 365 (1983).
[2] L. P. Levy *et al.*, Phys. Rev. Lett. **64**, 2074 (1990); V. Chandrasekhar *et al.*, Phys. Rev. Lett. **67**, 3578 (1991); D. Mailly, C. Chapelier, and A. Benoit, Phys. Rev. Lett. **70**, 2020 (1993); W. Rabaud *et al.*, Phys. Rev. Lett. **86**, 3124 (2001); E. M. Q. Jariwala *et al.*, Phys. Rev. Lett. **86**, 1594 (2001).
[3] Y. Imry, *Introduction to Mesoscopic Physics* (Oxford University Press, Oxford, 2002), 2nd ed.
[4] O. Entin-Wohlman, Y. Imry, A. G. Aronov, and Y. Levinson, Phys. Rev. B **51**, 11 584 (1995).
[5] T. Holstein, Phys. Rev. **124**, 1329 (1961).
[6] V. E. Kravtsov and V. I. Yudson, Phys. Rev. Lett. **70**, 210 (1993); V. E. Kravtsov and B. L. Altshuler, Phys. Rev. Lett. **84**, 3394 (2000).
[7] Some authors [P. Mohanty, Ann. Phys. (Berlin) **8**, 549 (1999); P. Schwab, Eur. Phys. J. B **18**, 189 (2000); P. Cedraschi, V. V. Ponomarenko, and M. Büttiker, Phys. Rev. Lett. **84**, 346 (2000); P. Cedraschi and M. Büttiker, Phys. Rev. B **63**, 165312 (2001)] discuss the possible connections between the enhanced persistent current and dephasing at equilibrium with a boson bath. As in Ref. [6], we concentrate on a *nonequilibrium* source of bosons and do not discuss dephasing.
[8] O. L. Chalaev and V. E. Kravtsov, Phys. Rev. Lett. **89**, 176601 (2002).
[9] M. Moskalets and M. Büttiker, Phys. Rev. B **66**, 245321 (2002).
[10] S. A. Cavill *et al.*, Phys. Rev. B **66**, 235320 (2002). See also B. A. Glavin *et al.*, JETP Lett. **51**, 191 (2000).
[11] M. Büttiker, Phys. Rev. B **32**, 1846 (1985).
[12] O. Entin-Wohlman, C. Hartzstein, and Y. Imry, Phys. Rev. B **34**, 921 (1986).
[13] E. Akkermans, A. Auerbach, J. E. Avron, and B. Shapiro, Phys. Rev. Lett. **66**, 76 (1991).
[14] D. C. Langreth, in *Linear and Nonlinear Electron Transport in Solids*, Nato Advanced Study Institute, Ser. B, edited by J. T. Devreese and V. E. Van Doren (Plenum, New York, 1976), Vol. 17.
[15] T. K. Ng and P. A. Lee, Phys. Rev. Lett. **61**, 1768 (1988).
[16] Details will be given elsewhere.
[17] B. R. Bulka and P. Stefański, Phys. Rev. Lett. **86**, 5128 (2001); W. Hofstetter, J. König, and H. Schoeller, Phys. Rev. Lett. **87**, 156803 (2001); O. Entin-Wohlman, A. Aharony, Y. Imry, Y. Levinson, and A. Schiller, Phys. Rev. Lett. **88**, 166801 (2002); H. A. Weidenmüller, Phys. Rev. B **65**, 245322 (2002).
[18] A. Aharony, O. Entin-Wohlman, and Y. Imry, Phys. Rev. Lett. **90**, 156802 (2003).
[19] A-P. Jauho, N. S. Wingreen, and Y. Meir, Phys. Rev. B **50**, 5528 (1994).
[20] L. I. Glazman and R. I. Shekhter, Sov. Phys. JETP **67**, 163 (1988).
[21] In the nonequilibrium case, n_d is determined by both the acoustic intensity and the relaxation processes. We have extended the relations given in Refs. [19,20] to include $n_d \neq 0$.
[22] However, at thermal electronic equilibrium, $\mu_\ell = \mu_r$, the wideband approximation is not required for the derivation of I_c, and the result (2) is exact.