

Gurarie and Lobkovsky Reply: A particle which undergoes random motion usually moves diffusively. Indeed, given the correlation between velocities of the particle at different times,

$$K(t_1 - t_2) = \langle v(t_1)v(t_2) \rangle, \quad (1)$$

we can calculate its average displacement at large T :

$$\begin{aligned} \langle r^2(T) \rangle &= \int_0^T dt_1 \int_0^T dt_2 \langle v(t_1)v(t_2) \rangle \\ &= \int_0^T dt_1 \int_0^T dt_2 K(t_1 - t_2). \end{aligned} \quad (2)$$

In most practical problems, the correlation function $K(t)$ is proportional to $\exp(-t/\tau)$, where τ is the correlation time inherent to the system under study. For a Brownian particle, τ is the mean scattering time. Substituting this definition of K into Eq. (2), we find

$$\langle r^2(T) \rangle \approx T \int_0^\infty dt K(t) \equiv DT, \quad (3)$$

where

$$D = \int_0^\infty dt K(t). \quad (4)$$

We find therefore that the average square displacement of the particle is proportional to time, i.e., normal diffusion, with D being diffusion constant. A random walk along a spiral, suggested by the authors of the Comment [1], is of this type.

However, the logic which led to this conclusion is crucially based on the convergence of the integral in Eq. (4). For an exponentially decaying correlation function $K(t)$ it is indeed correct. On the other hand, for a class of problems where the integral over $K(t)$ is not convergent, the motion of the particle will no longer be diffusive. For example, if $K(t) \propto 1/t^\alpha$, where $\alpha < 1$, we find

$$\langle r^2(T) \rangle \approx T \int_0^T K(t) \propto T^{2-\alpha}. \quad (5)$$

The motion of the particle is superdiffusive.

In our Letter [2], we calculated the average displacement of a particle moving along 2D planes with screw dislocations. One of our results was that the correlation function $K(t)$ behaves as a power law,

$$K(t) \propto \frac{1}{t}, \quad t \gg \epsilon, \quad (6)$$

as follows from Eq. (10) of our Letter. Here ϵ is the time it takes for a particle to enclose a single dislocation. As a result, the motion in the perpendicular direction was no longer diffusion but replaced by a more complex formula

$$\langle r^2 \rangle \propto T \log(T). \quad (7)$$

This formula follows from Eq. (5) as $\alpha \rightarrow 1$.

The long time velocity correlations mean that in this problem a diffusive particle possesses something akin to memory; its movement now is correlated with what it did in the past. This memory is due to the trajectory dependence of motion in a dislocated crystal. On a simple surface, any modification of only the past segments of the trajectory cannot lead to movement of the final point. However, changing segments of a trajectory on a dislocated surface can lead to significant motion of its final point in the vertical direction. This trajectory dependence is crucial for the derivation of Eq. (7).

The authors of the Comment to our Letter do not consider these long time correlation effects. According to their logic, for an even smaller time interval $\Delta T \ll \epsilon$, the particle will not “know” that the surface it is moving on is dislocated. Therefore, the usual formula for a particle moving on a surface without any topological defects will apply, $\langle (\Delta r)^2 \rangle \propto \Delta T$ [Eq. (1) of the Comment]. Then the authors claim that diffusion at times much *bigger* than ϵ will automatically follow. This, however, is not true as at these longer times the memory effects encoded in Eq. (6) have to be taken into account.

The second argument of the Comment’s authors has to do with the fact that when a particle moves in a close vicinity of a screw dislocation, the projection of its motion onto the plane perpendicular to the dislocation’s axis is no longer simple. Indeed, the 2D plane is strongly curved nearby a dislocation. However, for our purposes it is enough to consider trajectories which do not come closer than some cutoff distance a , the core size of the dislocation. a is also typically of the order of the distance between the layers connected by the dislocations. It is such trajectories which result in superdiffusion, and for them the curvature of the layers can be neglected.

In short, the memory effects in this problem are non-trivial and must be carefully taken into account via the calculation presented in our Letter.

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[1] D. Constantin and R. Holyst, preceding Comment, Phys. Rev. Lett. **91**, 039801 (2003).

[2] V. Gurarie and A. E. Lobkovsky, Phys. Rev. Lett. **88**, 178301 (2002).