# Generation of a Self-Chirped Few-Cycle Optical Pulse in a FEL Oscillator

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We study the generation of a self-chirped optical pulse in a free-electron laser (FEL) oscillator. In a high-gain FEL oscillator, the frequency chirp is induced in the slippage region as a result of superradiant FEL resonance, and this time-frequency correlation evolves continuously into a few-cycle regime, if the optical cavity length is perfectly synchronized to the electron bunch interval. Numerical simulations based on the slowly evolving wave approximation and experimental results are presented.

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The frequency chirp in an optical pulse is one of the key technologies in laser source development, because it is indispensable for the generation of an ultrashort laser pulse in a few-cycle regime and high peak power [1]. Frequency-chirped laser pulses are also useful for the resonant excitation of atomic or molecular systems, which have an anharmonic potential ladder [2].

A free-electron laser (FEL) oscillator operated with bunched electrons from a radio frequency accelerator is capable of producing optical pulses with picoseconds or subpicoseconds duration. A chirped-pulse generation in a FEL oscillator by using an electron bunch with energy chirp was proposed previously [3]. This energy chirp can be realized by accelerating an electron bunch at an off-crest phase of a sinusoidal rf field and results in a frequency chirp of the laser pulse according to the relation between the electron energy $mc^2\gamma$ and the laser wavelength $\lambda$: $\lambda = \lambda_u(1 + a_w^2)/2\gamma^2$, where $\lambda_u$ is the undulator pitch, and $a_w$ the undulator parameter. The energy and time correlation of the electron bunch, however, cannot be copied exactly to the laser frequency chirp. This is because (1) an electron bunch has a longitudinal velocity slightly less than the vacuum speed of light due to the undulation and slips backward in the rest frame of the laser pulse during the FEL interaction, and (2) in order to compensate for the FEL lethargy effect [4], optical cavity length shortening from a perfectly synchronized condition is necessary, in which the laser pulse is pushed forward with respect to the electron bunch every round-trip. Each part of the laser pulse, consequently, scans many electrons during the evolution, and the frequency chirp becomes diluted. Chirped-pulse generation in this manner and the following pulse compression were demonstrated at 8.5 $\mu$m [5] and 3.2 $\mu$m [6].

A chirped-pulse FEL is possible by another method. If we have an energy sweep in the train of electron bunches, the electron energy at the beginning of the FEL interaction has a different value on each successive round-trip. This energy difference combined with the cavity length shortening, which pushes the laser pulse forward, induces a frequency chirp in the FEL pulse. A frequency chirp of 0.2% on a 1.5 ps pulse was demonstrated by this method [7].

So far, frequency-chirped FELs have been investigated based on an energy chirp in an electron bunch or a bunch train, in which the amount of frequency chirp is restricted to a few percent at most. In this Letter, we propose a novel approach to generate a frequency-chirped FEL pulse, which is self-chirping of a FEL pulse without energy chirp of an electron bunch. We see that a self-chirped pulse is generated in a high-gain FEL oscillator, where the amount of frequency chirp inside the pulse duration (FWHM) becomes nearly 10% and the FEL pulse is as short as two optical cycles after appropriate pulse compression. We discuss the generation of a self-chirped FEL pulse in the context of a superradiant process appearing in a high-gain FEL and show numerical and experimental results.

Optical pulse evolution in a high-gain FEL depends on three characteristic parameters: electron bunch length $L_b$, slippage length $L_s \equiv \lambda N_w$, and cooperation length $L_c \equiv \lambda/4\pi\rho$, where $N_w$ is the number of undulator periods, and $\rho$ is the FEL parameter for the high-gain regime [8]. In single pass FELs for UV and x-ray starting from a noise signal (self-amplified spontaneous emission), these parameters have a relation: $L_c < L_s \ll L_b$. In this long-bunch and long-undulator limit, a FEL pulse after saturation consists of many spikes separating at a distance $\sim 2\pi L_c$ and longitudinal phase correlation is only established within each spike [9].

Let us consider a FEL amplifier with a different regime, short-bunch limit: $L_b < L_s$ and $L_b < 2\pi L_c$. If we put an optical seed pulse having resonant wavelength and temporal duration comparable to the electron bunch, a FEL interaction is initiated and amplification occurs. The electrons soon slip out from the seed pulse before reaching saturation, but still continue to emit radiation for the whole slippage distance. At the undulator exit, the FEL pulse is a single spike of whole duration $\sim L_s$, which has a peak at the tail edge and an exponential-like leading slope. This type of lasing is called weak superradiance and a linear solution and nonlinear analyses were shown previously [8].

  

Similar superradiant pulses are produced in a short-bunch FEL oscillator with perfectly synchronized optical cavity length ($\delta L = 0$), in which the cavity round-trip time of an optical pulse is exactly the same as the electron bunch interval. In such a FEL oscillator, FEL interaction starting from shot noise forms a superradiant pulse which has an exponential-like leading edge and a peak at the tail part. An analytical study for the linear regime [10] shows the evolution of the pulse is a function of the round-trip number, $n$, where the peak intensity grows as $n^2$ and the width decreases as $n^{-1/2}$. There exists a linear solution of the same form as the weak superradiance in high-gain FEL amplifiers [8,10]:

$$A(z_2, n) \simeq (A_0/2\sqrt{3\pi})(2/y)^{1/3} \exp[(3/2)(\sqrt{3} + i)(y/2)^{2/3} - i\pi/12 - \alpha_0 n/2], \quad (1)$$

where $A_0$ is a uniform initial excitation, $z_2 \equiv (ct - z)/L_c$ is the local variable for the rest frame of the vacuum speed of light, $\alpha_0$ is the cavity round-trip loss, $y \equiv \sqrt{\sigma n} z_2$ is a self-similar variable, and $\sigma$ is the bunch length in units of $L_c$.

If we fix the round-trip number and observe the field along $z_2$, we see that the superradiant pulse has a nonlinear frequency chirp: $\partial\phi/\partial z_2 = (4z_2/\sigma n)^{-1/3}$. The frequency chirp is, thus, a general property of the superradiant FEL pulses and a key to the generation of few-cycle pulses in a FEL oscillator, although frequency chirp in superradiant FELs has never been investigated explicitly. We show numerical and experimental results of the self-chirped FEL pulse generation in a high-gain FEL oscillator of the superradiant regime.

Since the duration of a superradiant FEL pulse becomes shorter as the FEL goes into deep saturation, we need to analyze FEL dynamics in the nonlinear regime. For this purpose, we derive a wave equation applicable to a few-cycle FEL pulse. The Maxwell equations to describe the evolution of the electric field in a FEL oscillator of Compton regime can be reduced into a single equation:

$$\left(\nabla^2 - \frac{1}{c^2}\partial_t^2\right)E(\vec{r}, t) = \mu_0 \partial_t J(\vec{r}, t), \quad (2)$$

where $E(\vec{r}, t)$ is a time-varying electric field and $J(\vec{r}, t)$ is a time-varying transverse current source driven by undulating electrons: $E(\vec{r}, t) = A(\vec{r}_\perp, z, t)\exp[i(\omega t - kz)] +$ c.c., $J(\vec{r}, t) = S(\vec{r}_\perp, z, t)\exp[i(\omega t - kz)] +$ c.c., where $A(\vec{r}_\perp, z, t)$ and $S(\vec{r}_\perp, z, t)$ are complex envelopes of the electric filed and the current source, respectively. We choose the resonance frequency of undulator radiation as the carrier. This carrier and envelope expression is known to be assigned, with unambiguous definition, to ultrashort light wave packets that contain at least one carrier cycle within the FWHM of their intensity envelope [11]. Following Brabec's study [11], we define a local variable $\tau$ and a propagation variable $\xi$ in the rest frame with speed $v_\parallel$: $\tau = t - z/v_\parallel$, $\xi = z$, and rewrite Eq. (2) using the carrier and envelope expression into

$$\left(\partial_\xi + \frac{i}{2k}\nabla_\perp^2\right)A + \frac{\mu_0 c}{2}\left(1 - \frac{i}{\omega}\partial_\tau\right)S = \left(1 - \frac{v_\parallel}{c}\right)\left[1 - \frac{i}{2k}\left(1 + \frac{v_\parallel}{c}\right)\frac{\partial_\tau}{v_\parallel}\right]\frac{\partial_\tau}{v_\parallel}A - \frac{i}{2k}\left(\partial_\xi - \frac{2\partial_\tau}{v_\parallel}\right)\partial_\xi A. \quad (3)$$

The first term on the right side vanishes for $v_\parallel = c$, and the second term becomes small as compared with the terms on the left side, if we choose the rest frame with a group velocity of the optical packet $v_\parallel = v_g$ and the complex amplitude $A$ does not excessively change during propagation: $|\partial_\xi A| \ll k|A|$. These requirements for neglecting the terms on the right side are, therefore, combined into two conditions: $|(v_g - c)/c| \ll 1$ and $|\partial_\xi A| \ll k|A|$ in the rest frame with speed $v_\parallel = c$. This is a similar result to the slowly evolving wave approximation (SEWA) derived for the propagation of optical packets in a polarized medium [11]. The former condition is satisfied apparently for FEL oscillators with $\delta L = 0$ and is still valid for small cavity length shortening. The latter is also guaranteed, because the exponential power gain length of FEL oscillators contains many undulator periods even for high-gain parameters. Consequently, the higher-order terms on the right side of Eq. (3) can be neglected in FEL oscillators, and we find the envelope equation first order in the propagation coordinate $\xi$:

$$\left(\partial_\xi + \frac{i}{2k}\nabla_\perp^2\right)A + \frac{\mu_0 c}{2}\left(1 - \frac{i}{\omega}\partial_\tau\right)S = 0. \quad (4)$$

The derivative of the source term $\partial_\tau S$ is equivalent to the nonslowly varying envelope approximation correction discussed in a previous FEL analysis [12] and cannot be dropped in the calculations of the few-cycle FEL pulses propagation.

We have seen that the envelope equation based on the SEWA can be adopted to describe the pulse propagation in a FEL oscillator of the superradiant regime, in which FEL pulses may become as short as a few cycles. A numerical analysis using the first-order propagation equation is conducted to see the possible generation of few-cycle pulses in a high-gain FEL oscillator. We integrate the propagation equation by the finite differential method. The source term is given by tracking macroparticles, coarse grating electrons, in the phase-momentum space of the electrons' motion including the energy exchange with the optical field [13,14]. The shot noise of the electron bunch is also introduced by a small artificial disturbance on the initial phase of each macroparticle [15]. It has been shown that the leading edge of superradiant pulses in $\delta L = 0$ lasing is kept as an incoherent field generated from longitudinal mixing of shot noise due to slippage [16]. The calculation presented in this Letter is limited to one-dimensional propagation dropping the diffraction term. This one-dimensional treatment is reasonable to study frequency chirp in a FEL oscillator. This is because the FEL pulse evolution balances, after the onset of saturation, with optical cavity round-trip loss, which is typically less than 10%. The single pass gain becomes small after the saturation, and the optical guiding effect does not have a dominant role. We note that the guiding effect should be taken into account for high-gain single pass FEL amplifiers [17].

We choose simulation parameters similar to our previous experiments at JAERI-FEL [18,19]: FEL wavelength $\lambda = 23\ \mu$m, $\rho = 0.0044$, $L_s = 52\lambda$, electron bunch length (rms) $L_b = 36\lambda$, cavity loss $\alpha_0 = 0.06$, and calculate FEL pulse evolution by a one-dimensional FEL code based on the SEWA wave equation. In the following calculation, a seed of random numbers to simulate the shot noise during the FEL evolution is fixed every round-trip, because the gradual variation of the saturated FEL pulse arising from random shot noise [14] is out of our concern here. Figure 1 shows simulation results for the FEL pulse evolution: peak intensity normalized by the high-gain FEL parameter [9], and FWHM pulse duration $\tau_p$ expressed by carrier period $T$. The frequency chirp is evaluated as a frequency shift in the FWHM pulse duration divided by carrier frequency. The pulse duration becomes shorter as the pulse evolves and finally reaches 3.9 cycles at the 800th round-trip. This FEL pulse evolution also brings frequency chirp. The simulation shows that the frequency chirp increases as the FEL approaches saturation. The amount of frequency chirp is 8.6% at the 800th round-trip.

Figure 2 shows the intensity profile of a FEL pulse after saturation (the 800th round-trip). We also plotted instantaneous frequency as detuning from the resonance frequency of the undulator radiation. The intensity profile of the FEL pulse agrees with the previous analyses based on the self-similar variables [10]. The FEL pulse consists of a long lobe of leading edge and a large peak followed by small ringings. The instantaneous frequency gradually shifts down in the leading lobe of the linear regime and shows steep down-chirp within the main peak. In the trailing part of the FEL pulse, we see further down-chirp with discontinuities by the ringings.

Each electron begins the FEL interaction at the leading lobe and slips backward in the FEL pulse rest frame during the interaction. As the electrons lose their energy through the FEL interaction, the resonance frequency also goes down, but the electrons still continue to emit radiation at the down-shifted frequency. This is the origin of the frequency down-chirp and large gain bandwidth appearing in the superradiant FEL. The formation of such a time-frequency correlation in the FEL pulse requires precise synchronization of the FEL pulse and the electron bunches every round-trip. This is the reason why a perfectly synchronized FEL oscillator exhibits unique lasing behavior, quasistationary superradiance [14].

We see that the FEL pulse can be compressed by applying additional chirp compensation, that is frequency dependent phase shift. Figure 3 is the compressed pulse after numerical chirp compensation: $\phi(\omega) = 2.2(\omega - \omega_c)^2T^2 - 2.1(\omega - \omega_c)^3T^3$, where $\phi(\omega)$ is the phase shift and $\omega_c$ is the pulse central frequency. The compressed pulse has FWHM duration $\tau_p = 2.1T$.

Fitting a chirped FEL pulse by an elementary function is necessary to analyze experimental data shown later. We make a fitting with a chirped-sech$^2$ pulse defined as $E(t) = E_0 \mathrm{sech}^{1+i\alpha}(t/\tau_d)$, where $\alpha$ is the chirp parameter and the FWHM of the intensity envelope is given by $\tau_p = 1.7627\tau_d$. Instantaneous frequency is $\omega(t) = -(\alpha/\tau_d)\tanh(t/\tau_d)$. The Fourier transform limited pulse after chirp compensation has a duration of $\tau_{p0} = \tau_p \mathrm{arccosh}(3)/\mathrm{arccosh}[\cosh(\pi\alpha) + 2]$. In Fig. 2, we plot a fitting result. Here parameters for the chirped-sech$^2$ pulse are $\alpha = 1.1$, $\tau_d/T = 2.7$, frequency chirp inside the FWHM width $\Delta\omega/\omega_0 = 9.2\%$, and the FWHM pulse duration $\tau_p = 4.8T$ can be compressed, ideally, into

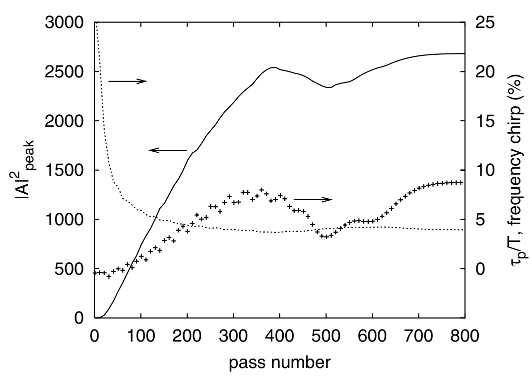


FIG. 1. Numerically calculated FEL pulse evolution for the JAERI-FEL parameters. Peak intensity (solid line), FWHM duration as optical cycles (dotted line), and a frequency chirp inside FWHM duration (+).

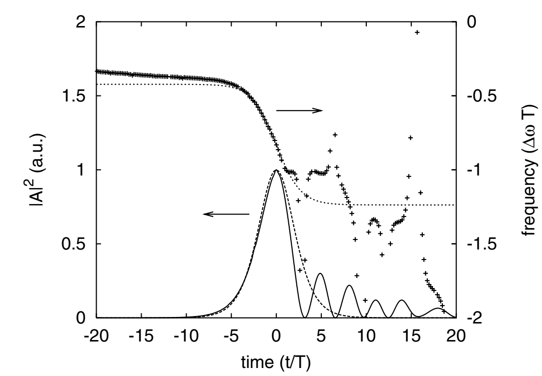


FIG. 2. A FEL pulse calculated for the 800th round trip. Intensity profile (solid line) and instantaneous frequency detuning normalized by the carrier period (+) are plotted for a temporal variable as optical cycles of carrier frequency. Dotted lines are results from chirped-sech$^2$ fitting, $\alpha = 1.1$ and $\tau_d/T = 2.7$.

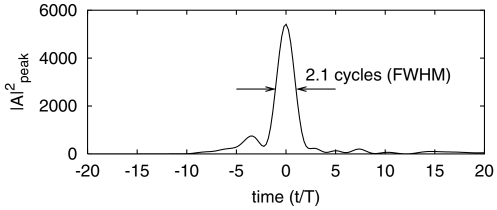


FIG. 3. Intensity profile of a FEL pulse after numerical chirp compensation applied to the pulse in Fig. 2.

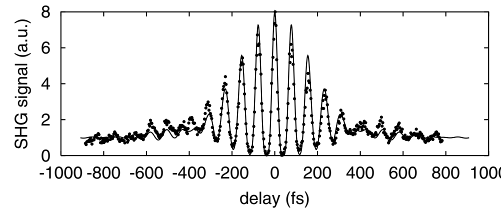


FIG. 4. A fringe-resolved SHG signal from the JAERI-FEL (dots). The solid line is a fitted curve by the chirped-sech$^2$ pulse, $\alpha = 1.47$ and $\tau_d/T = 2.32$.

$\tau_{p0} = 2.4T$ as the Fourier transform limit. The fitting with chirped-sech$^2$ has been found to be an appropriate assumption to estimate the duration of the compressed pulse.

In the JAERI free-electron laser facility, temporal structure of FEL pulses was measured by autocorrelation of second-harmonic-generation (SHG) signals from a Te crystal. The FEL pulse duration was evaluated as 3.4 optical cycles (FWHM) without taking frequency chirp into account [19]. Now we reconsider the experimental result including frequency chirp in a FEL pulse. An experimentally obtained fringe-resolved SHG autocorrelation signal is used for pulse form retrieval [20] with the assumption of a chirped-sech$^2$ pulse. A fitting result is presented in Fig. 4. The parameters for the fitted chirped-sech$^2$ pulse are found to be $\lambda = 23.3$ $\mu$m, $\alpha = 1.47$, and $\tau_d = 180$ fs $(2.32\lambda)$, which gives the FWHM of the intensity envelope $\tau_p = 319$ fs $(4.09\lambda)$ and the relative chirp inside the FWHM $\Delta\omega/\omega_0 = 14.3\%$. These obtained parameters are almost consistent with the numerical simulations presented above. The autocorrelation measurement does not distinguish between up-chirp and down-chirp, but we determine the measured pulse as down-chirp from the simulations. If we compensate frequency chirp in the measured FEL pulse, we can compress it, ideally, to a Fourier transform limited pulse of 121 fs $(1.55\lambda)$ under the assumption of a chirped-sech profile.

In conclusion, we have presented a self-chirped few-cycle pulse generation in a high-gain FEL oscillator of the short-bunch regime. Frequency chirp, which is an intrinsic property of superradiant FELs, is established in the slippage region and evolves through a number of round-trips. The perfect synchronism of the optical cavity length is necessary to keep the time-frequency correlation in the FEL pulse. A numerical study based on the SEWA and an experimental result from JAERI-FEL agree with each other and show that even a pulse as short as 2 cycles can be generated in a FEL oscillator followed by chirp compensation.

We note that the generation of self-chirped few-cycle FEL pulses is due to the large gain bandwidth of a superradiant FEL, which exceeds the bandwidth in a long-bunch limit $(\sim 1/2N_w)$ [14,21]. The formation of a single superradiant spike discussed in this Letter requires the short-bunch condition: $L_b < L_s$ and $L_b < 2\pi L_c$, otherwise more than two spikes evolve independently. Although our experiment is in an infrared FEL oscillator, a self-chirped few-cycle FEL pulse can be generated in shorter or longer wavelength regions in the same manner as long as a high-gain short-bunch FEL oscillator is available.

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