Brownian Particle in an Optical Potential of the Washboard Type

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(Received 13 September 2002; published 15 July 2003)

Experimental observations of the fluctuation-driven net transport of silica microspheres are presented in a two-dimensional optical potential of circular symmetry created by a Bessel light beam. The optical field is tailored to break symmetry and create a static tilted periodic (washboard) potential. As the tilt of potential exceeds a threshold, a transition between locked and running modes is observed. The running mode manifests itself by the rapid accumulation of particles at the beam center.

DOI: 10.1103/PhysRevLett.91.038101 PACS numbers: 87.80.Cc, 02.50.Ey, 05.40.Jc, 87.17.–d

Motor proteins incorporated within the cytoskeleton meshwork play a key role in processes of intracellular communications [1]. In attempts to understand the mechanical aspect of intracellular molecular transport, the concept of a thermal forced ratchet was introduced earlier [2]. The relevance to biological systems [3] and perspectives for construction of universal molecular driven device [4] have motivated substantial theoretical studies of fluctuation-induced transport of Brownian particles in asymmetric periodic potentials [5–9]. Unidirectional net transport can emerge in an asymmetric ratchet due to ambient noise that intrinsically links this effect to phenomenon of stochastic resonance (SR) [10]. The theory has been tested experimentally in simple model systems—for a single colloidal particle in an optical trap with sawtooth potential [11] and for an atomic cloud driven in an optical lattice by a weak moving modulation [12]. However, the quest of these studies is to identify the conditions when fluctuation-driven net transport emerges in overdamped systems without resorting to time-dependent forcing terms. Risken [5] first studied the Brownian dynamics in such a potential. Later on there were many "hot" discussions [6] regarding observation of SR in overdamped systems to which most of the experimental conditions might be related, notably in mesoscopic physics.

In this Letter we describe the Brownian dynamics of a microscopic particle in a two-dimensional optical washboard potential created using a zeroth-order Bessel light beam [13]. We observe and characterize the transition from a locked to a running mode in an overdamped spatially distributed *static* optical trap potential for parameters, which were previously attributed solely to underdamped systems. The Bessel beam can also offer extended distances for transporting microscopic particles, biological molecules, and even realize novel forms of dipole traps for cold atoms.

First, we overview briefly the basics of the diffusion dynamics of an overdamped Brownian particle in a one-dimensional space-periodic potential [5,6]. While this is not directly related to our experimental situation, the model serves to illustrate the relevant dynamical behavior that can be observed. The dynamics can be studied by the Langevin equation with a stochastic noise source $\xi(t)$ [5]:

$$\gamma \frac{dx}{dt} = -U'(x) + \xi(t), \tag{1}$$

where $\gamma = k_B T/D$, γ is a friction coefficient, D is free diffusion constant. The spatially periodic (washboard) tilted potential profile $U(x)$ is described by $U(x) = d\cos x - Fx$ and $\xi(t)$ is Gaussian white noise with zero mean and correlation function $\langle \xi(t)\xi(0)\rangle = 2\gamma k_B T \delta(t)$: k_B is the Boltzmann constant and T is the temperature.

The solution of Eq. (1) is considered in terms of the average particle drift velocity or probability current, $\langle v(t)\rangle = \langle dx/dt\rangle$. The time evolution of the Brownian motion of the particle is characterized by random switches between what is termed a locked state with zero average velocity (the particle is confined at the bottom of single potential well) and a running state with average drift velocity F/γ. In the running state, particles make transitions at a rapid rate (apparently exceeding free diffusion) between the various potential wells of the system. In the overdamped limit ($\gamma \gg d$) as long as $F < d$ a drift is solely due to thermal activation. The transition from locked to running mode occurs for $F > d$ when minima of potential disappear. In the underdamped limit (with no noise), the opposite transition occurs when F is reduced below $(4/\pi)\gamma\sqrt{d}$ due to inertia. This implies the existence of bistability in the intermediate region [4,6].

To create our two-dimensional periodic optical potential we use a zeroth-order Bessel light beam [13]. The beam is generated when illuminating a conically shaped optical element (axicon) of internal angle 1° by a collimated parallel beam with Gaussian transverse distribution from a cw laser operating at either 1064 or 780 nm with a power of up to 500 mW. This creates a circularly symmetric pattern of concentric rings (up to 40) whose intensity profile is a good approximation to a Bessel function [Fig. 1(a)]. An optical telescope scales the beam size. The beam propagation distance after the telescope is 1 cm. The spatial period is approximately 5 μm with a central spot diameter 5.6 μm and the Bessel beam

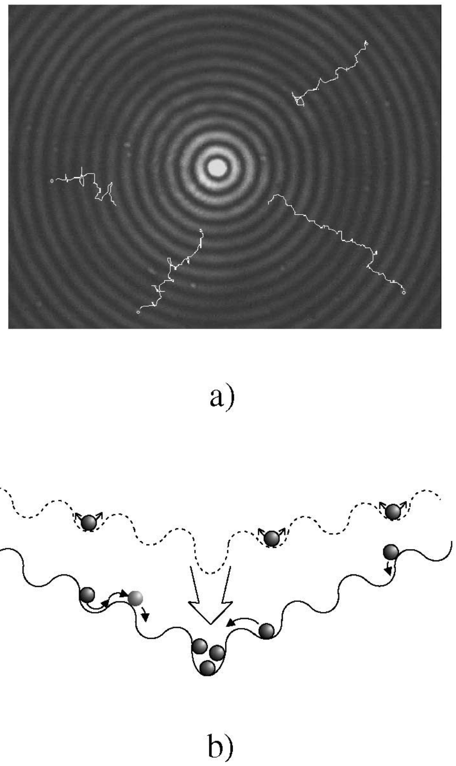

FIG. 1. (a) Singled particle (1 μm) trajectories taken by particle tracking technique. The background shows intensity profile of the transverse cross section of Bessel beam; (b) Schematic representation of a locked and a running mode in washboard potential with central symmetry.

intensity as a whole varied between 150–350 mW. The beam was directed horizontally into a cuboid sample chamber with optical quality walls (outer dimensions 5 mm $\times$ 5 mm $\times$ 20 mm) such that the beam is propagation invariant along the length of the cell. (The beam was directed horizontally for most of our experiments, though the results of vertical propagation, as on Fig. 1, are also discussed.) We studied the diffusion transport of Brownian particles [1 μm diameter silica microspheres, refractive index (589 nm) $n = 1.37$(Bangs Lab)] monodispersed in water within the cell. The diluted microspheres were loaded from the open top of the sample cell at a concentration of less than 0.2% by volume. This concentration is such that we can neglect particle-particle interactions in our system. Observation of the particles was performed using a microscope objective (20$\times$, NA $= 0.4$, Newport) mounted at the front of a CCD camera and placed orthogonally to the propagation direction of the Bessel beam.

Notably the Bessel beam can be experimentally modified to produce a washboard profile with adjustable tilt [Fig. 1(b)]. In particular, the tilt of the optical potential is realized and can be varied if the axicon is illuminated by slightly converging laser beam. This results in a beam structure that approaches the potential profile described by Eq. (1) where U now represents a Bessel function

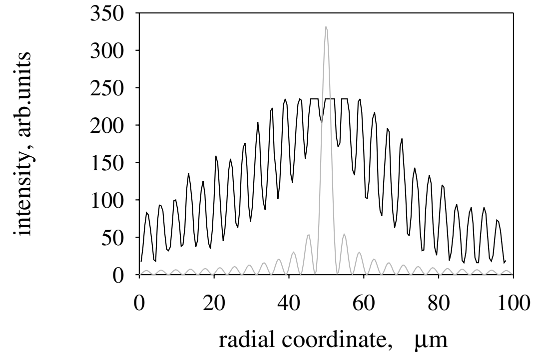

FIG. 2. Bessel function (gray line) and experimental profile of tilted Bessel beam (black line). By simply changing the divergence of the incident beam on the axicon we can alter the parameter F which is defined in the bifurcation diagram below.

minus Fx. The comparison between a theoretical Bessel beam profile and its experimental tilted realization is shown in Fig. 2.

Brownian particles diffuse freely within a single potential well formed by each ring of the Bessel beam, with the random hopping between wells (rings) as described by Kramer's theory [14]. The running mode in our horizontal geometry results in a spontaneously stable accumulation of particles in the central maximum (core) of the Bessel beam. The experimental observation of this running mode is presented in Fig. 3. The figure shows a panoramic view across the cell. Spheres tend to accumulate in the central maximum of the Bessel beam, where they are transversely localized and are subsequently guided due to optical radiation pressure. These observations show that the running mode is stable and the accumulation is continuous over a time of about half an hour after which the sample is depleted of available spheres to load into the beam. Without tilt added to the optical potential, there would be no strong tendency for the multiple, rapid inwardly directed transitions between the rings of the Bessel beam that are demonstrated in Fig. 3.

We studied the transition from a locked to a running mode by adiabatically changing the static tilt of potential profile while other parameters remain unchanged (Fig. 4, points AB). This was achieved by slowly varying the convergence of the incident beam upon the axicon. When a certain critical value F_3 has reached a few spheres appeared in the beam center. Stable multiple-particle

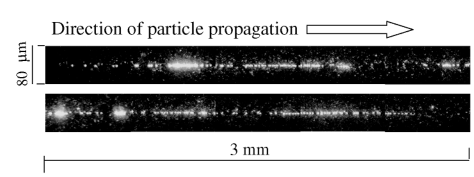

FIG. 3. Observation of a running mode in the sample cell. The view is taken orthogonally to the Bessel beam propagation direction. The particles are guided in the beam core with velocity 8 μm/s.

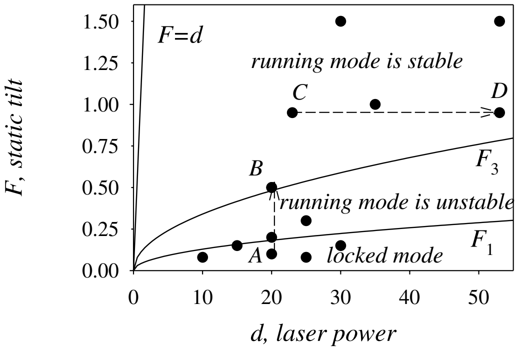

FIG. 4. Bifurcation diagram showing observation of tilt F versus laser power (d). The diagram shows the three main regimes that we observe, namely, the locked, running, and instability (bistability) regions.

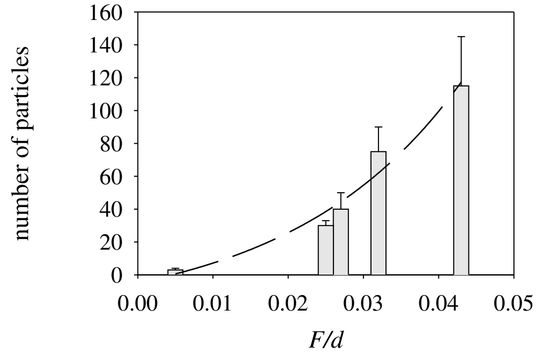

FIG. 5. Particle accumulation as a function of control parameters in the central maximum of the Bessel beam.

accumulation (running mode) was observed when the static tilt exceeded F_3. The reduction of the tilting force F at this stage did not stop the running mode abruptly but led instead to a rather bistable behavior. A bifurcation diagram on the plane of control parameters is presented in Fig. 4. The curves are plotted for arbitrary $F_{1,3} = k_{1,3}\sqrt{d}$ where $k_1 = 0.04$ and $k_3 = 0.1$, indicating the experimentally observed transitions between the modes. Points on this diagram correspond to the experimentally evaluated parameters of the Bessel beam. The region between F_1 and F_3 is designated as a region supporting both locked and running modes (bistable). The running mode was not as stable as for parameters above curve F_3 and could simply disappear. These observations support our belief that a critical value of control parameters (tilt and power) is required to investigate the onset of the running mode. The line $F = d$ is indicated on the diagram as the transition to the running mode for an overdamped system according to the simplified theory outlined above.

We have found also that the average number of particles in the beam center depends upon the ratio of control parameters (tilt and laser power). Moving further into the running mode region on Fig. 4, we observed an exponential growth of particle accumulation in the center of the Bessel beam, with a characteristic rate as shown in Fig. 5. According to [14], the average residence time of a particle in a metastable state is a function of potential well parameters. In a Bessel beam the total energy of the beam is distributed equally between the rings, while the circumference of each ring increases with distance from the center. As a result the depth of individual potential wells is unequal. This inequality grows as rings shrink in size ranging from 0.1 k_BT for the ring of radius 100 μm, to 10 k_BT at the beam center. The particle walking across the (untilted) potential barriers, at each new step falls into a deeper well than the previous one. It has to overcome an even higher potential barrier to continue its progress towards the beam center. As a result the mean squared displacement of this particle is a function of particle position and time and it exponentially decays as the particle approaches the beam center. Thus, any directed motion to the center of the beam will be rapidly bogged down by the soil of the potential landscape. The particle stops or turns over before reaching the center of the beam.

Application of the tilt leads to the transformation of beam profile from Bessel to sinusoidal function—Fx due to tilt (Fig. 2). The power redistributes between separate wells, making them approximately equivalent in depth in expense of total number of rings, at the same time introducing an asymmetry to the heights of the barriers on either side of each well, which favors motion towards the center of the beam. The continuous net motion results in the accumulation of particles at the beam core. The running mode is stable for the range of power indicated on Fig. 4 by points CD. For very high barrier heights (parameter d), the running stops.

Initially the particle is attracted to the Bessel beam due to the optical gradient force. Particles cross the shallow outer wells of the beam (the depth is smaller than thermal energy of particle, i.e., close to the critical threshold $F = d$). The particles accelerate in the direction of field gradient (or tilt F), achieving a large amount of kinetic energy, allowing them to pass over a number of potential barriers, before they stop in a well. Each successful transition between two minima of the potential landscape consists of a ballistic run from one potential well to another accompanied by fast relaxation to a local minimum. At the bottom of the well the particles are restricted by potential walls, and during this time their diffusion is essentially localized. In Fig. 1(a) the trajectories of a few particles walking across the Bessel landscape are shown—here the tilted Bessel beam is directed upwards crossing a thin (100 μm deep) sample cell containing 1 μm particles in water. As indicated by these trajectories, the direction of net current for all particles regardless of their initial positions is towards the maximum tilt. (This supports our heuristic use of one-dimensional model.) For any nonradial direction the potential profile remains periodic but the tilt is rapidly dropping down which prevents particles from changing the direction of the drift.

The explanation of the running mode can be retrieved from the assumption of the finite mass of a particle, which was neglected in the oversimplified stochastic model [4,8]. Equation (1) then includes, on the left side, the term md^2x/dt^2 and associated with the mass the colored noise source $\chi(t)$ with zero mean and correlation function $\langle\chi(t)\chi(t+\tau)\rangle = (P/\tau)\exp(-t/\tau)$, where τ is correlation time and P is constant (noise strength). The validity of neglecting mass was originally justified by the condition $m \ll \gamma$ ($\gamma/m = 10^{-6}$). However, the presence of colored noise changes the dynamical features of stochastic Eq. (1) nontrivially. For such inertial ratchets, for any correlation time $\tau > 0$ the diffusion net current is generically nonzero for nonsymmetric potentials even for $F = 0$ [8].

Instability between locked and running modes could potentially be attributed to the variation of the microscopically space-dependent friction coefficient [9]. It has been shown that the system (1) with inhomogeneous friction can exhibit the enhancement of diffusion near critical tilt $F = d$. The physical interpretation is as follows. The periodic variation of the friction parameter at antiphase with effective potential will assist in effective lowering barrier heights and slowing down the relaxation motion to the next potential minima [9]. Although we cannot exclude the possibility of varying fluid viscosity due to local heating, the use of a low water absorption laser wavelength reduces this effect to minimum and is thus unlikely to generate the above described dynamics. Moreover, the experimentally observed transition to running mode occurs at a considerably lower tilt than theory predicts for an overdamped system with inhomogeneous friction [9].

The influence of inertia forces can explain the bistability effect. While adiabatically changing the tilt of potential profile the running mode does not stop abruptly but rather continues for some time even when the tilt is below the critical value F_3 (points BA on Fig. 4).

We have presented the first experimental results of stochastic dynamics of a Brownian particle with finite mass in an overdamped optical trap. We have shown enhanced diffusion mobility in a static optical potential of the washboard type. When the tilt of the potential exceeds a critical value, particles move across the potential barriers in the direction of tilt and accumulate in the Bessel beam core. The interplay of thermal noise, the finite mass, and washboard potential created by Bessel beam explains the observed effects. The most significant impact of our results is in understanding the molecular, in particular, motor protein transport in the cells [1,3,15–17]. Primarily motor proteins that use biopolymer tracks as the transport substrate support activity of intracellular transport networks. These tracks are heavily symmetry broken [1,17] and have structural polarity making the two ends inequivalent. The motion of a particle along our optical potential accompanied by temporal trapping of the particle within the single potential well which can imitate transient binds of protein on microtubules network observed *in vivo* in which a protein spends a significant time during the walk. Although the reconstitution of the microtubule network *in vitro* [17] or tracking protein in living cells *in vivo* [15,16] gives a useful insight of protein-microtubule associated dynamics, some mechanical aspects of protein dynamics remain unclear. We believe that our study of particle dynamics in a washboard potential will shed light on similar processes in a cell environment as well as helping to develop a new generation of molecular driven devices. Recently, we have shown that a chromosome attached to a microsphere can be dragged across this potential and guided in the Bessel beam core while a free chromosome could not.

We thank the U.K. Engineering and Physical Sciences Research Council for their support of this work.

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