

Zeeman Splitting of Zero-Bias Anomaly in Luttinger Liquids

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(Received 14 November 2002; published 22 July 2003)

Tunneling density of states (DOS) in Luttinger liquid has a dip at zero energy, commonly known as the zero-bias anomaly. In the presence of a magnetic field, in addition to the zero-bias anomaly, the DOS develops two peaks separated from the origin by the Zeeman energy. We show that these finite-bias anomalies are characterized by a power-law behavior of the DOS and the differential conductance, and find the corresponding exponents at arbitrary strength of the electron-electron interaction. The developed theory is applicable to various kinds of quantum wires, including carbon nanotubes.

DOI: 10.1103/PhysRevLett.91.046801

PACS numbers: 73.63.-b, 71.10.Pm, 73.21.Hb, 73.22.-f

Interaction between electrons in a conductor leads to the formation of an anomaly in the tunneling density of states (DOS) at the Fermi level. The one-particle DOS is directly related to the differential conductance of a tunnel junction, and the anomaly in DOS translates to the zero-bias anomaly (ZBA) of the tunneling conductance. This anomaly gets stronger if the conductor is disordered, and if the dimensionality of the electron system is reduced. The perturbative treatment of the DOS anomaly in disordered conductors is well developed [1]. In a disordered wire or film, the perturbation theory in the interaction strength is divergent at the Fermi level, and therefore a nonperturbative treatment is needed to describe the DOS at low energies [2–4]. In one-dimensional conductors with one or a few propagating electron modes, the suppression of the DOS due to the electron-electron repulsion is strong even in the absence of disorder. The density of states in this case is adequately described within the Luttinger liquid theory [5]. The ZBA was observed in experiments with higher-dimensional disordered systems [7,8]. The recently measured [9,10] strong suppression of the tunneling in a single-wall carbon nanotube at low bias showed that electrons in a nanotube indeed form a Luttinger liquid.

The zero-bias anomaly thus provides important information about strongly correlated electron systems. However, the ZBA is sensitive mostly to the dynamics of electron charge but not to that of spin. To probe the spin physics, one may study the effect of a magnetic field on the properties of the electron system. The perturbative calculation shows [1] that the application of a magnetic field modifies the anomaly in the DOS. It acquires, in addition to the zero-bias dip, two peaks at energies $\varepsilon = \pm g\mu_B B$, where $g\mu_B B$ is the Zeeman energy. The peaks heights are equal and proportional to the electron-electron interaction constant in the triplet channel [1], which is not accessible in a measurement of the conventional ZBA. The described Zeeman splitting of the ZBA in disordered normal conductors was not observed yet. A possible obstacle for the observation is that the quasipar-

ticle lifetime at energy $\varepsilon \sim g\mu_B B$ is short enough to smear the singularity [11].

The perturbative theory of Zeeman splitting of ZBA in a clean 1D system was developed in [12]. There, the anomalies in the conductance were ascribed to the physics of Bragg reflection of electrons off the Friedel oscillation. Magnetic field B splits the standard Friedel oscillation in two. The difference between the two corresponding wave vectors is proportional to B . Electron scattering off the two components of the Friedel oscillation results in the conventional DOS anomaly at zero energy and two additional peaks in the DOS at $\pm g\mu_B B$. This single-electron picture is valid for a weak electron-electron interaction only, and is not applicable to the Luttinger liquid with a strong repulsion between electrons. However, the strongest manifestation of the Luttinger liquid behavior is found in carbon nanotubes, where the interaction is not weak. Keeping in mind that the very notion of an electronlike quasiparticle is not adequate in a Luttinger liquid, one may question the existence of peaks in the DOS at finite energy $|\varepsilon| = g\mu_B B$ in such systems.

In this paper, we demonstrate that the tunneling density of states in a Luttinger liquid is singular at energies $\varepsilon = \pm g^* \mu_B B$, independent of the interaction strength in the charge channel. It allows us to expect an observation of such singularities in carbon nanotubes. The effective Landé factor g^* here is renormalized by the interaction. The overall magnitude of the singular correction to DOS is proportional to the constant of electron-electron interaction in the triplet channel. The energy dependence of the DOS around the singularities is given by a power law, $\delta\nu(\varepsilon) \sim |\varepsilon \pm g^* \mu_B B|^\gamma$. We calculate the exponent γ in terms of the Luttinger liquid parameters.

Consider the one-dimensional Luttinger liquid filling the half line $x > 0$ and confined by a barrier at $x = 0$. We decompose the electron creation operator $\psi_s^\dagger(x)$ into the left- and right-moving parts: $\psi_s^\dagger = \psi_{+,s}^\dagger + \psi_{-,s}^\dagger$. Here \pm denote left and right movers, and $s = \pm 1$ denote two spin states. Then, we bosonize electrons [6]:

$$\psi_{\pm,s}^{\dagger}(x) = \frac{1}{\sqrt{2\pi a}} \exp\left\{\mp i p_F x \pm i \frac{\pi}{2} + \frac{i}{\sqrt{2}} \left\{ \pm [\varphi_{\rho}(x) + s \varphi_{\sigma}(x)] + \vartheta_{\rho}(x) + s \vartheta_{\sigma}(x) \right\}\right\} \quad (1)$$

Bosonic variables φ_i and ϑ_i describe charge ($i = \rho$) and spin ($i = \sigma$) fluctuations, and a is the short-distance cut-off. The fields φ_i and ϑ_j are canonically conjugate: $[\varphi_i(x), \vartheta_j(x')] = -i\pi\delta_{ij}\theta(x-x')$. Since the electron wave function is zero at the barrier ($x = 0$), the fields φ_i satisfy the boundary condition:

$$\varphi_{\rho}(0) = \varphi_{\sigma}(0) = 0. \quad (2)$$

The Hamiltonian can be divided into four parts:

$$H = H_{\rho} + H_{\sigma} + H' + H_B, \quad (3)$$

where the first two terms include the density-density interactions:

$$H_i = \frac{u_i}{2\pi} \int dx \left[K_i (\partial_x \vartheta_i)^2 + \frac{1}{K_i} (\partial_x \varphi_i)^2 \right], \quad (4)$$

and H' represents spin-flip backscattering:

$$H' = \frac{2g_{\perp}}{(2\pi a)^2} \int dx \cos\sqrt{8}\varphi_{\sigma}.$$

The last term of Eq. (3) describes Zeeman splitting:

$$H_B = -\frac{g\mu_B B}{2} \int dx \rho_{\text{spin}}(x) = \frac{g\mu_B B}{2\pi\sqrt{2}} \int dx \partial_x \varphi_{\sigma},$$

where $\rho_{\text{spin}}(x)$ is spin density. The parameters u_i , K_i , and g_{\perp} can be expressed in terms of interaction potential $V(x)$, but here we treat them as phenomenological constants. For free electrons, $K_{\rho} = K_{\sigma} = 1$, while for repulsive interaction $K_{\rho} < 1$ and $K_{\sigma} > 1$. Also, the bare parameters K_{σ} and g_{\perp} are not independent. For $g_{\perp} \ll 1$, they are related as $K_{\sigma} \approx 1 + g_{\perp}/2\pi u_{\sigma}$.

For convenience, we absorb the magnetic field term H_B into the quadratic part by shift $\varphi_{\sigma} \rightarrow \varphi_{\sigma} + g\mu_B B x K_{\sigma}/\sqrt{2}u_{\sigma}$. Then, the backscattering term transforms into

$$H' = \frac{2g_{\perp}}{(2\pi a)^2} \int dx \cos(\sqrt{8}\varphi_{\sigma} + bx), \quad (5)$$

with $b = 2g\mu_B B K_{\sigma}/u_{\sigma}$.

The Hamiltonian (3) decouples into charge and spin sectors. While the charge excitations do not interact and their Hamiltonian H_{ρ} is quadratic, the spin sector is described by the sine-Gordon model $H_{\sigma} + H'$. In zero magnetic field, the constant g_{\perp} is renormalized at low energies [6],

$$g_{\perp}(D) = \frac{g_{\perp}(W)}{1 + \frac{g_{\perp}(W)}{\pi v_F} \log \frac{W}{D}}, \quad (6)$$

where W is the initial, D is the running bandwidths, and $g_{\perp}(W)$ is the ‘‘bare’’ interaction constant. The renormalization group (RG) flow occurs along the line $K_{\sigma} \approx 1 + g_{\perp}/2\pi v_F$ toward the fixed point $K_{\sigma}^* = 1$, $g_{\perp}^* = 0$. The finite magnetic field does not affect the RG flow for

energies larger than $g\mu_B B$. For smaller energies, K_{σ} becomes essentially independent of ε , while g_{\perp} flows toward zero [13]. In this way, the nonlinear term H' is not relevant at large times, and can be treated as a perturbation. Since we are interested in the DOS at $\varepsilon \rightarrow g\mu_B B$, in the following we take D somewhat exceeding $2g\mu_B B$.

Before developing a rigorous calculation, we provide a hint to the origin of the singularity in the DOS. Tunneling of an electron may be viewed as spreading of charge and spin densities, which initially at $t = 0$ were formed near the barrier ($x = 0$). The charge propagates freely, while the propagation of the spin density is affected by the backscattering term Eq. (5). To demonstrate qualitatively its effect, we expand H' in φ_{σ} to the second order and then derive the linear equation of motion for the field φ_{σ} . The first-order expansion term only shifts by a small amount the solution of that equation. The second-order term generates a contribution $\propto g_{\perp} \cos(bx)\varphi_{\sigma}$ in the equation of motion, and leads to the phenomenon of Bragg reflection with wave vector $b/2$. As the result, the backscattered component of φ_{σ} oscillates with frequency $\omega_z = u_{\sigma} b/2 = K_{\sigma} g\mu_B B$. These oscillations give rise to features in the DOS at energies $\varepsilon = \pm \omega_z$.

We start with retarded Green’s function

$$G_s^R(x, x', \varepsilon) = -i \int_0^{\infty} dt e^{i\varepsilon t} \langle \{ \psi_s^{\dagger}(x, t), \psi_s(x', 0) \} \rangle$$

(here $\{ \dots \}$ is anticommutator) and compute the tunneling density of states as

$$\nu(\varepsilon) = \frac{1}{4\pi(ik_F)^2} \sum_s \text{Im} \frac{\partial^2}{\partial x \partial x'} \Big|_{x=x'=0} G_s^R(x, x', \varepsilon). \quad (7)$$

For slowly varying $\varphi_i(x)$ and $\vartheta_i(x)$, one may neglect their derivatives, and differentiate only the factors $e^{\pm ik_F x}$ in the electron operators (1). Equation (7) can be rewritten as

$$\begin{aligned} \nu(\varepsilon) &= \frac{1}{2\pi^2 a} \text{Re} \int_0^{\infty} dt [\mathcal{G}_{\rho}(t) e^{i\varepsilon t} + \mathcal{G}_{\rho}(-t) e^{-i\varepsilon t}] \\ &\quad \times [\mathcal{G}_{\sigma}(t) + \mathcal{G}_{\sigma}(-t)], \end{aligned} \quad (8)$$

where

$$\mathcal{G}_i(t) = \left\langle T \exp \frac{i\vartheta_i(x=0, t)}{\sqrt{2}} \exp \frac{-i\vartheta_i(x=0, 0)}{\sqrt{2}} \right\rangle \quad (9)$$

are time-ordered Green’s functions of charge and spin, and T denotes time ordering.

To compute these correlation functions at $g_{\perp} = 0$, we express the fields φ_i and ϑ_i in terms of bosonic eigenmodes $a_{\rho}, a_{\rho}^{\dagger}$ of the Hamiltonian (4). Because of the boundary condition (2), only odd modes contribute to φ_i :

$$\varphi_i(x, t) = \sum_q c_{q,i} \sin qx (a_{q,i}^\dagger e^{-iu_i qt} + a_{q,i} e^{iu_i qt}), \quad (10)$$

$$\vartheta_i(x, t) = \sum_q \frac{c_{q,i}}{K_i} \cos qx \frac{a_{q,i}^\dagger e^{-iu_i qt} - a_{q,i} e^{iu_i qt}}{i}.$$

Here $c_{q,i} = e^{-qa/2} \sqrt{\pi K_i/q}$, and the short-distance cutoff $a = u_\sigma/D$ is related to the reduced bandwidth D . The summation in Eq. (10) involves wave vectors $q > L^{-1}$, where L is the length of the system. One can compute the average in Eq. (9) using the relations

$$e^A e^B = e^{A+B} e^{1/2[A,B]} \quad \text{and} \quad \langle e^A \rangle = e^{1/2\langle A^2 \rangle}, \quad (11)$$

valid for any operators A and B linear in a_q and a_q^\dagger . At zero temperature, the only nonzero average is $\langle a_q a_q^\dagger \rangle = 1$, and one finds

$$\delta \mathcal{G}_\sigma(t) = -\frac{2ig_\perp}{(2\pi a)^2} \int_{-\infty}^{\infty} dt' \int_0^{\infty} dx \langle T \exp \left[\frac{i[\vartheta_\sigma(t) - \vartheta_\sigma(0)]}{\sqrt{2}} \right] \rangle \times \{ \cos[\sqrt{8}\phi_\sigma(x, t') + bx] - \langle \cos[\sqrt{8}\phi_\sigma(x, t') + bx] \rangle_0 \},$$

we use Eqs. (10) and (11), and obtain

$$\begin{aligned} \delta \mathcal{G}_\sigma(t) = & -\frac{2ig_\perp}{(2\pi a)^2} \int_{-\infty}^{\infty} dt' \int_0^{\infty} dx \left(\frac{a^2}{a^2 + 4x^2} \right)^{K_\sigma} \\ & \times \mathcal{G}_\sigma^{(0)}(t) [\delta \mathcal{G}_+(x, t, t') e^{ibx} \\ & + \delta \mathcal{G}_-(x, t, t') e^{-ibx}], \end{aligned} \quad (14)$$

where

$$\begin{aligned} \delta \mathcal{G}_\pm(x, t, t') = & \pm \frac{2xu_\sigma t}{u_\sigma(t-t') \mp x + ia \operatorname{sgn}(t-t')} \\ & \times \frac{1}{u_\sigma t' \mp x + ia \operatorname{sgn}(t')}. \end{aligned} \quad (15)$$

Unlike Eq. (12), the correction $\delta \mathcal{G}_\sigma(t)$ contains an oscillating part. We will retain only this part, since we are interested in singularities at nonzero energies. The oscillation originates from the point of enhanced singularity in Eq. (15), $x = u_\sigma t' = u_\sigma t/2$. The oscillating contribution to $\delta \mathcal{G}_\sigma(t)$ is

$$\frac{\delta \nu(\varepsilon)}{\nu^{(0)}(\omega_z)} = -\frac{g_\perp(\omega_z)}{4u_\sigma} \frac{1}{\sin \pi \gamma} \left| \frac{\varepsilon - \omega_z}{\omega_z} \right|^\gamma \times \frac{\Gamma(1+\alpha)}{\Gamma(1+\gamma)} \times \begin{cases} \cos \frac{\pi}{2}(\alpha - \gamma) & \text{for } \varepsilon > \omega_z \\ \cos \frac{\pi}{2}(\alpha + \gamma) & \text{for } \varepsilon < \omega_z \end{cases} \quad (18)$$

with the exponent $\gamma = \alpha + 2(K_\sigma - 1)$, i.e.,

$$\gamma = \frac{1}{2}(K_\rho^{-1} + K_\sigma^{-1} - 2) + 2(K_\sigma - 1). \quad (19)$$

Equation (18) is valid for arbitrarily strong interaction in the charge channel, and confirms the existence of singularity in DOS centered at energy

$$\omega_z = g^* \mu_B B; \quad \frac{g^*}{g} = 1 + \frac{g_\perp(D \sim \mu_B B)}{2\pi u_\sigma}. \quad (20)$$

$$\mathcal{G}_i^{(0)}(t) = \left(\frac{ia}{u_i|t| + ia} \right)^{1/(2K_i)}. \quad (12)$$

Substituting this expression into Eq. (8) and evaluating the integral for $\varepsilon \ll D$, one arrives at the well-known formula [5],

$$\nu^{(0)}(\varepsilon) = \frac{C_0}{\Gamma(1+\alpha)} |\varepsilon|^\alpha, \quad (13)$$

with the anomalous exponent $\alpha = (K_\rho^{-1} + K_\sigma^{-1})/2 - 1$, and the prefactor

$$C_0 = \frac{1}{\pi a} \left(\frac{a}{u_\rho} \right)^{1/(2K_\rho)} \left(\frac{a}{u_\sigma} \right)^{1/(2K_\sigma)}.$$

Expanding \mathcal{G}_σ defined by Eq. (9) to the first order in g_\perp ,

$$\frac{\delta \mathcal{G}_\sigma(t)}{\mathcal{G}_\sigma^{(0)}(t)} = -\frac{ig_\perp}{2u_\sigma} \left(\frac{a}{u_\sigma|t|} \right)^{2(K_\sigma-1)} \theta(t) e^{i\omega_z t}, \quad (16)$$

with $\omega_z = g^* \mu_B B$, and renormalized Landé factor $g^* = K_\sigma g$. The preexponential time-dependent factor in Eq. (16) is related, via integration in Eq. (14), to the asymptotic ($|x| \rightarrow \infty$) power-law behavior of the Green function in the spin sector. This power-law asymptote is a generic property of the Luttinger liquid, and is preserved at any g_\perp . This gives a reason to believe that the functional form Eq. (16) of $\delta G(t)$ is valid beyond the limit of small g_\perp .

Using Eqs. (8) and (12) at $t > D^{-1}$, one finds the correction to the DOS

$$\delta \nu(\varepsilon) = \pi C_0 \operatorname{Re} e^{i(\pi/2)(\alpha+1)} \int_{D^{-1}}^{\infty} \frac{\cos \varepsilon t dt}{t^{\alpha+1}} \frac{\delta \mathcal{G}_\sigma(t)}{\mathcal{G}_\sigma^{(0)}(t)}, \quad (17)$$

which is singular at $\varepsilon = \pm \omega_z$. Since Eq. (17) is even in ε , we consider further only $\varepsilon \approx \omega_z$. Integrating over the time domain $t \sim |\varepsilon - \omega_z|^{-1}$, we find the following for the singular part:

At small g_\perp , which implies $K_\sigma \approx 1$, the main contribution to the exponent in Eq. (18) comes from the charge mode. Therefore, the singularities of the DOS at $\varepsilon = 0$ and $\varepsilon = \omega_z$ have nearly identical exponents, $\gamma \approx \alpha$. The contribution (18) was found for zero temperature, and its energy dependence is nonanalytic at any γ . However, it may be easily distinguished from the regular part of $\nu(\varepsilon)$ only at $\gamma < 1$. Also, finite temperature T smears the singularity at $|\varepsilon - \omega_z| \approx T$.

The DOS anomaly (18) is directly related to the bias dependence of the tunneling conductance $G(V)$ between a conventional metal and a one-dimensional conductor. The corresponding singular contributions are related as $\delta G(V)/G = \delta \nu(eV)/\nu$. Tunneling between the ends of two identical one-dimensional conductors (such as an intramolecular junction between carbon nanotubes [10]) also has a singularity at Zeeman energy. The tunneling conductance can be calculated as $G(V) = dI/dV$, where the current $I(V)$ between the two conductors is proportional to the convolution of the two corresponding DOS:

$$I(V) \propto \int_0^{eV} d\varepsilon \nu(\varepsilon)\nu(\varepsilon - eV). \quad (21)$$

Calculating this integral, one finds the singular contribution to the differential conductance,

$$\frac{\delta G(V)}{G(V)} = -\frac{g_{\perp}}{4u_{\sigma}} \frac{1}{\sin^{\frac{\alpha}{2}}(\alpha + \gamma)} \left| \frac{eV - \omega_z}{\omega_z} \right|^{\alpha + \gamma} \times \frac{\Gamma(1 + 2\alpha)}{\Gamma(1 + \alpha + \gamma)}. \quad (22)$$

The exponent $\alpha + \gamma$ here coincides with 2γ up to a small term of the order of $K_{\sigma} - 1$. This ‘‘exponent doubling’’ at $eV = \omega_z$ is similar to that occurring at zero bias [10].

It is interesting to analyze Eq. (22) in the limit of weak interactions, in which [6]

$$K_{\sigma} - 1 \approx \frac{g_{\perp}}{2\pi v_F} \approx \frac{U(2k_F)}{2\pi v_F}, \quad K_{\rho} \approx K_{\sigma} - \frac{U(0)}{\pi v_F}.$$

To the first order in the interaction potential $U(q)$, Eq. (22) yields $\delta G/G = [U(2k_F)/4\pi u_{\sigma}] \ln(|eV - \omega_z|/\omega_z)$. This result is in agreement with the first-order expansion of the tunneling conductance obtained in [12]. However, beyond this order, there is a difference between Eq. (22) and Eq. (47) in [12]. It stems apparently from the inapplicability of the RG approach developed in [12] for the treatment of tunneling at energies close to ω_z .

We derived Eqs. (18) and (19) for the single-mode Luttinger liquid. In the case of carbon nanotubes, one has to take into account the degeneracy between the two conic points in the Brillouin zone [14]. Treating the interaction in the charge channel nonperturbatively, as we did before, we find the exponent of the singularity at $\epsilon = \omega_z$,

$$\gamma_{\text{nt}} \approx \alpha_{\text{nt}} \approx \frac{1}{4}(K_{\rho}^{-1} - 1). \quad (23)$$

Thus, the exponent γ_{nt} again nearly coincides with the ZBA exponent α_{nt} . The reported values of α_{nt} in the experiments with carbon nanotubes were $\alpha_{\text{nt}} = 0.3 \div 0.6$, and therefore the peak at $\epsilon = \omega_z$ should be sharp and easy to observe.

In conclusion, the application of a magnetic field to a Luttinger liquid creates additional singularities (two peaks) in the tunneling density of states. The power law characterizing these singularities is related to the universal long-range behavior of the spin and charge excitations in a Luttinger liquid. Because of this relation, the power law persists at any interaction strength in the charge channel. The magnitude of the peaks is determined by the short-range interaction, which plays a minor role in the charge physics of a Luttinger liquid and therefore is hardly accessible in the measurements of the conventional zero-bias anomaly of the tunneling conductance.

We are grateful to L. S. Levitov, P. B. Wiegmann, K. A. Matveev, M. P. A. Fisher, A. G. Abanov, and I. L. Aleiner for useful discussions. This research was supported by NSF Grants No. PHY99-07949, No. DMR97-31756, No. DMR02-37296, and No. EIA02-10736, and by the Research Corporation Grant No. CC5491.

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